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Smooth feasible solutions to a dual Monge – Kantorovich problem with applications to best approximation and utility theory in mathematical economics

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Abstract. Given a (closed or open) subset X in $\mathbb{R}^n$, which is stable with respect to shifts in positive directions, we consider inequalities $u(x) - u(y) \leq c(x,y)$, $x,y \in X$, and for a wide class of functions c on $X \times X$, derive a smooth solution to these inequalities from a Lebesgue measurable one. Applications are given to a best approximation problem and to several problems of mathematical economics relating to preferences that admit smooth (or Lipschitz continuous) utility functions, smooth-utility-rational choice, and smooth representations of interval orders.

Key words: axioms of revealed preference, dual Monge – Kantorovich problem, exact solutions to a best approximation problem, Lipschitz continuous and smooth utility functions, preference, representations of interval orders, smooth-utility-rational choice, strengthened acyclicity assumption orders