Search and Knightian Uncertainty Revisited: Roles of Optimism and Pessimism*

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Abstract

This paper considers a one-sided search problem in the presence of ambiguity (i.e., Knightian uncertainty). Our novelty is to depart from the assumption of uncertainty-aversion by adopting the Choquet expected utility with a JP capacity. This setting allows us to separate the degree of ambiguity from ambiguity-attitudes. By exploiting this feature, we examine whether higher ambiguity induces an agent to search shorter or longer. We first show that higher ambiguity induces the agent to search shorter if and only if the agent's degree of optimism is sufficiently low. Second, the effect of higher ambiguity also depends on the discount factor. Focusing on a subclass of JP capacities, called neo-additive capacities, we find that even slight contamination of optimism leads to ambiguity-loving behaviors when the agent is sufficiently patient.

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1 Introduction

Should I go or should I stay? Timing is a key decision in dynamic economic environments characterized by uncertainty. Consider an unemployed worker who is searching for a job. She receives a wage offer and decides whether to stop the search or not. The difficulty lies in uncertainty about the future. Without knowing future payoffs, she must judge whether this is the good timing to stop the search. This type of decision-making problem, which is called the optimal stopping problem, is everywhere in economics, not limited to job search. We aim at contributing to the literature of optimal stopping problems under ambiguity by relaxing the assumption of uncertainty-aversion in the framework of a one-sided job search problem.

Suppose that "uncertainty" about labor market conditions has increased. Does this change induce an unemployed worker to search longer, or shorter? The literature has shown that the answer depends on the type of uncertainty. On the one hand, when the worker precisely understands the wage distribution (i.e., the worker faces *risk*), higher uncertainty in the sense of a mean-preserving spread encourages the worker to search longer.

On the other hand, when the worker does not know even the distribution itself, which is called *ambiguity* or *Knightian uncertainty*, higher uncertainty rather induces the worker to search shorter. That is, the effect of higher ambiguity is in contrast to that of a mean-preserving spread. This effect was first found by Nishimura and Ozaki (2004). Since then, subsequent studies have extended this analysis to various situations, such as a general framework of optimal stopping problems (e.g., Riedel 2009; Miao and Wang 2011; Chudjakow and Riedel 2013), experimentation (e.g.. Li 2019), a game-theoretic situation (e.g., Kishishita 2020), and option problems in finance (e.g., Nishimura and Ozaki 2007; Cheng and Riedel 2013).

Despite such attempts, the role of ambiguity has not been fully understood yet. In modeling the worker's decision-making under ambiguity, the existing studies adopt the Maxmin expected utility (MEU) theory, where she has a set of probabilities and evaluates the expected utility based on the worst probability (Gilboa and Schmeidler 1989). This modeling has two shortcomings. First, both the worker's attitudes toward ambiguity and the degree of ambiguity itself are represented by the size of the set of priors. Consequently, we cannot conduct comparative statics concerning the degree of ambiguity, keeping the ambiguity-attitude fixed. Second, this model assumes that the worker dislikes ambiguity, which is called uncertainty-aversion. Although this property is motivated by the famous experiment by Ellsberg (1961), uncertainty-aversion is neither necessary nor sufficient to explain his paradox (Epstein 1999). Furthermore, massive experimental studies find that people's behavior can be both ambiguity-averse and ambiguity-loving depending on situations (e.g., Kocher, Lahno, and Trautmann 2018). Due to these limitations, the existing studies cannot answer the following fundamental question: How does the effect of higher ambiguity depend on the ambiguity-attitude? The contribution of our study is to give the complete answer to this unresolved question.

For this purpose, we adopt the Choquet expected utility (CEU) theory (Schmeidler 1989), where the preference is represented by the Choquet integral of utility numbers with respect to a probability capacity. When the capacity is convex, the worker is uncertainty-averse, and the preference is equivalent to the MEU, whereas she is uncertainty-loving when the capacity is concave. Hence, CEU describes a broad class of ambiguity-attitudes. In particular, we adopt its specific class, which is called CEU with a JP-capacity (Jaffrey and Philippe 1997).[1] This is a class of capacities, which are represented by a convex combination of a convex capacity and its conjugate (a concave capacity). Thus, the worker endowed with a JP capacity is characterized by the mixture of uncertainty-aversion and uncertainty-loving. The weight for the concave capacity, say α, can be interpreted as the degree of optimism. As such, this capacity allows us to separate ambiguity-attitudes and the degree of ambiguity.[2]

By focusing on this class of CEU, we show that the previous finding hinges on the assumption that the worker is uncertainty-averse. In particular, we find that higher ambiguity makes the search shorter if the agent is sufficiently pessimistic (i.e., α is close to zero), while the opposite holds if the agent is sufficiently optimistic (i.e., α is close to one). Nishimura and Ozaki (2004) focus on convex capacities (i.e., the case where $\alpha = 0$). By extending their analysis to a broader class of capacities, we find that the previous finding holds if and only if the worker is sufficiently pessimistic.

A next natural question would be how economic environments affect the effect of higher ambiguity. The existing studies have shown that the effect is independent of them when $\alpha = 0$. Hence, it might be expected that the economic environments such as the discount factor do not matter. This is not the case. To see this, we restrict our attention to a subclass of JP-capacities, which is called neo-additive capacities, whose axiomatic foundation is given by Chateauneuf, Eichberger, and Grant (2007).[3]

We show that the effect of higher ambiguity crucially depends on the discount factor as long as $\alpha \in (0, 1)$. In particular, for any positive α, there exists a high discount factor under which more ambiguity makes the search longer. Since longer search is ambiguity-loving behavior, our result indicates that slight contamination of optimism induces ambiguity-loving behavior when the agent is sufficiently patient. This result uncovers the overlooked role of the discount factor in determining the effect of higher ambiguity in optimal stopping problems.

In summary, whether ambiguity about labor market conditions induces shorter search or not depends on the worker's ambiguity-attitude. It encourages the worker to stop the

[1] Eichberger and Kelsey (2014) adopt this class of capacities in the analysis of games under ambiguity.

[2] CEU with this capacity can be also interpreted as a variant of MEU. In parallel to the relationship between CEU with a convex capacity and MEU, CEU with a JP capacity can be rewritten as α-Maxmin expected utility, where preferences are represented by a weighted average of the minimum and the maximum expected utility over the set of probabilities.

[3] This capacity has been widely used in applied researches (e.g., Zimper 2012; Giraud and Thomas 2017; Kishishita and Ozaki 2020).

search if and only if her degree of optimism is sufficiently low. Furthermore, when the worker is endowed with a neo-additive capacity, even slight contamination of optimism leads to ambiguity-loving behaviors when the worker is patient.

In analyzing a dynamic decision-making problem beyond the assumption of uncertainty-aversion, we face two methodological difficulties. First, ambiguity creates a dynamic inconsistency problem. In the case of MEU and CEU with a convex capacity, this problem is easily resolved by assuming the independent and indistinguishable distribution because it guarantees the rectangular property (Epstein and Schneider 2003a; 2003b). However, once we depart from uncertainty-aversion, this approach is no longer applicable (Schröder 2011). We resolve this problem by employing recursive preferences, which enables us to use dynamic programming techniques. This approach is also taken by Beissner, Lin, and Riedel (2020) in the analysis of a continuous-time consumption and asset pricing model.[4] To the best of our knowledge, the present study is one of the first studies to analyze an optimal stopping problem beyond the assumption of uncertainty-aversion. Recently, Huang and Yu (2021) analyze a different class of optimal stopping problems in continuous time by adopting $\alpha-$MEU preferences. Their approach is to allow dynamic inconsistency and model decision-making as a game between the present self and the future self. They do not conduct comparative statics with respect to the degree of ambiguity.[5]

The second difficulty lies in the dynamic programming theory. Whether standard dynamic programming techniques are applicable to the situation under ambiguity is not straightforward. Nishimura and Ozaki (2004) show that the dynamic programming techniques are applicable in a quite general setting where the state space in each period could be *un*bounded by relying the convexity of a capacity.[6] We show that their techniques can be extended to non-convex capacities. We provide a rigorous theoretical foundation for how we can apply dynamic programming techniques when the agent is not necessarily uncertainty-averse.

The remainder of the paper is organized as follows. Section 2 presents a motivating example. Section 3 provides a formal model and Section 4 characterizes the optimal decision. Sections 5 and 6 conduct comparative statics. Section 7 concludes.

[4]See also Saghafian (2018) as another study of a dynamic decision-making problem in the class of $\alpha-$MEU preferences.

[5]A continuous-time irreversible investment problem (a different class of optimal stopping problems) is introduced but not solved by Schröder (2011) in the class of $\alpha-$MEU preferences. Due to the dynamic inconsistency problem, he solves the problem only for extreme cases where $\alpha = 0, 1$. For intermediate values of α, he assumes that the investor faces an all-or-nothing decision: if the investor does not invest in period t, he cannot investment forever. That is, the investor's decision is only once (i.e., it is not dynamic), which is in contrast to optimal stopping problems.

[6]Allowing the unboundedness is important especially for other applications of search problems such as in finance. For a bounded or finite state space, some studies develop the dynamic programming techniques by applying the contraction mapping theorem (e.g., Miao and Wang 2011, Saghafian 2018).

2 A Motivating Example

As a motivating example, we consider a simple one-sided job search model with $t = 0, 1, \ldots$. In each period, an unemployed worker draws a wage offer, w, from a wage distribution F.[7] If the unemployed worker accepts the offer, she earns that wage from this period. If she decides to reject the offer, she gets unemployment compensation, $c > 0$, in this period and will make a draw again in the next period. The discount factor is $\beta \in (0, 1)$.

The optimal stopping rule is to accept the wage offer if it is no smaller than the reservation wage R^* and to wait for another offer if otherwise. Furthermore, the reservation wage R^* is determined by a choice between accepting this period's offer or waiting for the next period's offer. In particular, when there is no ambiguity,

$$\frac{R^*}{1-\beta} = c + \frac{\beta}{1-\beta}\left(\int_0^{R^*} R^*\, dF + \int_{R^*}^{\infty} x\, dF\right),$$

where the left-hand side is the utility when accepting the current wage offer, and the right-hand side is the utility when continuing the search. At the reservation wage, these two must be equalized. By rewriting this equality, we get

$$R^* = c + \frac{\beta}{1-\beta}\int_{R^*}^{\infty} [1 - F(x)]\, dx, \tag{1}$$

which characterizes the reservation wage.

We next introduce ambiguity into this setup. That is, we allow the worker to face uncertainty about the true distribution F. To describe decision-making under ambiguity, we adopt the Choquet expected utility (CEU), where the worker evaluates the expected utility by using not a probability measure but a probability capacity, $\hat{\theta}$. Under this setting, the analogy of (1) holds; that is,

$$R^* = c + \frac{\beta}{1-\beta}\int_{R^*}^{\infty} \hat{\theta}(\{w \geq x\})\, dx \tag{2}$$

(see Corollary 1).

To examine the effect of ambiguity further, we specify the capacity θ as follows: for any $E \notin W \cup \emptyset$,

$$\hat{\theta}(E) := (1-\varepsilon)\pi(E) + \varepsilon\alpha\,,$$

where π is a probability measure, $\varepsilon \in (0, 1)$, and $\alpha \in [0, 1]$. This is called a *neo-additive capacity*, whose axiomatic foundation is given by Chateauneuf, Eichberger, and Grant (2007). The CEU with a neo-additive capacity has a remarkable feature as seen in the following equivalence:

$$\int f(w) d\hat{\theta} = \alpha \max_{p\in\mathscr{D}} \int f(w)\, dp + (1-\alpha) \min_{p\in\mathscr{D}} \int f(w)\, dp\,,$$

[7] $F(x)$ denotes the probability that the wage offer is no greater than x.

where

$$\mathscr{D} = \{p \in \Delta(W) | \, (\forall E) \; p(E) \geq (1-\varepsilon)\pi(E)\}.$$

On the one hand, an increase of ε leads to the dilation of $\mathscr{D}$ (the set of probability measures), and hence, its increase can be interpreted as an increase of the degree of ambiguity. On the other hand, α may be understood as the degree of optimism because it represents the weight for the best-case payoff. As such, the ambiguity itself and the players' attitudes toward it are successfully separated.

When we restrict our attention to this class of capacities, (2) can be further simplified to

$$R^* = c + \frac{\beta}{1-\beta}\int_{R^*}^{\infty}(1-\varepsilon)\pi\left(\{w\,|w \geq x\}\right)dx + \frac{\beta}{1-\beta}\int_{R^*}^{\infty}\alpha\varepsilon\,dx\,, \tag{3}$$

where the last term is $+\infty$ and is not well-defined yet.

To let equality (3) make sense (as well as by the motivation of getting a simple example), we assume that according to the principal probability measure π, the wage offer w should follow the *uniform distribution* with its support given by $[a, b]$, where $0 \leq a \leq c < b < +\infty$ so that $\pi\left(\{w\,|w \geq x\}\right) = (b-x)/(b-a)$. Finally, we then obtain

$$R^* \;\; = \;\; c + \frac{\beta}{1-\beta}\left[\frac{(1-\varepsilon)(b-R^*)^2}{2(b-a)} + \alpha\varepsilon(b-R^*)\right]. \tag{4}$$

Note that $R^* < b$ (see Lemma A.6).

2.1 The Exact Form of the Reservation Wage R^*

We have derived the equation, *i.e.,* Equation (4), the reservation wage R^* must satisfy when the unemployed worker believes that the ambiguity she faces is characterized by the neo-additive capacity with the principal probability measure being the uniform distribution over $[a, b]$.

By noting that Equation (4) is a quadratic equation,[8] we can solve it to find the exact form of R^*:

$$R^* = \frac{1}{1-\varepsilon}\left[b(1-\varepsilon) + (b-a)\alpha\varepsilon + (b-a)\frac{1-\sqrt{D}}{\hat{\beta}}\right], \tag{5}$$

where $\hat{\beta} := \beta/(1-\beta) \in (0, +\infty)$ and

$$D := 1 + \frac{2\hat{\beta}}{b-a}(1-\varepsilon)(b-c) + 2\alpha\hat{\beta}\varepsilon + \alpha^2\hat{\beta}^2\varepsilon^2\,.$$

Note that it can be verified that $R^* \in (c, b)$ (see Appendix A.1).

Before proceeding further, we make two important observations about the reservation

[8] The other solution of the quadratic equation, name it R, can be obtained if we set the sign right before the square root as plus. However, such an R would be greater than b and thus it is not plausible as a reservation wage of the current model.

wage R^* here.

The first one is regarding the effect of α. From (5), we can show by a direct computation that

$$\begin{aligned}\frac{dR^*(\alpha)}{d\alpha} &= \frac{\varepsilon(b-a)}{1-\varepsilon}\left[1-\sqrt{\frac{1+2\alpha\hat{\beta}\varepsilon+\alpha^2\hat{\beta}^2\varepsilon^2}{1+2\hat{\beta}(1-\varepsilon)\frac{b-c}{b-a}+2\alpha\hat{\beta}\varepsilon+\alpha^2\hat{\beta}^2\varepsilon^2}}\,\right] \\ &> 0\,,\end{aligned}$$

where the positivity holds true because $2\hat{\beta}(1-\varepsilon)(b-c)/(b-a) > 0$. Recall that the parameter α measures the degree of the worker's optimism, and hence, its increase urges her to anticipate a "better" wage offer in the future, which should be reflected by the *higher* reservation wage, R^*.

The second observation concerns the behavior of R^* with respect to the discount factor, β. Let $\alpha \in (0,1)$ be arbitrarily fixed. We observe that by varying the value of β within $(0,1)$, equivalently, by varying the value of $\hat{\beta}$ within $(0,+\infty)$, R^* can become as close as possible to b (from below). Now, let $\beta \uparrow 1$, which corresponds to $\hat{\beta} \uparrow +\infty$. If we look at a relevant part of (5), $(1-\sqrt{D})/\hat{\beta}$, its "limit" as $\hat{\beta} \uparrow +\infty$ will be $-\infty/+\infty$, which is an indeterminate expression, and thus, we can invoke l'Hôpital's rule to find out its limit. Therefore,

$$\begin{aligned}\lim_{\hat{\beta}\uparrow+\infty}\frac{1-\sqrt{D}}{\hat{\beta}} &= \lim_{\hat{\beta}\uparrow+\infty} -\frac{\partial}{\partial\hat{\beta}}\sqrt{D} \\ &= -\alpha\varepsilon\,, \end{aligned} \tag{6}$$

the last term of which can be plugged back to (5) to obtain

$$R^* = \frac{1}{1-\varepsilon}\left[\,b(1-\varepsilon)+(b-a)\alpha\varepsilon-(b-a)\alpha\varepsilon\,\right] = b\,.$$

We thus conclude that R^* can be made as close as possible to b (from below) by means of letting β as large and close as possible to 1. In addition, we can also easily verify that R^* goes to c as β goes to zero.

2.2 Comparative Statics with Respect to an Increase in Ambiguity

We conclude this section by conducting comparative statics with respect to an increase in ambiguity.

To this end, suppose that ambiguity increases. In the current setup, it corresponds to an increase in ε.

We regard (4) as an identity of ε and take a differentiation of its both sides. After some rearrangement of terms, we obtain

$$\frac{dR^*(\varepsilon)}{d\varepsilon} = -\frac{\beta(b-R^*)\,(b-R^*-2\alpha(b-a))}{2(b-a)(1-\beta+\alpha\beta\varepsilon)+2\beta(1-\varepsilon)(b-R^*)}\,. \tag{7}$$

Given that $R^* < b$ from an observation made in the previous subsection, it is immediate that

$$\frac{dR^*(\varepsilon)}{d\varepsilon} < 0 \;\Leftrightarrow\; b - R^*(\alpha,\beta) > 2\alpha(b-a)\,, \tag{8}$$

where we emphasized the fact that R^* depends on both α and β in ways clarified by a series of observations in the previous subsection.

The equivalence relation (8) tells us two interesting results on the comparative statics. By the first observation in the previous subsection, we know that R^* is increasing in α. Thus, there exists a threshold of α, say $\bar{\alpha}$, such that once that α exceeds $\bar{\alpha}$, (8) never happens again. Therefore, we obtain the following result.

Result 1. On the one hand, if the worker is optimistic in the sense that her α exceeds $\bar{\alpha}$, an increase in ambiguity increases the reservation wage. On the other hand, if the worker is pessimistic in the sense that her α is beneath $\bar{\alpha}$, an increase in ambiguity reduces the reservation wage.

Nishimura and Ozaki (2004) show that higher ambiguity always decreases the reservation wage when $\alpha = 0$. By extending the analysis to the cases where $\alpha \neq 0$, we find that it increases the reservation wage when α is close to one.[9]

Next, consider the role of the discount factor. Assume that $\alpha \in (0,1)$. Then, by the second observation in the previous subsection, R^* can be made as close as possible to b by means of letting β as close as possible to 1. Note also that R^* is increasing in β (see Appendix A.1). Hence, we obtain the following result.

Result 2. For any $\alpha \in (0,1)$, there exists a threshold value $\bar{\beta} \in [0,1)$ such that an increase in ambiguity increases the reservation wage if and only if $\beta > \bar{\beta}$.[10]

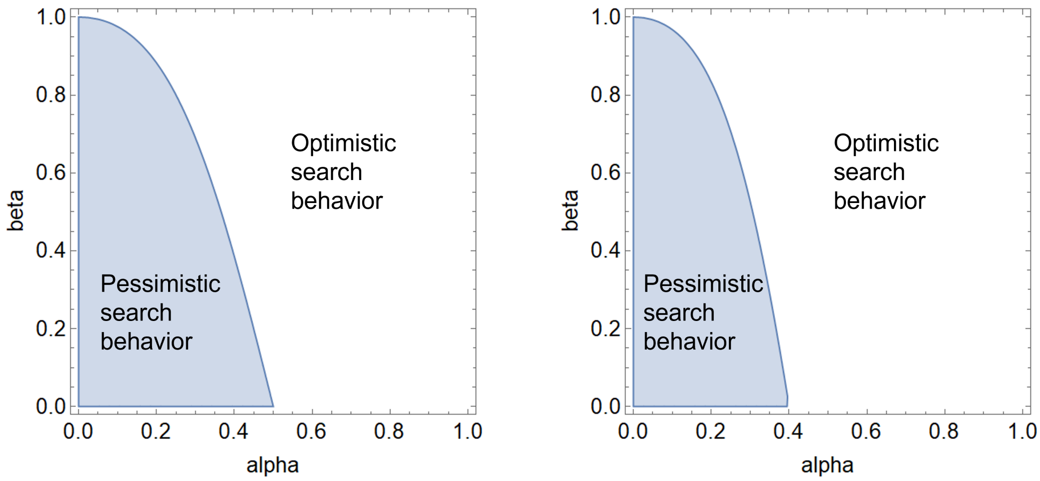
That is, as long as the worker is not fully pessimistic, we can make higher ambiguity increase the reservation wage by letting the worker sufficiently patient. In other words, $\bar{\alpha}$ converges to zero as β goes to one. Even if α is close to zero, the worker may rather increase the reservation wage when the discount factor is close to one.

[9] Note that higher ambiguity could have the same effect independently of ambiguity-attitude in certain situations (e.g., Kishishita and Ozaki 2020). In contrast, the current result indicates that the effect of higher ambiguity is heterogeneous depending on ambiguity-attitudes in the case of an optimal stopping problem.

[10] R^* goes to c as β goes to zero (see Appendix A.1). Hence, $\bar{\beta} > 0$ as long as

$$b - c > 2\alpha(b-a) \Leftrightarrow \alpha < \frac{b-c}{2(b-a)}.$$

Although these are interesting findings, the current motivating example has several drawbacks. First, it lacks some theoretical rigors; deriving (2) as an analogy to the risk case is not trivial at all. For this purpose, we need to develop dynamic programming techniques. Second, the comparative statics results depend on various assumptions on capacities. As a result, its generalizability is uncertain. In the following sections, we resolve these problems one by one and provide the complete view on the effect of higher ambiguity in the one-sided search problem.



(a) $a = 0, b = 1, c = 0$ **(b)** $a = 1, b = 2, c = 1.2$

Figure 1: Effect of higher ambiguity. Notes: Each figure compares the values of R^* between the case with $\varepsilon = 0$ and the case with $\varepsilon = 0.1$. In the blue region, the former value is greater than the latter one, corresponding to pessimistic search behaviors.

3 The Formal Model

3.1 Stochastic Environment

Let $(W, \mathscr{B}_W)$ be a measurable space, where W is a Borel subset of $\mathbb{R}_+$ and $\mathscr{B}_W$ is the Borel σ-algebra on W. Note that W does not have to be bounded. We regard each element $w \in W$ as an offer of wage in each single period. For any $t \geq 0$, we construct the t-dimensional product measurable space $(W^t, \mathscr{B}_{W^t})$ (we let $\mathscr{B}_{W^0} := \{\phi, W^\infty\}$) and embed it in the infinite-dimensional product measurable space $(W^\infty, \mathscr{B}_{W^\infty})$ as follows.

First, let $W^\infty = W \times W \times \cdots$ be the countably infinite Cartesian product of W with itself, and let $W^t = w \times \cdots \times W$ be the $(t-1)$-time Cartesian product of W with itself. That is, W^∞ is the set of infinite sequences $(w_1, w_2, \ldots)$, and W^t is the set of finite sequences $(w_1, w_2, \ldots, w_t)$, where $(\forall i)\ w_i \in W$.

Second, let $\mathscr{B}_{w^\infty}$ be the σ-algebra on W^∞ generated by the family of sets of the form $E_1 \times E_2 \times \cdots$, and let $\mathscr{B}_{(W^t)}$ be the σ-algebra on W^t generated by the family of sets of the form $E_1 \times \cdots \times E_t$, where for each i, $E_i \in \mathscr{B}_W$; that is, E_i is a Borel set. Because W is a separable metric space, $\mathscr{B}_{(W^t)}$ is identical to $(\mathscr{B}_W)^t := \mathscr{B}_W \otimes \cdots \otimes \mathscr{B}_W$, the $(t-1)$-time product measurable space of $\mathscr{B}_W$.

Third and finally, we define the σ-algebra $\hat{\mathscr{B}}_{W^t}$ on W^∞ (*not* on W^t) as the σ-algebra generated by the family of cylinder sets $E_1 \times \cdots \times E_t \times W \times W \times \cdots$, where $(\forall i)$ E_i is a Borel set. In particular, $\hat{\mathscr{B}}_{W^0} := \{\phi, W^\infty\}$ represents *no* information. Then, any function define on W^∞ that is $\hat{\mathscr{B}}_{W^t}$-measurable takes on the same value given the realization of $(w_1, w_2, \ldots, w_t)$ regardless of the realization of $(w_{t+1}, w_{t+2}, \ldots)$; hence, it can be identified with the function defined on $\mathscr{B}_{(W^t)}$. In this manner, we embed $\mathscr{B}_{(W^t)}$ in $\hat{\mathscr{B}}_{W^t}$. Therefore, we do not distinguish these two objects and use the notation $\mathscr{B}_{W^t}$ to represent both. This convention is convenient when consider stopping rules that are defined on W^∞. We write a history of realized offers as ${}_1\boldsymbol{w}_t := (w_1, w_2, \ldots, w_t) \in W^t$, ${}_1\boldsymbol{w} := (w_1, w_2, \ldots) \in W^\infty$, and so on.

3.2 Capacitary Kernel

We adopt CEU in describing a worker's decision-making under ambiguity. In this framework, beliefs, ambiguity, and ambiguity-attitude are represented as capacities. This subsection defines a capacitary kernel the worker employs when she evaluates uncertain future wage offers. See Appendix C for the basic concepts and properties of CEU.

3.2.1 Preliminaries

Let θ be a *capacitary kernel*; that is, let $\theta : W \times \mathscr{B}_W \to [0,1]$ be a function such that $(\forall w \in W)$ θ_w is a probability capacity on $(W, \mathscr{B}_W)$ and $(\forall E \in \mathscr{B}_W)$ $\theta_{\cdot}(E)$ is $\mathscr{B}_W$-measurable.

As a preliminary, we introduce several concepts about θ_w. The first concept is the convexity, which is defined as follows:

Definition 1 *θ_w is convex if*

$$(\forall A, B \in \mathscr{B}_W)\; \theta_w(A \cup B) + \theta_w(A \cap B) \geq \theta_w(A) + \theta_w(B)\,.$$

When an agent is a CEU maximizer with a capacity θ, the agent is uncertainty-averse if and only if θ is convex (Schmeidler 1989). Note that θ_w is concave if for all A and B, $\theta_w(A \cup B) + \theta_w(A \cap B) \leq \theta_w(A) + \theta_w(B)$. In contrast to a convex capacity, a concave capacity implies uncertainty-loving.

The next concept is the continuity, which is defined as follows. The importance of this property will be explained in the next subsection.

Definition 2 *θ_w is continuous if the following two conditions hold:*

$$(\forall \langle A_i \rangle_i \subseteq \mathscr{B}_W)\;\; A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots \Rightarrow \theta(\cup_i A_i) = \lim_{i\to\infty} \theta(A_i)\,.$$

$$(\forall \langle A_i \rangle_i \subseteq \mathscr{B}_W) \quad A_1 \supseteq A_2 \supseteq A_3 \supseteq ... \Rightarrow \theta(\cap_i A_i) = \lim_{i \to \infty} \theta(A_i)\,.$$

A capacitary kernel θ is *convex* (resp. *continuous*) if $(\forall w)$ θ_w is convex (resp. continuous). Throughout this paper, we keep to assume:

Assumption 1 The capacitary kernel θ is convex and continuous.

We define the *conjugate* of θ, denoted θ', by $(\forall w)(\forall E)$ $\theta'(E)_w := 1 - \theta_w(E^c)$, where E^c denotes the complement of E in W. It immediately turns out that θ' is concave because θ is assumed to be convex.

3.2.2 JP-Capacities

We aim to model both ambiguity-averse and ambiguity-seeking behaviours. To this end, we focus on the class of *JP-capacities*, which was first introduced by Jaffrey and Philippe (1997).

Assumption 2 Our agent in this paper is exclusively endowed with the capacitary kernel, denoted $\hat{\theta}^\alpha$, which is defined by

$$(\forall w \in W)(\forall E \in \mathscr{B}_W) \quad \hat{\theta}^\alpha_w(E) := \alpha\theta'_w(E) + (1-\alpha)\theta_w(E)\,, \tag{9}$$

where $\alpha \in [0,1]$.

As discussed in the above, θ_w is convex and represents uncertainty-aversion, whereas θ'_w is concave and represents uncertainty-loving. Hence, the worker endowed with a JP capacity is characterized by the mixture of uncertainty-aversion and uncertainty-loving, and the weight for uncertainty-loving is α. That is, α is interpreted as a parameter measuring the degree of *optimism* of the worker.[11] Note that neo-additive capacities introduced in Section 2 belong to this class of capacities.

To see the properties of this class of capacities further, we observe that the CEU with a JP-capacity is equivalent to a weighted average of the minimum and the maximum expected utility over the set of probabilities core(θ_w). Let $B(W, \mathscr{B}_W)$ be the space of all bounded real-valued functions on W which is $\mathscr{B}_W$-measurable. Then, we have $(\forall f \in$

[11] Ghirardato and Marinacci (2002) developed a notion of comparative ambiguity aversion and defined more ambiguity-aversion as follows. Let $\hat{\theta}^1$ and $\hat{\theta}^2$ be two capacitary kernels. The capacitary kernel $\hat{\theta}^2$ *exhibits more ambiguity-aversion* if

$$(\forall w)(\forall E) \quad \hat{\theta}^1_w(E) \geq \hat{\theta}^2_w(E)\,.$$

It is observed that higher α implies less ambiguity-aversion in this sense.

$B(W, \mathscr{B}_W))(\forall w \in W)$

$$\begin{aligned} & \int_W f(w')\,\hat{\theta}^{\alpha}_w(dw') \qquad (10) \\ = \; & \alpha \int_W f(w')\,\theta'_w(dw') + (1-\alpha)\int_W f(w')\,\theta_w(dw') \\ = \; & \alpha \max_{p \in \mathrm{core}(\theta_w)} \int_W f(w')\,p(dw') + (1-\alpha) \min_{p \in \mathrm{core}(\theta_w)} \int_W f(w')\,p(dw')\,, \end{aligned}$$

where the *core* of θ, denoted $\mathrm{core}(\theta)$, is defined by

$$\mathrm{core}(\theta) := \{p \in \mathscr{M}(W, \mathscr{B}_W) \mid (\forall E \in \mathscr{B}_W)\;\; \theta'(E) \geq p(E) \geq \theta(E)\}$$

and $\mathscr{M}(W, \mathscr{B}_W)$ is the set of all probability charges on $(W, \mathscr{B}_W)$. It is well-known that the core is non-empty when θ is convex and that any element of the core is not only finitely additive but also countably additive, and hence, it turns out to be a measure when θ is continuous. That is, the CEU with a JP capacity can be rewritten as the $\alpha-$Maxmin expected utility ($\alpha-$MEU).[12] This preferences lies in the intersection between CEU and $\alpha-$MEU.

Equation (10) naturally suggests α's interpretation we adopt here. When $\alpha = 1$, the worker is totally optimistic in the sense that she takes care of only the "best" probability measure in the core to compute the expectation. In contrast, when $\alpha = 0$ there, the worker is totally pessimistic in the sense that she takes care of only the "worst" probability measure in the core to compute it. When α is strictly between them, she has both aspects of optimism and pessimism with higher α indicating more optimism.

Two novel features of this class of capacities should be noted. First, $\hat{\theta}^{\alpha}$ can be neither convex nor concave, and thus this preference allows the worker to be neither uncertainty-averse nor uncertainty-loving. Second, this enables us to separate the degree of ambiguity from the degree of ambiguity-attitudes. As seen before, on the one hand, α can be regarded as the degree of optimism i.e., the degree of ambiguity-attitudes. On the other hand, the dilation of $\mathrm{core}(\theta_w)$ implies that the set of probabilities expands. Hence, it can be interpreted as an increase of the degree of ambiguity keeping the degree of ambiguity-attitudes fixed. This separation enables us to conduct comparative statics with respect to the degree of ambiguity.

[12] $\alpha-$MEU was first suggested by Hurwicz (1951) and has been adopted by various applied studies. Its axiomatic foundations are given by Ghirardato, Maccheroni, and Marinacci (2004), Hill (2019), and so on. Note that Eichberger et al. (2011) prove that the approach by Ghirardato, Maccheroni, and Marinacci (2004) has only been successfully demonstrated for infinite state spaces. In finite state spaces, α takes only zero or one. Hill (2019) takes a different approach and provides an axiomatic foundation. Note that α represents the weight for the minimum expected utility in the standard terminology of $\alpha-$MEU. We use a different notation to make it consistent with CEU with a neo-additive capacity, which will be examined later.

3.3 Lifetime Income for Unemployed Worker

An *income process* ${}_0\boldsymbol{y} = (y_0, y_1, y_2, \ldots)$ is an $\mathbb{R}_+$-valued stochastic process that is $\langle \mathscr{B}_{W^t} \rangle$-adapted; that is, it satisfies $(\forall t \geq 0)$ y_t is $\mathscr{B}_{W^t}$-measurable. Given an income process ${}_0\boldsymbol{y}$, we denote the *continuation* of ${}_0\boldsymbol{y}$ after the realization of a history ${}_1\boldsymbol{w}_t$ by ${}_t\boldsymbol{y}_{{}_1\boldsymbol{w}_t} := (y_t({}_1\boldsymbol{w}_t), y_{t+1}({}_1\boldsymbol{w}_t, \cdot), y_{t+1}({}_1\boldsymbol{w}_t, \cdot, \cdot), \ldots)$. Obviously, the continuation ${}_1\boldsymbol{w}_t$ is $\langle \mathscr{B}_{W^t} \rangle$-adapted given ${}_1\boldsymbol{w}_t$.

Given any adapted income process ${}_0\boldsymbol{y}$ and an initial wage offer $w_0 \in W$, we define the *lifetime expected income* $I_{w_0}({}_0\boldsymbol{y})$ by[13]

$$I_{w_0}({}_0\boldsymbol{y}) = \lim_{t\to\infty} y_0 + \beta \int_W \left(y_1 + \beta \int_w \left(y_2 + \cdots \right.\right.$$
$$\left.\left. \beta \int_W y_t \, \hat{\theta}(dw_t) \cdots \right) \hat{\theta}(dw_2) \right) \hat{\theta}_{w_0}(dw_1) \,, \quad (11)$$

where $\beta \in (0,1)$ is the discount factor and $\int_W \cdot \, d\hat{\theta}$ is the *Choquet integral with respect to* a capacitary kernel defined by $\hat{\theta}$, that is, (10). Note that each element of the sequence defining I is well-defined by the continuity of $\hat{\theta}$ and by the Fubini property (Nishimura and Ozaki, 2017, p.45, Theorem 2.5.1), and that the limit exists (allowing $+\infty$) because the sequence is nondecreasing by the non-negativity of y_t values and by the monotonicity of the Choquet integral (Nishimura and Ozaki, 2017, p.40, Proposition 2.4.2).

The objective function that resembles (11) and possess a dynamically consistent intertemporal structure has been axiomatized for a finite horizon by Nishimura and Ozaki (2003, and Chapter 3 of 2017) and for an infinite horizon by Wang (2003). These treat an objective function defined via general capacities while the one with the specific capacity used in this paper is not yet done as far as we know. For further discussions, see Appendix D.

The assumed continuity of $\hat{\theta}$ and the monotone convergence theorem (Nishimura and Ozaki, 2017, p.44, Theorem 2.4.6) imply that

$$(\forall\, {}_0\boldsymbol{y})(\forall w_0) \quad I_{w_0}({}_0\boldsymbol{y}) = y_0 + \beta \int_W I_{w_1}({}_1\boldsymbol{y}|_{w_1})\, \hat{\theta}_{w_0}(dw_1) \,,$$

which is called *Koopmans' equation.*

3.4 Stopping Rule and Optimization Problem

In each period, the prospective worker is given an offer w. Upon observing the value of w, she has two alternatives: to accept it or to reject it. If she accepts the offer, she will obtain w for each period from that period on; if she rejects the offer, she will get the unemployment compensation $c > 0$ for that period and will be given a random offer again

[13]In (11), we suppressed the arguments of the integrand: it should be kept in mind that for every t, $y_t = y_t({}_1\boldsymbol{w}_t)$.

in the next period.

A $\{0,1,2,\dots\}\cup\{+\infty\}$-valued random variable d on $(W^\infty, \mathscr{B}_{W^\infty}$ is called a *stopping time* if it satisfies $(\forall t=0,1,2,\dots)\ \{d=t\}\in\mathscr{B}_{W^t}$, where $\{d=t\}$ abbreviates $\{\,{}_1\boldsymbol{w}\,|\,d({}_1\boldsymbol{w})=t\,\}$. We allow d to be $+\infty$ for some history. We denote the set of all stopping rules by Δ. Given any stopping rule $d\in\Delta$, we define a process ${}_0\boldsymbol{y}^d=(y_0^d,y_1^d,y_2^d,\dots)$ by

$$(\forall t\geq 0)\quad y_t^d := \begin{cases} c & \text{if} \quad d>t \\ w_T & \text{if} \quad d=T\in\{0,1,\dots,t\}\,. \end{cases}$$

It turns out that ${}_0\boldsymbol{y}^d$ is in fact $\langle\mathscr{B}_{W^t}\rangle$-adapted (see, for instance, Nishimura and Ozaki, 2017, Lemma A.4.1, p.278), and hence, it is certainly an income process. Given an initial wage offer $w_0\in W$, we denote the lifetime expected income under a stopping rule $d\in\Delta$ by the symbol $I(d)$ for notational simplicity: $I_{w_0}(d):=I_{w_0}(\,{}_0\boldsymbol{y}^d)$. Similarly, given any $t\geq 1$ and any history ${}_1\boldsymbol{w}_t\in W^t$, we denote the income under $d\in\Delta$ after the realization of ${}_1\boldsymbol{w}_t$ by $I_{w_t}(d\,|_{{}_1\boldsymbol{w}_t})$: that is, $I_{w_t}(d\,|_{{}_1\boldsymbol{w}_t}):=I_{w_t}(\,{}_t\boldsymbol{y}^d\,|_{{}_1\boldsymbol{w}_t})$, where ${}_t\boldsymbol{y}^d\,|_{{}_1\boldsymbol{w}_t}$ is the continuation of ${}_0\boldsymbol{y}^d$ after the realization of ${}_1\boldsymbol{w}_t$ as is defined in the previous subsection.

It can be shown that

$$I_{w_0}(d)=\frac{w_0}{1-\beta}\chi_{\{d=0\}}+\left(c+\beta\int I_{w_1}(d\,|_{w_1})\,d\hat{\theta}_{w_0}\right)\chi_{\{d>0\}} \tag{12}$$

(for instance, see Nishimura and Ozaki, 2017, Lemma A.4.2, p.279). Here, χ denotes the *indicator function* on W^∞.[14] Equation (12) is a version of Koopmans' equation with a stopping rule d.

A stopping rule $d\in\Delta$ is *optimal from* w_0 if $d\in\arg\max\{\,I_{w_0}(d)|\,d\in\Delta\}$. A stopping rule d is *admissible* if it dictates searching more as long as the observed offer is strictly less than c. Any stopping rule that is *not* admissible is suboptimal because is is dominated by the stopping rule which never stops, and hence, it can be safely ignored. When an optimal stopping rule exists, we define the *value function* $V^*:W\to\bar{\mathbb{R}}_+$ by $(\forall w\in W)\ V^*(w):=I_w(d_w^*)$, where we denote an optimal stopping rule from w by d_w^*.

[14]For example,

$$\chi_{\{d=t\}}=\begin{cases} 1 & \text{if} \quad \omega\in\{d=t\},\ i.e.,\ d(\omega)=t \\ 0 & \text{if} \quad \omega\notin\{d=t\},\ i.e.,\ d(\omega)\neq t\,. \end{cases}$$

Because d is a stopping rule, $\chi_{\{d=t\}}$ and $\chi_{\{d>t\}}$ are $\mathscr{B}_{W^t}$-measurable.

4 Existence and Characterization of Optimal Stopping Rule

4.1 Assumptions and Admissibility

Given random variables $w, w_0, \ldots, w_t$, let $c \vee w$ and $\vee_{i=0}^t w_i$ denote the random variables defined by $\max\{c, w\}$ and $\max\{w_1, \ldots, w_t\}$.

Assumption 3 Throughout the rest of this paper, we assume that the primitives of the model satisfy the following two conditions: $(\forall w_0)$

$$\textbf{E1.} \quad (\forall t > 0) \;\; \bar{W}^t(w_0) := \int \cdots \int \left(c \vee \vee_{i=0}^t w_i\right) \theta'(dw_t) \cdots \theta'_{w_0}(dw_1) < +\infty$$

$$\textbf{E2.} \quad \overline{\lim}_{t \to \infty} \left(\bar{W}^t(w_0)\right)^{1/t} < \beta^{-1} \, ,$$

where θ' is the conjugate of θ defined in Subsection 3.2.

The integrand in E1, $c \vee \vee_{i=0}^t w_i$, is the overly optimistic income the worker expects in time t. It is *overly* optimistic because it is the highest offer up to time t (note that our model is on search *without* recall). The integral in E1 is its overly optimistic "expectation" evaluated at time 0. This is *overly* optimistic because it is evaluated by the *conjugate* of θ, rather than $\hat{\theta}$ which is a convex combination of θ itself and its conjugate (recall that θ is assumed to be convex).[15] The assumption E1 requires that this should be finite for any t.

This optimistic "expected" income, $\bar{W}^t$, grows as t increases because it takes the maximum offer up to time t. The left-hand side of E2 defines the time-average of the rate of growth in $\bar{W}^t$. Hence, E2 as a whole assumes that this time-average is lower than the worker's impatience. When E2 holds, the effect caused by the high income in the far future can be safely ignored because the worker's impatience dominates the income growth along any optimistic path. This is an analogue to the condition for the dynamic programming technique introduced in Ozaki and Streufert (1996). If θ were simply a probability measure, the left-hand side of E2 is unity as long as the expectation of w is finite (Chung, 1974, p.49), and hence, E2 would be automatically satisfied.

Define a (constant) function $V^- : W \to \mathbb{R}_+$ by $(\forall w)$ $V^-(w) := c/(1-\beta)$ and a function

[15] When we prove that the solution to Bellman's equation (specified later) is the value function, we apply the method of "squeezing," which was first developed by Ozaki and Streufert (1996) for maximizing a stochastic objective function which is *not* defined via a standard expectation. To "squeeze," we need to bound the increment of the Choquet integral with respect to $\hat{\theta}$ by the one with respect to θ's conjugate. This is why we need to define the "expected" income via the conjugate of θ in E1. For more details, see Appendix A.

$V^+ : W \to \bar{\mathbb{R}}_+$ by

$$(\forall w_0) \;\; V^+(w_0) := \lim_{T\to\infty} (c \vee w_0) + \beta \int \Bigg(\big(c \vee \vee_{i=0}^{1} w_i\big) + \cdots$$
$$\beta \int \big(c \vee \vee_{i=0}^{T} w_i\big)\, \hat{\theta}(dw_T) \cdots \Bigg) \hat{\theta}_{w_0}(dw_1)\,, \quad (13)$$

which is a well-defined $\mathscr{B}_W$-measurable function (let $y_T := c \vee \vee_{i=0}^{T} w_i$ in (11)). Clearly, $V^- \le V^+$, and it can be shown that $(\forall w_0)\; V^+(w_0) < +\infty$ by Nishimura and Ozaki (2017, Lemma A.4.3, p.279) and by the fact that a convex combination of a convex capacity and its conjugate is dominated by the latter.

A $\mathscr{B}_W$-measurable function $V : W \to \mathbb{R}_+$ is *admissible* if it satisfies $V^- \le V \le V^+$. Note that for any admissible stopping rule d, $I.(d)$ is admissible. Let $\mathscr{V}$ be the space of all admissible functions from W into $\mathbb{R}_+$, and let B be the operator from $\mathscr{V}$ into itself defined by

$$(\forall V \in \mathscr{V})(\forall w \in W) \;\; BV(w) := \max\left\{ \frac{w}{1-\beta}\,,\, c + \beta \int_W V(w')\hat{\theta}_w(dw') \right\}. \qquad (14)$$

By Nishimura and Ozaki (2017, Lemma A.4.4, p.280), BV is admissible for any admissible function V, and hence, the operator B is well-defined. Finally, an admissible function $V \in \mathscr{V}$ *solves Bellman's equation* if $BV = B$.

4.2 Main Results on the Reservation Wage

We are now ready to state the main results of this section, which is summarized in the following theorem. The proof is contained in Appendix A.

Theorem 1 (Bellman's Principle of Optimality) *The value function V^* exists and is the unique admissible solution to Bellman's equation. Furthermore, V^* is attained by the stopping rule d^* such that for all $t \ge 0$, $d^* = t$ as soon as*

$$\frac{w_t}{1-\beta} \ge c + \beta \int_W V^*(w_{t+1})\hat{\theta}_{w_t}(dw_{t+1})$$

holds; and $d^* > t$ if otherwise.

Let $R^* : W \to \mathbb{R}_+$ be a $\mathscr{B}_W$-measurable function defined by

$$(\forall w) \quad R^*(w) := (1-\beta)\left(c + \beta \int_W V^*(w')\hat{\theta}_w(dw') \right).$$

We call $R^*(w)$ the *reservation wage* at state w, which is a threshold in the sense that as soon as the current wage offer equals or exceeds $R^*(w)$, the unemployed worker stops the search and takes the wage offer; if otherwise, she continues the search.

4.3 The *i.i.d.* Case

For a while, we assume that the capacitary karnel $\hat{\theta}$ is independent of the current wage offer, w. That is, w is *independently and indistinguishably distributed* as proposed by Epstein and Schneider (2003b).[16] In this case, the characterization of the reservation wage is given by a simple formula as seen in the following two corollaries.

For the proof of the next corollary, see Nishimura and Ozaki (2017, A.4.4, p.285).[17]

Corollary 1 *Suppose that the capacitary karnel $\hat{\theta}$ is independent of the current wage offer, w, then the reservation wage $R^*(w)$ will be constant and is given by the solution R^* to the next equation:*

$$R^* = c + \frac{\beta}{1-\beta} \int_{R^*}^{\infty} \hat{\theta}\left(\{w\,|w \geq x\}\right) dx\,.$$

The next corollary is also immediate:

Corollary 2 *Under the same assumption as Corollary 1, the reservation wage R^* will be constant and is given by the solution R^* to the next equation:*

$$\begin{aligned} R^* = c + \frac{\beta}{1-\beta} \int_{R^*}^{\infty} \theta\left(\{w\,|w \geq x\}\right) dx \\ + \frac{\beta}{1-\beta} \int_{R^*}^{\infty} \left[\alpha\Big(1 - \theta\left(\{w\,|w < x\}\right) - \theta\left(\{w\,|w \geq x\}\right)\Big)\right] dx\,. \end{aligned} \tag{15}$$

5 Effect of Higher Ambiguity

This section conducts a sensitivity analysis exclusively with respect to the *reservation wage* upon the change of the degree of ambiguity. For other comparative statics results on, say α, see Appendix B. We denote the reservation wage defined in the previous section by $R^*(w|\alpha,\theta)$ when the current wage offer (*i.e.*, the current state) is w and the worker employs the JP capacity $\hat{\theta}^\alpha := \alpha\theta' + (1-\alpha)\theta$.

5.1 Defining "Being More Ambiguous"

As a first step, we define "being more ambiguous" in a precise manner. As seen in Section 3, the dilation of core(θ_w) represents higher ambiguity because it means that the set of probabilities enlarges. That is, "being more ambiguous" is defined as follows:

Definition 3 *Let θ^1 and θ^2 be two convex capacitary kernels. θ^2 exhibits more ambiguity than θ^1 if*

$$(\forall w) \quad \mathrm{core}(\theta^2_w) \supseteq \mathrm{core}(\theta^1_w)\,,$$

[16] They originally proposed this concept in the framework of the Maxmin expected utility, but it can be straightforwardly extended to our case.

[17] Replace θ there by $\hat{\theta}$ here.

which is equivalent to

$$(\forall w)(\forall E) \quad \theta^1_w(E) \geq \theta^2_w(E)\,. \tag{16}$$

In particular, θ^2 exhibits strictly more ambiguity than θ^1 if (16) holds with a strict inequality for any $E \neq W$ such that $\theta^1_w(E) > 0$.

The above equivalence comes from the convexity of capacitary kernels.

5.2 General Results

Given the above definition of higher ambiguity, we obtain several remarkable properties regarding the effect of higher ambiguity as follows:

Proposition 1 *Fix α and suppose that θ^2 is strictly more ambiguous than θ^1. Then, the following properties hold.*

(i). Fix w. There exists $\bar{a} \in (0,1)$ such that for any $\alpha \in [0, \bar{a})$, $R^*(w|\alpha, \theta^1) > R^*(w|\alpha, \theta^2)$.

(ii). Fix w. There exists $\underline{a} \in (0,1)$ such that for any $\alpha \in (\underline{a}, 1]$, $R^*(w|\alpha, \theta^1) < R^*(w|\alpha, \theta^2)$.

(iii). Suppose that θ^1 is additive. Fix w. There exists a unique $\bar{\alpha} \in (0,1)$ such that

- *$R^*(w|\alpha, \theta^1) \leq R^*(w|\alpha, \theta^2)$ for all $\alpha > \bar{\alpha}$*
- *$R^*(w|\alpha, \theta^1) > R^*(w|\alpha, \theta^2)$ for all $\alpha < \bar{\alpha}$.*

The proposition shows how the effect of higher ambiguity depends on the degree of optimism. When $\alpha = 0$ (i.e., the worker is purely pessimistic), higher ambiguity reduces the reservation wage. This is the result obtained by Nishimura and Ozaki (2004, Theorem 2). From this result, the exsiting studies have understood that higher uncertainty in the sense of ambiguity always discourages the worker to continue the search, which is in contrast to the effect of higher risk.

(i) and (ii) demonstrate that this result hinges on the assumption that the worker is uncertainty-averse. On the one hand, (i) shows that the result of Nishimura and Ozaki (2004) remains when α is close to zero. On the other hand, (ii) shows that the effect is opposite when α is close to one; that is, higher ambiguity rather increases the reservation wage when the worker is relatively optimistic. This is because higher ambiguity on the future wage offers increases the expected value of continuing search when the worker is optimistic.

From (i) and (ii), it would be expected that there is a threshold value $\bar{\alpha}$, and higher ambiguity reduces the reservation wage if and only if the degree of optimism is lower than $\bar{\alpha}$. Although this seems to be straightforward, it is hard to prove this property. (iii) confirms this conjecture by restricting our attention to the compariosn with and without ambiguity. When the capacity is additive, its core is singleton, thus such a capacity represents the absence of ambiguity. (iii) argues that compared with no ambiguity cases, the existence of ambiguity reduces the reservation wage if and only if the degree of optimism is lower than $\bar{\alpha}$.

These results together confirm the findings in the motivating example (Section 2).

6 Time Discounting and Effect of Higher Ambiguity

We will next examine how the effect of ambiguity depends on the agent's discount factor when the agent is neither purely optimistic nor pessimistic (i.e., $\alpha \in (0,1)$). To this end, we focus on the i.i.d. case examined in Section 4.3. That is, $\hat{\theta}_w$ is independent of w.

6.1 Neo-Additive Capacities and Their $\delta-$Approximation

To obtain a clear-cutting property, from now own, we exclusively focus on the class of *neo-additive capacities*, which was examined in Section 2. This is a well-known special class of JP-capacities with a nice axiomatic foundation (Chateauneuf, Eichberger, and Grant 2007) and has been widely used in applied researches.

Formally, the neo-additive capacities are defined as follows:

Definition 4 *A capacity $\hat{\theta}$ is called a neo-additive capacity if for any $E \notin S \cup \emptyset$,*

$$\hat{\theta}(E) := (1-\varepsilon)\pi(E) + \varepsilon\alpha\,,$$

where π is a probability measure and $\varepsilon \in (0,1)$.

Let the cumulative distribution function of π be F. We assume that F is continuous and differentiable and π has a full-support. This is the JP-capacity, where the convex capacity θ has a particular formula:

$$\theta(E) := (1-\varepsilon)\pi(E)\,.$$

When $\alpha = 0$, this class of capacities is reduced to the ε-contamination, a well-known capacity in the context of MEU.[18] Hence, neo-additive capacities can be regarded as the generalization of ε-contamination to the cases where the agent is not necessarily uncertainty-averse.

This class of capacities has two advantages in analyzing the relationship between time discounting and the effect of higher ambiguity. First, an increase of ε leads to the dilation of core(θ), and hence, its increase can be interpreted as an increase of the degree of ambiguity in the sense of Definition 3. That is, the degree of ambiguity is modelled as a single parameter ε, which makes comparative statics analysis much easier. Second,

[18]Axiomatic foundations of ε-contamination are provided by Nishimura and Ozaki (2006) and the subsequent works.

and most importantly, it is known that this is neither convex nor concave if and only if $\alpha \in (0,1)$. Hence, this capacity allows us to analyze the case where the worker is neither uncertainty-averse nor uncertainty-loving.[19]

Neo-additive capacities are, however, not continuous, which violates our initial assumption. Thus, we need to consider the approximation by a continuous capacity.[20]

Definition 5 *Capacity $\hat{\theta}_\delta$ is called a $\delta-$approximation of a neo-additive capacity if for any $E \in \mathscr{B}_W$,*

$$\theta_\delta(E) = \begin{cases} (1-\varepsilon)\pi(E) & (\pi(E) \le 1-\delta) \\ (1-\varepsilon)\pi(E) + \varepsilon[(\pi(E)-1)/\delta + 1] & (\pi(E) > 1-\delta) \end{cases} .$$

If δ goes to zero, this capacity approximates a neo-additive capacity.

Although this capacity looks complicated, we have another simple representation. From the definition of the core, we have

$$\text{core}(\theta_\delta) = \{(1-\varepsilon)\pi + \varepsilon\mu |\ \mu \in \mathcal{M}(\pi,\delta)\} \ ,$$

where

$$\mathcal{M}(\pi,\delta) := \{\mu \in \mathcal{M} |\ (\forall E)\ \delta\mu(E) \le \pi(E)\} \ .$$

Note that $\mathcal{M}$ is the set of all probability charges. Consequently, by using equation (10), we obtain the following representation. Let $B(W, \mathscr{B}_W)$ be the space of all bounded real-valued functions on W which is $\mathscr{B}_W$-measurable. Then, we have $(\forall f \in B(W, \mathscr{B}_W))(\forall w \in W)$

$$\begin{aligned} &\int_W f(w)\, \hat{\theta}_\delta(dw) \\ =&\alpha \max_{p\in \text{core}(\theta_\delta)} \int_W f(w)\, p(dw) + (1-\alpha) \min_{p\in \text{core}(\theta_\delta)} \int_W f(w)\, p(dw) \\ =&(1-\varepsilon) \int_W f(w)\, \pi(dw) \\ &+ \varepsilon \left[\alpha \max_{p\in\mathcal{M}(\pi,\delta)} \int_W f(w)\, p(dw) + (1-\alpha) \min_{p\in\mathcal{M}(\pi,\delta)} \int_W f(w)\, p(dw)\right] \ . \end{aligned}$$

When $\delta = 0$ (i.e., in the case of neo-additive capacities), $\mathcal{M}(\pi,\delta)$ becomes equivalent to the set of all probability charges, and thus, this can be further rewritten as

$$(1-\varepsilon) \int_W f(w)\, \pi(dw) + \varepsilon \left[\alpha \max_{w\in W} f(w) + (1-\alpha) \min_{w\in W} f(w)\right] \ . \qquad (17)$$

[19]JP-capacities do not always lead the worker to be neither uncertainty-averse nor uncertainty-loving for some α. To see this, consider a capacity such that $\hat{\theta}_w(E) = \pi(E)_w + (\alpha - 0.5)\gamma$ for $E \neq S, \emptyset$, where $\gamma > 0$. This capacity is a JP capacity, but it is convex if $\alpha < 0.5$ and concave if $\alpha > 0.5$.

[20]Nishimura and Ozaki (2004) originally consider the $\delta-$approximation of $\varepsilon-$contamination.

6.2 Bounded Case

We start the analysis with the case where W is bounded: $W := [0, B]$ where $B > c$. Let the reservation wage for a $\delta-$approximation of a neo-additive capacity be $R^*(\delta)$. Our interest is $R^*_{neo} := \lim_{\delta \searrow 0} R^*(\delta)$.

Proposition 2 *Let the expected value of w when the true distribution is π be $E_\pi(w)$ and let*

$$\bar{\alpha} := 1 - \frac{B - E_\pi(w) - \int_0^c F(w)dw}{B - c} \in (0, 1).$$

(i). For any $\alpha \in [\bar{\alpha}, 1]$, R^*_{neo} is increasing in ε.

(ii). For any $\alpha \in (0, \bar{\alpha})$, there exists $\bar{\beta} \in (0, 1)$ such that R^*_{neo} is increasing in ε if and only if $\beta \in (\bar{\beta}, 1)$. Furthermore, $\bar{\beta}$ is decreasing in c.

(i) is for the case where the worker is sufficiently optimistic, though α can be strictly lower than one. When α is strictly lower than one, the worker is no longer uncertainty-loving. That is, with a small fraction, $1 - \alpha$, she takes the worst case into account. (i) argues that higher ambiguity always leads to higher reservation wage *independently* of the discount factor β even in that case. Hence, for a sufficiently optimistic worker, higher ambiguity always encourages the worker to continue search.

On the contrary, (ii) states the case where the worker is sufficiently pessimistic, though α can be strictly higher than zero. When α is strictly positive, the worker is no longer uncertainty-averse in the sense that she takes the best case into account with a small fraction, while she is still sufficiently pessimistic. In this case, the discount factor, β, matters. The result indicates that for *any* positive α, there exists a high discount factor under which higher ambiguity increases the reservation wage. That is, as long as the worker is not purely pessimistic, the worker behaves as if she were ambiguity-loving, when the discount factor is sufficiently high. The striking part of the result is that this is true even if α is close to zero.

The mechanism can be understood as follows. Consider whether the worker has an incentive to accept a wage offer R^*. Let the expected payoff from the next period when continuing search under the probability measure π be v_π. Then, the payoff difference between accepting the offer and continuing search can be rewritten as

$$\varepsilon[R^* - \beta((1-\alpha)R^* + \alpha B)] + (1-\varepsilon)\left[R^* - \beta v_\pi\right] .$$

The first term is the payoff difference when π is not the true distribution and the second term is that when π is the true distribution. Suppose that β is close to one so that the first term is negative. Hence, at the equilibrium, the second term is positive because the payoff difference should be zero. What will happen if ε increases in this environment? Then, the payoff difference decreases and becomes negative. That is, the worker is encouraged to continue search. In this discussion, the key is the fact that $R^* - \beta((1-\alpha)R^* + \alpha B) < 0$

when β is close to one. Because of the infinite-horizon structure, the worst-case payoff is still βR^* when continuing search. Hence, the convex combination of the best-case and the worst-case exceeds R^* when β is large even if α is close to zero. This mechanism creates the above results.

This result implies that the discount factor affects whether the result for an uncertainty-averse agent is applicable to agents who are not perfectly pessimistic. When the discount factor is high, even slight contamination of optimistic aspects could have a drastic change. This might provide a rationale for why experimental results are mixed. In a job search experiment, Asano, Okudaira, and Sasaki (2015) find that higher ambiguity decreases the reservation wage. On the other hand, Della Seta, Gryglewicz, and Kort (2014) experimentally analyze the effect of ambiguity on a similar optimal stopping problem. They find that higher ambiguity rather increases the reservation wage, which contradicts to the theoretical prediction under uncertainty-aversion. In short, the two studies obtain different results about whether the theoretical prediction under uncertainty-aversion holds in an experimental setting. This mixed conclusion might come from a difference in the subjects' discount factor. Our result indicates that we should carefully control the subjects' discount factor in experiments.

6.3 Unbounded Case

We turn to the analysis of the unbounded case where $W := [0, \infty)$.

Remember that when $\delta = 0$, $\mathcal{M}(\pi, \delta)$ is the set of all probability charges, and thus it includes the probability charge giving probability one to the best outcome. As a result,

$$\max_{p \in \mathcal{M}(\pi,0)} \int_W f(w) = \max_{w \in W} f(w) \ .$$

This simplifies the analysis in the bounded case, but creates a technical challenge when W is unbounded. When the return is unbounded, the best outcome is also unbounded. Hence, $\max_{p \in \mathcal{M}(\pi,0)} \int_W f(w)$ takes infinity . Consequently, (17) is not well-defined and E1 is never satisfied when W is unbounded and $\delta = 0$. Indeed, in Section 2, we assumed that W is bounded to guarantee that (3) is well-defined.

This problem can be resolved by assuming $\delta > 0$. The above problem comes from the property that $\mathcal{M}(\pi, 0)$ includes the probability charge giving a positive probability to the best outcome. When δ is strictly positive, any distribution included in $\mathcal{M}(\pi, \delta)$ has no mass point, because $\mu(w) = 0$ for any $w \in W$ and $\mu \in \mathcal{M}(\pi, \delta)$. As a result, as long as π is well-behaved, $\max_{p \in \mathcal{M}(\pi,0)} \int_W f(w)$ does not diverge to infinity, and thus (17) is well-defined and E1 is satisfied. In short, our $\delta-$approximation of neo-additive capacities enables us to consider unbounded returns by using neo-additive capacities.

Since the utility goes to infinity as δ goes to zero, we can no longer focus on the limit case i.e., $\lim_{\delta \to 0} R^*(\delta)$. Instead, we need to fix δ and analyze the effect of ambiguity on $R^*(\delta)$. This complicates the analysis, but it is proven that the main property in

Proposition 2 still holds.

Proposition 3 *Fix $\alpha \in (0,1)$. Suppose that $c > 0$ and δ is sufficiently small, but strictly positive. Then, there exists $\bar{\beta} \in (0,1)$ such that $R^*(\delta)$ is increasing in ε if $\beta \in (\bar{\beta}, 1)$.*

Hence, as long as the worker is not purely pessimistic, the worker behaves as if she were ambiguity-loving, when the discount factor is sufficiently high. This property is preserved even in the unbounded case.

7 Concluding Remarks

This paper considered a one-sided search problem in the presence of ambiguity (i.e., Knightian uncertainty). Our novelty is to depart from the assumption of uncertainty-aversion by adopting the Choquet expected utility with a JP capacity. This setting allows us to separate the degree of ambiguity from ambiguity-attitudes. By exploiting this feature, we examined whether higher ambiguity induces an agent to search shorter or longer. We first showed that higher ambiguity induces the agent to search shorter if and only if the agent's degree of optimism is sufficiently low. We next examined how the effect of ambiguity depends on the agent's discount factor when the agent is neither purely optimistic nor pessimistic. Focusing on a subclass of JP capacities, called neo-additive capacities, we found that even slight contamination of optimism leads to ambiguity-loving behaviors when the agent is sufficiently patient.

APPENDICES

A Lemmas and Proofs

A.1 Derivations in Section 2

(i). We first verify that $R^* \in (c, b)$. By simple rearrangement of the terms,

$$D = \left(1 + \alpha\hat{\beta}\varepsilon\right)^2 + \frac{2\hat{\beta}}{b-a}(1-\varepsilon)(b-c) > \left(1 + \alpha\hat{\beta}\varepsilon\right)^2 =: D'$$

because $2\hat{\beta}(1-\varepsilon)(b-c)/(b-a) > 0$, which in turn follows from $\varepsilon < 1$, $b > a$ and $b > c$, the last of which can be seen as an appropriate assumption in the current model because if otherwise, the unemployed worker should keep declining the wage offer.

Therefore, it immediately follows that

$$R^* < \frac{1}{1-\varepsilon}\left[b(1-\varepsilon) + (b-a)\alpha\varepsilon + (b-a)\frac{1-\sqrt{D'}}{\hat{\beta}}\right] = b\,, \tag{18}$$

where the inequality holds because $D > D'$, the observation we have just made.

Because the worker cannot expect any wage offer exceeding b, which is the least upper bound of the possible wage offers, this inequality is exactly what we expect for this model.

Symmetrically, define D'' by

$$D'' := D + (1-\varepsilon)\hat{\beta}^2\frac{b-c}{b-a}\left[(1-\varepsilon)\frac{b-c}{b-a} + 2\alpha\varepsilon\right]\,.$$

Obviously, it holds that $D'' > D$ under the current configuration of the parameter values, and hence, we have

$$R^* > \frac{1}{1-\varepsilon}\left[b(1-\varepsilon) + (b-a)\alpha\varepsilon + (b-a)\frac{1-\sqrt{D''}}{\hat{\beta}}\right] = c\,. \tag{19}$$

Because the worker can always decline the wage offer and get at least the unemployment compensation, c, the relation seen in Inequality (19) can be reasonably expected to hold for the current model.

(ii). We next derive Equality (6).

$$
\begin{aligned}
\lim_{\hat{\beta}\uparrow+\infty} \frac{1-\sqrt{D}}{\hat{\beta}} &= \lim_{\hat{\beta}\uparrow+\infty} -\frac{\partial}{\partial\hat{\beta}}\sqrt{D} \\
&= -\lim_{\hat{\beta}\uparrow+\infty} \frac{(1-\varepsilon)(b-c)/(b-a)+\alpha\varepsilon+\alpha^2\hat{\beta}\varepsilon^2}{\sqrt{1+2\hat{\beta}(1-\varepsilon)(b-c)/(b-a)+2\alpha\hat{\beta}\varepsilon+\alpha^2\hat{\beta}^2\varepsilon^2}} \\
&= -\lim_{\hat{\beta}\uparrow+\infty} \frac{\alpha^2\hat{\beta}\varepsilon^2}{\sqrt{1+2\hat{\beta}(1-\varepsilon)(b-c)/(b-a)+2\alpha\hat{\beta}\varepsilon+\alpha^2\hat{\beta}^2\varepsilon^2}} \\
&= -\lim_{\hat{\beta}\uparrow+\infty} \frac{\alpha^2\varepsilon^2}{\sqrt{\frac{1}{\hat{\beta}^2}+\frac{2(1-\varepsilon)(b-c)}{\hat{\beta}(b-a)}+\frac{2\alpha\varepsilon}{\hat{\beta}}+\alpha^2\varepsilon^2}} \\
&= -\alpha\varepsilon\,.
\end{aligned}
$$

(iii). Next, we prove that R^* is increasing in β. For notational ease, define the three formulas as follows:

$$
\begin{aligned}
A &:= \sqrt{\frac{1}{\hat{\beta}^2}+\frac{2(1-\varepsilon)(b-c)}{\hat{\beta}(b-a)}+\frac{2\alpha\varepsilon}{\hat{\beta}}+\alpha^2\varepsilon^2}\,; \\
B &:= \frac{1}{\hat{\beta}}+\frac{(1-\varepsilon)(b-c)}{b-a}+\alpha\varepsilon\,; \quad \text{and} \\
C &:= \sqrt{\frac{1}{\hat{\beta}}+\frac{2(1-\varepsilon)(b-c)}{b-a}+2\alpha\varepsilon+\alpha^2\hat{\beta}\varepsilon^2}\,.
\end{aligned}
$$

Note that A, B and C are all positive. Also, a direct computation shows that $B^2 > A^2$, and hence, that $B > A$ by their positivity. Furthermore, we will have[21]

$$
\frac{\partial}{\partial\hat{\beta}}\left(\frac{1-\sqrt{D}}{\hat{\beta}}\right) = \frac{-A+B}{C} > 0\,,
$$

which shows the reservation wage R^* is increasing in $\hat{\beta}$, and hence, in β regardless of the values of the other parameters (as far as they satisfy the aforementioned conditions).

(iv). Lastly, we prove that R^* can become as close as possible to c by letting β to be close to zero. Let $\beta \downarrow 0$, which corresponds to $\hat{\beta} \downarrow 0$. If we look at a relevant part of (5), $(1-\sqrt{D})/\hat{\beta}$, its "limit" as $\hat{\beta} \downarrow 0$ will be $0/0$, which is an indeterminate expression, and

[21] Please trust us unless you are quite skeptical.

thus, we can invoke l'Hôpital's rule again to find out its limit. We have

$$\begin{aligned}
\lim_{\hat{\beta}\downarrow 0} \frac{1-\sqrt{D}}{\hat{\beta}} &= \lim_{\hat{\beta}\downarrow 0} -\frac{\partial}{\partial\hat{\beta}}\sqrt{D} \\
&= -\lim_{\hat{\beta}\downarrow 0} \frac{(1-\varepsilon)(b-c)/(b-a)+\alpha\varepsilon+\alpha^2\hat{\beta}\varepsilon^2}{\sqrt{1+2\hat{\beta}(1-\varepsilon)(b-c)/(b-a)+2\alpha\hat{\beta}\varepsilon+\alpha^2\hat{\beta}^2\varepsilon^2}} \\
&= -\frac{(1-\varepsilon)(b-c)}{b-a} - \alpha\varepsilon\,,
\end{aligned}$$

the last term of which can be plugged back to (5) to obtain

$$R^* = \frac{1}{1-\varepsilon}\left[\, b(1-\varepsilon) + (b-a)\alpha\varepsilon - (1-\varepsilon)(b-c) - (b-a)\alpha\varepsilon \,\right] = c\,,$$

which confirms the third observation: R^* can be made as close as possible to the unemployment compensation by making β as small and close as possible to 0.

A.2 Proof of Theorem 1

Lemma 1 *Let θ be a convex capacity. Then, for any $a, b \in B(W, \mathscr{B}_W)$, it holds that*

$$\left| \int a\, d\theta' - \int b\, d\theta' \right| \leq \int |a-b|\, d\theta'\,.$$

Proof Because θ' is concave and because the Choquet integral with respect to a concave capacity is sub-additive (see Lemma 9), it follows that

$$\int (a+b) d\theta' - \int a\, d\theta' \leq \int b\, d\theta'\,.$$

Here, if we set $a' := a + b$ and $b' := a$, then $a' - b' = b$ and it holds that

$$\int a'\, d\theta' - \int b'\, d\theta' \leq \int (a'-b') d\theta' \leq \int |a'-b'|\, d\theta'\,,$$

where we applied the monotonicity of the Choquet integral (see Lemma 8) for the second inequality.

Symmetrically, we have

$$\int b'\, d\theta' - \int a'\, d\theta' \leq \int |a'-b'|\, d\theta'\,.$$

Because we could have chosen a' and b' arbitrarily, the proof is complete. □

Lemma 2 *Let θ be a convex capacity. Then, for any $a, b \in B(W, \mathscr{B}_W)$, it holds that*

$$\left| \int a \, d\theta - \int b \, d\theta \right| \leq \int |a - b| \, d\theta' \, .$$

Proof See Nishimura and Ozaki (2017, Theorem 2.4.3, p.43). □

Lemma 3 *Let θ be a convex capacity and let $\hat{\theta}$ be a capacity defined by* (9)*. Then, for any $a, b \in B(W, \mathscr{B}_W)$, it holds that*

$$\left| \int a \, d\hat{\theta} - \int b \, d\hat{\theta} \right| \leq \int |a - b| \, d\theta' \, .$$

Proof By the definition of $\hat{\theta}$, note that, for any $\alpha \in [0, 1]$,

$$\begin{aligned}
& \left| \int a \, d\hat{\theta} - \int b \, d\hat{\theta} \right| \\
= & \left| \left(\alpha \int a \, d\theta' + (1 - \alpha) \int a \, d\theta \right) - \left(\alpha \int b \, d\theta' + (1 - \alpha) \int b \, d\theta \right) \right| \\
= & \left| \alpha \left(\int a \, d\theta' - \int b \, d\theta' \right) + (1 - \alpha) \left(\int a \, d\theta - \int b \, d\theta \right) \right| \\
\leq & \ \alpha \left| \int a \, d\theta' - \int b \, d\theta' \right| + (1 - \alpha) \left| \int a \, d\theta - \int b \, d\theta \right| \\
\leq & \ \alpha \int |a - b| \, d\theta' + (1 - \alpha) \int |a - b| \, d\theta' \\
= & \int |a - b| \, d\theta' \, ,
\end{aligned}$$

where we invoked Lemmas 1 and 2 for deriving the last inequality. □

Proof of Theorem 1 The proof can be conducted by imitating verbatim the proof of Theorem 9.4.1 in Nishimura and Ozaki (2017, p.156) except that Lemma 2 should be replaced by Lemma 3. □

A.3 Proof of Corollary 2

By substituting (9) into Corollary 1, we have

$$\begin{aligned} R^* &= c + \frac{\beta}{1-\beta} \int_{R^*}^{\infty} \hat{\theta}\left(\{w\,|w \geq x\}\right) dx \\ &= c + \frac{\beta}{1-\beta} \int_{R^*}^{\infty} \Big(\alpha\theta'\left(\{w\,|w \geq x\}\right) + (1-\alpha)\theta\left(\{w\,|w \geq x\}\right) \Big) dx \\ &= c + \frac{\beta}{1-\beta} \int_{R^*}^{\infty} \Big(\alpha \left(1 - \theta\left(\{w\,|w < x\}\right)\right) \\ &\qquad\qquad + (1-\alpha)\theta\left(\{w\,|w \geq x\}\right) \Big) dx\,, \end{aligned}$$

where the third line holds by definition of the conjugate of θ. Rearranging the terms completes the proof. □

A.4 Proof of Proposition 1

We prove only (ii) and (iii).

Proof of (i). For the case of $\alpha = 0$, the statemet is easily proven as in Nishimura and Ozaki (2004, Theorem 2). Since V^* is continuous with respect to α, $R^*(w|\alpha, \theta^1) - R^*(w|\alpha, \theta^2)$ is also continuous. By utilizing this fact, we can extend the result for $\alpha = 0$ to the cases for positive α that is sufficiently close to zero.

Proof of (ii). For each i, let B^i and V^{i*} be the operator and the value function corresponding to θ^i. Then, for any V and w,

$$\begin{aligned} B^1V(w) =& \max\left\{ \frac{w}{1-\beta}, c + \beta \int_W V(w') {\theta^{1}}'_w(dw') \right\} \\ \leq& \max\left\{ \frac{w}{1-\beta}, c + \beta \int_W V(w') {\theta^{2}}'_w(dw') \right\} \\ =& B^2V(w). \end{aligned}$$

The second inequality comes from the fact that θ'_w is concave. Hence, by this, the fact that $B^iV' \geq B^iV$ whenever $V' \geq V$, and that $V^{i*} = \lim_{n\to\infty}(B^i)^n V^-$, it follows that

$V^{1*} \leq V^{2*}$. Finally, we conclude that for any w,

$$\begin{aligned}\frac{R^*(w|1,\theta^1)}{1-\beta} =& c+\beta\int_W V^{1*}(w')\theta^{1'}{}_w(dw') \\ \leq& c+\beta\int_W V^{2*}(w')\theta^{1'}{}_w(dw') \\ <& c+\beta\int_W V^{2*}(w')\theta^{2'}{}_w(dw') \\ =& \frac{R^*(w|1,\theta^2)}{1-\beta} \ .\end{aligned}$$

As in (i), this can be extended to the cases for α that is sufficiently close to one.

Proof of (iii). Fix w.

First, we prove that $R^*(w|\alpha,\theta^1) - R^*(w|\alpha,\theta^2)$ is weakly decreasing in α. Because θ^1 is additive, $R^*(w|\alpha,\theta^1)$ is independent of α. Furthermore, from Proposition 4, $R^*(w|\alpha,\theta^1)$ is weakly increasing in α. By combining these two properties, it is shown that $R^*(w|\alpha,\theta^1) - R^*(w|\alpha,\theta^2)$ is weakly decreasing in α.

Second, from (i) and (ii), $R^*(w|1,\theta^1) - R^*(w|1,\theta^2) < 0$, whereas $R^*(w|0,\theta^1) - R^*(w|0,\theta^2) > 0$.

Lastly, V^* is continuous with respect to α.

By combining these results, we obtain the proposition. □

A.5 Proof of Proposition 2

Lemma 4 *Let $\hat{\theta}$ be a neo-additive capacity. Then, the reservation wage R^* will be constant and is given by the solution R^* to the next equation:*

$$\begin{aligned}g(R) :=& (1-\beta)c - R + \beta E_\pi(w) + \beta\int_0^R F(w)dw \\ &- \frac{\varepsilon}{1-\varepsilon}\left[(1-(1-\alpha)\beta)R - \alpha\beta B - (1-\beta)c\right] = 0 \ . \qquad (20)\end{aligned}$$

The solution R^* is unique and $R^* \in (0, B)$.

Proof By rewriting (15) based on the definition of a neo-additive capacity, we have

$$
\begin{aligned}
&R = c + \frac{\beta}{1-\beta}\int_R^B (1-\varepsilon)(1-F(x))dx \\
&+ \frac{\beta}{1-\beta}\int_R^\infty \left[\alpha\Big(1-(1-\varepsilon)F(x)-(1-\varepsilon)(1-F(x))\Big)\right]dx \\
&\Leftrightarrow R = c + \frac{\beta}{1-\beta}(1-\varepsilon)\int_R^B (1-F(x))dx + \frac{\beta}{1-\beta}\varepsilon\alpha\int_R^B dx \\
&\Leftrightarrow (1-\beta)R = (1-\beta)c + \beta(1-\varepsilon)\int_R^B (1-F(x))dx + \beta\varepsilon\alpha(B-R) \\
&\Leftrightarrow -\frac{\varepsilon}{1-\varepsilon}\left[(1-(1-\alpha)\beta)R - \alpha\beta B - (1-\beta)c\right] \\
&+\beta\int_R^B (1-F(x))dx + (1-\beta)c - (1-\beta)R = 0 \ . \qquad (21)
\end{aligned}
$$

Here,

$$
\begin{aligned}
\int_R^B (1-F(x))dx &= B - R - \int_R^B F(x)dx \\
&= B - R - \left[\int_0^B F(x)dx - \int_0^R F(x)dx\right] \\
&= B - R - \left[BF(B) - E_\pi(w) - \int_0^R F(x)dx\right] \\
&= -R + E_\pi(w) + \int_0^R F(x)dx \ . \qquad (22)
\end{aligned}
$$

The third equality comes from the integration by parts:

$$
\int_0^B F(w)dw = BF(B) - \int_0^B w dF(w) = B - E_\pi(w) \ . \qquad (23)
$$

Substituting (22) into (21), we have

$$
\begin{aligned}
&-\frac{\varepsilon}{1-\varepsilon}\left[(1-(1-\alpha)\beta)R - \alpha\beta B - (1-\beta)c\right] \\
&+\beta\int_0^R F(x)dx + \beta E_\pi(w) + (1-\beta)c - R = 0 \ . \qquad (24)
\end{aligned}
$$

The left-hand side is $g(R)$. Hence, $g(R^*) = 0$ is equivalent to (15).

For the second part,

$$
g'(R) = -(1-(1-\alpha)\beta) + \beta F(R) - 1 < 0 \ ;
$$

$$
g(0) = \frac{\varepsilon}{1-\varepsilon}\left[\alpha\beta B + (1-\beta)c\right] + \beta E_\pi(w) + (1-\beta)c > 0 \ ;
$$

$$g(B) = -\left(\frac{\varepsilon}{1-\varepsilon}+1\right)(1-\beta)(B-c) < 0 \ .$$

Note that for the calculation of $g(B)$, we use (23). Hence, we have the second part. $\square$

Given this lemma, we prove the proposition.

Proof of Proposition 2 Since (15) is continuous with respect to δ, R^*_{neo} is equal to the value of R^* when $\delta = 0$. Hence, it suffices to analyze the case where $\delta = 0$. When $\delta = 0$,

$$\frac{\partial g}{\partial \varepsilon} = -\left(\frac{\varepsilon}{1-\varepsilon}\right)' [(1-(1-\alpha)\beta)R - \alpha\beta B - (1-\beta)c] \ .$$

Hence, the comparative statics depends on the sign of $\frac{\partial \tilde{g}}{\partial \varepsilon}$ at $R = R^*$. That is,

$$\text{sign}\left(\frac{\partial R^*}{\partial \varepsilon}\right) = -\text{sign}\left((1-(1-\alpha)\beta)R^* - \alpha\beta B - (1-\beta)c\right) \ .$$

Therefore, it suffices to check the sign of $(1-(1-\alpha)\beta)R^* - \alpha\beta B - (1-\beta)c$.

Here,

$$(1-(1-\alpha)\beta)R^* - \alpha\beta B - (1-\beta)c > 0 \Leftrightarrow R^* > \frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta} \ ,$$

which holds if and only if

$$g\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) > 0 \ . \tag{25}$$

This equivalence comes from the fact that g is decreasing in R. Hence, when $R = \frac{\alpha\beta B+(1-\beta)c}{1-(1-\alpha)\beta}$:

$$g(R) > 0 \Leftrightarrow (1-\beta)c - R + \beta E_\pi(w) + \beta \int_0^R F(w)dw > 0 \ .$$

Using this, (25) is finally rewritten as follows:

$$\begin{aligned}(25) \Leftrightarrow k(\beta) := &-\frac{\alpha}{1-(1-\alpha)\beta}B - \frac{1-\beta}{1-(1-\alpha)\beta}(1-\alpha)c + E_\pi(w) \\ &+ \int_0^{\frac{\alpha\beta B+(1-\beta)c}{1-(1-\alpha)\beta}} F(w)dw > 0 \ .\end{aligned}$$

Since

$$\text{sign}\left(\frac{\partial R^*}{\partial \varepsilon}\right) = -\text{sign}(k(\beta)),$$

from now on, we analyze the sign of $k(\beta)$.

First, it is easily verified that

$$k(0) = E_\pi(w) - \alpha B - (1-\alpha)c + \int_0^c F(w)dw \ ;$$

$$k(1) = E_\pi(w) + B - E_\pi(w) - B = 0 \ .$$

Note that for the calculation of $k(1)$, we use the fact that from the integration by parts:

$$\int_0^B F(w)dw = BF(B) - \int_0^B wdF(w) = B - E_\pi(w) \ .$$

Furthermore,

$$\begin{aligned} k'(\beta) =& \frac{\partial}{\partial\beta}\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) F\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) \\ &- \frac{\partial}{\partial\beta}\left(\frac{\alpha}{1-(1-\alpha)\beta}B + \frac{1-\beta}{1-(1-\alpha)\beta}(1-\alpha)c\right) \\ =& \frac{\alpha(B-c)}{(1-(1-\alpha)\beta)^2} F\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) - \frac{(1-\alpha)\alpha(B-c)}{(1-(1-\alpha)\beta)^2} \\ =& \frac{\alpha(B-c)}{(1-(1-\alpha)\beta)^2}\left[F\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) - (1-\alpha)\right] \ . \end{aligned}$$

Hence,

$$k'(\beta) \gtreqless 0 \Leftrightarrow F\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) \gtreqless (1-\alpha) \ . \tag{26}$$

Since

$$\frac{\partial}{\partial\beta}\left(\frac{\alpha\beta B + (1-\beta)c}{1-(1-\alpha)\beta}\right) = \frac{\alpha(B-c)}{(1-(1-\alpha)\beta)^2} > 0$$

holds, (26) implies that there exists a threshold value $\bar{\beta} \in [0,1)$ such that $k'(\beta) > 0$ if and only if $\beta > \bar{\beta}$; that is, k is a U shape.

Proof of (i). When $\alpha \geq 1 - (B - E_\pi(w) - \int_0^c F(w)dw)/(B-c)$, $k(0) \leq 0$. Furthermore, as we have shown, $k(1) = 0$ and k is a U shape. Hence, for any β, $k(\beta) \leq 0$, which completes the proof.

Proof of (ii). When $\alpha < 1 - (B - E_\pi(w) - \int_0^c F(w)dw)/(B-c)$, $k(0) > 0$, while $k(1) = 0$. Furthermore, k is a U shape. Hence, there exists $\bar{\beta} \in (0,1)$ such that if and only if $\beta \in (\bar{\beta}, 1)$, $g(\beta) > 0$, implying the first part.

We next prove the latter part. By applying the implicit function theorem, we have

$$\frac{d\bar{\beta}}{dc} = -\frac{\frac{\partial k}{\partial c}}{\frac{\partial k}{\partial \beta}} \ .$$

Since $k(0) > 0$, $k(1) = 0$, and k is a U shape, $k'(\bar{\beta}) < 0$ i.e., the denominator is negative.

In addition,

$$\frac{\partial k}{\partial c} = \frac{1-\beta}{1-\alpha\beta}\left(F\left(\frac{(1-\alpha)\beta B + (1-\beta)c}{1-\alpha\beta}\right) - \alpha\right) ,$$

which is negative when $\beta = \bar{\beta}$. This is because $k'(\bar{\beta}) < 0$. Hence, $\frac{d\bar{\beta}}{dc} < 0$. □

A.6 Proof of Proposition 3

We first characterize the optimal stopping rule in a specific manner. Suppose that the worker rejects the current offer and continues the search. In this case, the payoff is:

$$\begin{aligned} & c + \beta(1-\varepsilon)\int_W u(w)\,\pi(dw) \\ & + \beta\varepsilon\left[\alpha \max_{p\in\mathcal{M}(\pi,\delta)}\int_W u(w)\,p(dw) + (1-\alpha)\min_{p\in\mathcal{M}(\pi,\delta)}\int_W u(w)\,p(dw)\right] , \end{aligned} \tag{27}$$

where

$$u(w) := \begin{cases} \frac{R^*}{1-\beta} \ (w < R^*) \\ \frac{w}{1-\beta} \ (w \geq R^*) \end{cases} .$$

To explicitly rewrite the third-term in (27), we first define two probability measures:

Definition 6 *(i). The following probability measure is denoted by* $\bar{\pi}$*:*

$$\bar{\pi}(\{\omega \geq x\}) := \min\left\{\frac{\pi(\{w \geq x\})}{\delta}, 1\right\} .$$

We also define $\bar{x}$ *as* x *satisfying* $\pi(\{w \geq x\}) = \delta$.

(ii). The following probability measure is denoted by $\underline{\pi}$*:*

$$\underline{\pi}(\{\omega \leq x\}) := \left\{\frac{\pi(\{w \leq x\})}{\delta}, 1\right\} .$$

We also define $\underline{x}$ *as* x *satisfying* $\pi(\{w \leq x\}) = \delta$.

From now on, we assume that δ is sufficiently small so that $\underline{x} < c$.

These two probability measures have the following property.

Lemma 5 $\bar{\pi}$ *first-order stochastically dominates any other probability charges contained in* $\mathcal{M}(\pi,\delta)$*, whereas* $\underline{\pi}$ *is first-order stochastically dominated by any other probability charges contained in* $\mathcal{M}(\pi,\delta)$*.*

Proof The proof is straightforward, and thus we omit it. □

Hence, $\max_{p\in\mathcal{M}(\pi,\delta)}\int_W u(w)\,p(dw)$ should be evaluated by using $\bar{\pi}$, whereas $\min_{p\in\mathcal{M}(\pi,\delta)}\int_W u(w)\,p(dw$ should be evaluated by using $\underline{\pi}$. Therefore,

$$\max_{p\in\mathcal{M}(\pi,\delta)}\int_W u(w)\,p(dw)=\frac{1}{1-\beta}B(R^*)\ ,$$

where

$$B(R^*):=\int_{\min\{\bar{x},R^*\}}^{R^*} R^* d\bar{\pi}+\int_{R^*}^{\infty} w d\bar{\pi}\ .$$

Furthermore, if $\underline{x}<c$, $R^*>\underline{x}$, and thus

$$\min_{p\in\mathcal{M}(\pi,\delta)}\int_W u(w)\,p(dw)=\frac{1}{1-\beta}R^*\ .$$

By using these notations, we can obtain the lemma corresponding to Lemma A.6.

Lemma 6 *Let $\hat{\theta}$ be a $\delta-$approximation of a neo-additive capacity. Suppose also that δ is sufficiently small so that $\underline{x}<c$. Then, the reservation wage R^* will be constant and is given by the solution R^* to the next equation:*

$$\begin{aligned}g(R):=&(1-\beta)c-R+\beta E_\pi(w)+\beta\int_0^R F(w)dw\\&-\frac{\varepsilon}{1-\varepsilon}\left[(1-(1-\alpha)\beta)R-\alpha\beta B(R)-(1-\beta)c\right]=0.\end{aligned}$$

The solution R^* is unique and $R^*\in(c,\infty)$.

Proof The proof is similar to that of Lemma . Thus, we omit it. □

We achieved the complete characterization of the optimal stopping rule. Before the proof of the proposition, we obtain another lemma.

Lemma 7 *Let*

$$h(R):=(1-(1-\alpha)\beta)R-\alpha\beta B(R)-(1-\beta)c\ .$$

There exists a unique $\bar{R}\in W$ such that $h(R)>0$ if and only if $R>\bar{R}$.

Proof First,

$$\frac{\partial h}{\partial R}=1-(1-\alpha)\beta-\alpha\beta\frac{\partial B}{\partial R},$$

which is positive because $\frac{\partial B}{\partial R}\leq 1$ by its construction. Furthermore, it is easily verified that $h(0)<0$, whereas $h(R)>0$ as $R\to\infty$. Thus, we obtain the lemma. □

Using these lemmas, we finally prove the proposition.

Proof of Proposition 3 As in the proof of Proposition 2,

$$\text{sign}\left(\frac{\partial R^*}{\partial \varepsilon}\right) = -\text{sign}\left((1-(1-\alpha)\beta)R^* - \alpha\beta B(R^*) - (1-\beta)c\right).$$

Therefore, it suffices to check the sign of $(1-(1-\alpha)\beta)R^* - \alpha\beta B(R^*) - (1-\beta)c$. From Lemma 7,

$$(1-(1-\alpha)\beta)R^* - \alpha\beta B - (1-\beta)c > 0 \Leftrightarrow R^* > \bar{R},$$

which holds if and only if $g(\bar{R}) > 0$. Hence, as in the proof of Proposition 2,

$$\begin{aligned} g(\bar{R}) > 0 \Leftrightarrow k(\beta) := &-\frac{\alpha}{1-(1-\alpha)\beta}B(\bar{R}) - \frac{1-\beta}{1-(1-\alpha)\beta}(1-\alpha)c + E_\pi(w) \\ &+ \int_0^{\frac{\alpha\beta B(\bar{R})+(1-\beta)c}{1-(1-\alpha)\beta}} F(w)dw > 0. \end{aligned}$$

Note that $B(\bar{R})$ depends on β, because $\bar{R}$ depends on β. Hence, this is different from $k(\beta)$ defined in the proof of Proposition 2.

Since

$$\text{sign}\left(\frac{\partial R^*}{\partial \varepsilon}\right) = -\text{sign}(k(\beta)),$$

our goal is to show that $g(\beta) < 0$ for sufficiently large β.

Here, define

$$\underline{B} := \int_{\bar{x}}^{\infty} w d\pi \ .$$

By its construction, $\underline{B}$ is independent of β and less than $B(\bar{R})$. In addition, define l by replacing $B(\bar{R})$ in $k(\beta)$ with $\underline{R}$:

$$l(\beta) := -\frac{\alpha}{1-(1-\alpha)\beta}\underline{B} - \frac{1-\beta}{1-(1-\alpha)\beta}(1-\alpha)c + E_\pi(w) + \int_0^{\frac{\alpha\beta\underline{B}+(1-\beta)c}{1-(1-\alpha)\beta}} F(w)dw \ .$$

Since $k(\beta)$ is decreasing in $B(R)$, $l(\beta) > k(\beta)$ holds. Thus, it suffices to prove that $l(\beta)$ is negative for sufficiently large β. This is easily verified because $l(\beta)$ is the same as $k(\beta)$ in the proof of Proposition 2. □

B The Effect of Higher Optimism

In the main analysis, we have focused on the comparative statics with respect to the degree of ambiguity. Another interest would be the effect of higher optimism (i.e., higher α). The following proposition shows that less optimism reduces the reservation wage and encourages the worker to accept a wage offer. As α decreases, the worker becomes more pessimistic about future wage offers. As a result, the worker becomes more likely to accept the current offer.[22]

Proposition 4 *For all w, $R^*(w)$ is weakly increasing in α.*

Proof Consider α^1 and α^2 such that $\alpha^1 < \alpha^2$. For each i, let B^i, V^{i*}, and R^{*i} be the operator, the value function, and the reservation wage corresponding to α^i. Then, for any V and w,

$$\begin{aligned} B^1V(w) =& \max\left\{\frac{w}{1-\beta}, c+\beta\left[(1-\alpha^1)\int_W V(w')\theta_w(dw') + \alpha^1\int_W V(w')\theta'_w(dw')\right]\right\} \\ \leq& \max\left\{\frac{w}{1-\beta}, c+\beta\left[(1-\alpha^2)\int_W V(w')\theta_w(dw') + \alpha^2\int_W V(w')\theta'_w(dw')\right]\right\} \\ =& B^2V(w). \end{aligned}$$

Note that the second inequality holds because

$$\int_W V(w')\theta_w(dw') \leq \int_W V(w')\theta'_w(dw'),$$

which is implied by the convexity of θ_w. Hence, it follows that $V^{1*} \leq V^{2*}$. Finally, we conclude that for any w,

$$\begin{aligned} \frac{R^{*1}(w)}{1-\beta} =& c+\beta\left[\alpha^1\int_W V^{1*}(w')\theta_w(dw') + (1-\alpha^1)\int_W V^{1*}(w')\theta'_w(dw')\right] \\ \leq& c+\beta\left[\alpha^1\int_W V^{2*}(w')\theta_w(dw') + (1-\alpha^1)\int_W V^{2*}(w')\theta'_w(dw')\right] \\ \leq& c+\beta\left[\alpha^2\int_W V^{2*}(w')\theta_w(dw') + (1-\alpha^2)\int_W V^{2*}(w')\theta'_w(dw')\right] \\ =& \frac{R^{*2}(w)}{1-\beta}. \end{aligned}$$

Hence, we have the proposition. □

C Basic Properties of CEU

In CEU, not a probability measure but a probability capacity is used.

Definition 7 *A probability capacity on $(W, \mathscr{B}_W)$ is a function θ: $\mathscr{B}_W \to [0,1]$ which satisfies the followings: (a) $\theta(\emptyset) = 0$; (b) $\theta(W) = 1$; (c) for any $C, D \in \mathscr{B}_W$, if $C \subseteq D$, then $\theta(C) \leq \theta(D)$.*

[22] In an extension, Miao and Wang (2011) analyze the one-sided search problem by using the smooth ambiguity model, where they show that higher ambiguity-aversion reduces the reservation wage. Besides, Huang and Yu (2021) analyze a continuous-time optimal stopping problem, which has a different structure. By adopting the non-iterated $\alpha-$MEU (see Appendix D), they show that the more ambiguity-averse, the more eager to stop. Our result is consistent with these findings.

(c) is called *monotonicity*. In a probability measure, a probability satisfies $\sigma-$additivity rather than (c).

By using this capacity, we calculate the expected utility based on not the Riemann integral but the Choquet integral. In the following, we present several properties of the Choquet integral. For the details of these properties, see, for example, Nishimura and Ozaki (2017).

Lemma 8 *Let θ be a probability capacity.*

(i). (Monotonicity). For $u, v \in B(W, \mathscr{B}_W)$,

$$u \leq v \Rightarrow \int u \, d\theta \leq \int v \, d\theta.$$

(ii). (Positive homogeneity). For $u \in B(W, \mathscr{B}_W)$, $a \in \mathbb{R}$, and $b \in \mathbb{R}_+$,

$$\int (a + bu) \, d\theta = a + b \int u \, d\theta.$$

(iii). For $u \in B(W, \mathscr{B}_W)$,

$$\int u \, d\theta' = - \int -u \, d\theta.$$

(iv). (Co-monotonic additivity). If $u, v \in B(W, \mathscr{B}_W)$ are comonotonic,

$$\int (u + v) \, d\theta = \int u \, d\theta + \int v \, d\theta.$$

When θ is convex, we additionally obtain the following properties.

Lemma 9 *Let θ be a convex capacity.*

(i). For $u \in B(W, \mathscr{B}_W)$,

$$\int u \, d\theta = \min \left\{ \int u \ dp | p \in \text{core}(\theta) \right\}.$$

(ii). (Super-additivity). For $u, v \in B(W, \mathscr{B}_W)$,

$$\int (u + v) \, d\theta \geq \int u \, d\theta + \int v \, d\theta.$$

Note that the inequality is reversed when θ is concave.

(iii). For $u \in B(W, \mathscr{B}_W)$,

$$\int u \, d\theta \leq \int u \, d\theta'.$$

D Iterated Utility vs. Non-iterated Utility

Consider a probability measure on $(W_\infty, \mathscr{B}_{W_\infty})$ of the form: $p = p_0 \times p_1 \times p_2 \times \cdots$, where p_0 is a probability measure in $\text{core}(\theta)$. p_t is a measurable selection of $w \mapsto \text{core}(\theta_w)$, that is, p_t is a stochastic kernel, for each $t \geq 1$, and the product measure is constructed in a standard manner. Denote the set of all such measures by $\mathcal{P}_{w_0}$. Note that in the construction of the product, any combination of a stochastic kernel is permitted. Though we have adopted the iterated utility in our analysis, another candidate could be

$$\alpha \max_{p \in \mathcal{P}_{w_0}} E^p \left[\sum_{t=0}^{\infty} \beta^t y_t \right] + (1-\alpha) \min_{p \in \mathcal{P}_{w_0}} E^p \left[\sum_{t=0}^{\infty} \beta^t y_t \right],$$

which we will refer to as the non-iterated $\alpha-$MEU.

In the expected utility framework, the iterated utility and the non-iterated utility always coincide with each other because the law of iterated expectation holds. However, in the presence of ambiguity, this does not necessarily hold. This problem is severe especially when α is strictly between zero and one. When α is either zero or one, there is a well-known condition under which both utilities are the same, which is called the rectangularity property (Epstein and Schneider 2003a). However, when $\alpha \in (0, 1)$, even the rectangurality property does not guarantee the law of iterated expectation, and thus these two utility concepts do not coincide with each other (see Schroder (2011) and Beissner, Lin, and Riedel (2020) for the further discussions). Indeed, the set of probability measures constructed above satisfies the rectangularity property, but this does not resolve the problem when $\alpha \in (0, 1)$. Hence, a choice must be made between two obvious approaches to model preferences.

When the approach based on the non-iterated utility is adopted, though simple, a dynamic inconsistency problem arises because the law of iterated expectation does not hold. Consequently, it has two drawbacks of modeling a worker's preferences. First, because dynamic consistency is not guaranteed, we cannot exploit the dynamic programming technique, which makes analysis quite difficult. Second, such a decision maker is naive in the sense that she fails to foresee potential dynamic inconsistency. Hence, in the present study, we take the approach based on the iterated utility. In this approach, the worker is rational in the sense that dynamic consistency holds. Furthermore, such dynamic consistency enables us to exploit dynamic programming technique. This approach is also adopted by Beissner, Lin, and Riedel (2020) in a continuous-time framework. Whether these two different approaches yield different results is a challenge for future researches.

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