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Law invariant risk measures have the Fatou property

Elyès Jouini[1], Walter Schachermayer[2] and Nizar Touzi[3]

[1] Université Paris Dauphine and CEREMADE, Place du Maréchal de Lattre de Tassigny, F-75775 Paris Cedex 16, France

[2] Vienna University of Technology, Wiedner Hauptstrasse 8-10/105, A-1040 Wien, Austria and Université Paris Dauphine, Place du Maréchal de Lattre de Tassigny, F-75775 Paris Cedex 16, France

[3] CREST, Laboratoire de Finance et Assurance, 15 Bd Gabriel Péri, F-92245 Malakoff Cedex, France and Imperial College London, Tanaka Business School.

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Abstract. S. Kusuoka [?, Theorem 4] gave an interesting dual characterization of law invariant coherent risk measures, satisfying the Fatou property. The latter property was introduced by F. Delbaen [?]. In the present note we extend Kusuoka's characterization in two directions, the first one being rather standard, while the second one is somewhat surprising. Firstly we generalize – similarly as M. Fritelli and E. Rossaza Gianin [?] – from the notion of coherent risk measures to the more general notion of convex risk measures as introduced by H. Föllmer and A. Schied [?]. Secondly – and more importantly – we show that the hypothesis of Fatou property may actually be dropped as it is automatically implied by the hypothesis of law invariance.
We also introduce the notion of the Lebesgue property of a convex risk measure, where the inequality in the definition of the Fatou property is replaced by an equality, and give some dual characterizations of this property.

Key words: law-invariance, cash-invariance, Fatou and Lebesgue properties