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Variational analysis and mathematical economics 1: Subdifferential calculus and the second theorem of welfare economics

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Abstract. The paper consists of two parts. The first is devoted to a general subdifferential theory based on an axiomatic approach. Along with the list of " axioms " which summarizes properties shared by all major subdifferentials studied in variational analysis, we also consider four " optional " properties which specific subdifferentials may or may not have, such as trustworthiness, robustness, tightness (validity of a certain fuzzy subdifferential inequality) and geometric compatibility (connection between subdifferentials and normal cones). The concluding result says that the approximate G-subdifferential is the only subdifferential that has the four properties on all Banach space. The second part is devoted to application of the general subdifferential theory to a model of welfare economics with a Banach commodity space. Here we begin with subdifferential characterization of nonconvex separation property in general and also for a special case of one of the sets being a shifted kernel of a linear epimorphism, and then apply the results to characterize Pareto and weak Pareto optimal allocations in welfare economics. The final result is a strengthening of earlier versions of the second welfare theorem due to Khan-Vohra, Cornet, Joffre and Mordukhovich. In particular, a weaker and more symmetric version of Cornet 's qualification condition appears in the characterization of Pareto optimality.

Key words: general subdifferential theory, nonconvex separation, Pareto optimality, qualification conditions, welfare economics