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On preference relations that admit smooth utility functions

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Abstract. We prove the existence of smooth utility functions for a class of preferences (closed preorders) on a subset X in $\mathbb{R}^n$ which satisfies $X = X + \mathbb{R}^n_+$. This class of preferences is given by the condition that adding one and the same positive vector to each of two comparable alternatives cannot affect the preference relation between them. Moreover, some its subclass consisting of total preferences admits linear utility functions. Also, we prove the existence of universal smooth utilities for preferences depending on a parameter. Our approach relies on our earlier results on continuous utilities for closed (non-total) preorders on metrizable spaces along with a particular device that enable to pass from a continuous utility to a smooth one.

Key words: closed preorder, utility function, stability with respect to shifts in positive directions, smooth utility function, linear utility, universal utility theorem