

Cyclical Shocks in a Model of Equilibrium Unemployment

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January 2003

Abstract

The paper shows a model which displays a procyclical movement of job creation as a rational expectations equilibrium. This conforms with a stylized fact that vacancy and unemployment shows anticlockwise movement around Beveridge curve over business cycles.

Keywords: Aggregate shocks; Equilibrium unemployment model; Rational expectations equilibrium.

JEL: E24; E32; J64

1 Introduction

It is observed that vacancies of job increase and unemployment decreases during boom and that vice versa during recession. In other words, there is a stylized fact that vacancies and unemployment trace anticlockwise loops around the Beveridge curve over the business cycles. The purpose of the paper is to show that this observation can be explained in a

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rational expectations equilibrium unemployment model with stochastic arrival of aggregate shocks.

What we gain from doing this analysis are the followings. (1) Pissarides (2000, Ch 1. p.32) explained this stylized fact as a comparative dynamics result in terms of once and for all change in aggregate productivity in a simple equilibrium unemployment model. It implies that the stylized fact is explained with cyclical arrivals of unexpected shocks. However, our analysis suggests that the same result can be attained as a rational expectations equilibrium path of the same model with cyclical aggregate shocks that arrive stochastically. (2) The analysis of the model shows that one must always consider the out of steady state dynamics of the model when we introduce cyclical shocks into the model. Mortensen and Pissarides (1994) look at the effect of cyclical aggregate shocks on job destruction and creation rates in a model of equilibrium unemployment with endogenous job destruction. However, their analysis misses the out of steady state dynamics of the model. Although the model in this paper does not endogenously determine the job destruction rate as in their model, it makes a first step toward completing their analysis.

2 The Model

The model is an extension of that shown in Ch. 1 of Pissarides (2001). Time is continuous and denoted by t . Each firm has a job that can be either filled and producing or vacant and searching for a worker. When a firm with a vacant job and a worker meet and start producing, it is said that job creation takes place. On the other hand, it is said that job destruction takes place when a filled job separates and stop producing.

There are continuum of workers whose population equals to unity. The workers can be either unemployed and searching for a job or employed and producing. It is assumed that employed workers can not search for a job. Wages are set so as to share the surplus from a job match in a fixed proportion.

Each job is characterized by a technology which produces a unit of a differentiated product at each moment of time whose price is p^i where i denotes the state in which the economy is in. The price p^i is fixed and common to all filled jobs. If the economy is in boom $i = B$ and if it is in recession $i = R$. We assume that $p^B > p^R$. The process of the change in state of the economy is Poisson with arrival rate μ . This is the new ingredient added to the original model of Pissarides and how cyclical aggregate shocks is described in our model.

The rate at which vacant jobs and unemployed workers meet is determined by the homogeneous-of-degree-one matching function $m(u_t^i, v_t^i)$, where v_t^i and u_t^i represent the number of vacancies and unemployed workers respectively. No job seeker turns down a vacancy, so the transition rate for vacancies is $q(u_t^i/v_t^i) = m(v_t^i, u_t^i)/v_t^i = m(1, u_t^i/v_t^i)$, with $q'(u_t^i/v_t^i) < 0$ and elasticity strictly between -1 and 0. Furthermore, we set $\theta_t^i \equiv u_t^i/v_t^i$.

The process that job destruction takes place for a filled job is also Poisson with arrival rate λ which is constant and common to all filled jobs.

It is assumed that vacancies cost c per unit time and that jobs are created at rate $q(\theta_t^i)$.

Hence we have,

$$rV_t^i = -c + q(\theta_t^i)[J_t^i - V_t^i] + \mu[V_t^j - V_t^i] + \dot{V}_t^i \quad \text{for } i, j \in \{B, R\} \quad \text{and } i \neq j, \quad (1)$$

where V_t^i and J_t^i are respectively the asset values of a vacancy and a filled job at time t , and the discount rate is denoted by r . Dot indicates the time derivative for the function in concern. Jobs are created until all rents are exhausted so that we have,

$$V_t^i = 0 \quad \text{and} \quad \dot{V}_t^i = 0 \quad \text{for } i \in \{B, R\}. \quad (2)$$

The condition (2) together with equation (1) implies,

$$J_t^i = \frac{c}{q(\theta_t^i)} \quad (3)$$

The value of a filled job is given by

$$rJ_t^i = p^i - w_t^i + \lambda[V_t^i - J_t^i] + \mu[J_t^j - J_t^i] + \dot{J}_t^i \text{ for } i \in \{B, R\} \text{ and } i \neq j, \quad (4)$$

where w_t^i is a state contingent wage. The wage is set to split the surplus of matched job in fixed proportion so that we have,

$$W_t^i - U_t^i = \beta[J_t^i + W_t^i - U_t^i] \text{ for } i \in \{B, R\}, \quad (5)$$

where W_t^i and U_t^i are values of a worker who is being employed and unemployed respectively. The parameter β is a constant between 0 and 1. The left hand side of the equation is a worker's gain from being matched and the right hand side is the worker's share on surplus created by the match.

By substituting (1) and (2) into (4) we obtain,

$$\dot{J}_t^i = (r + \lambda)J_t^i - \mu[J_t^j - J_t^i] - (p^i - w_t^i) \text{ for } i \in \{B, R\} \text{ and } i \neq j. \quad (6)$$

The value of employed worker is defined by,

$$rW_t^i = w_t^i + \lambda[U_t^i - W_t^i] + \mu[W_t^j - W_t^i] + \dot{W}_t^i \text{ for } i \in \{B, R\} \text{ and } i \neq j. \quad (7)$$

Moreover, the value of unemployed worker is defined by,

$$rU_t^i = z + \theta_{iQ}^i(\theta_i^i)[W_t^i - U_t^i] + \mu[U_t^j - U_t^i] + \dot{U}_t^i \text{ for } i \in \{B, R\} \text{ and } i \neq j, \quad (8)$$

where z is the exogenous value of leisure or unemployment income and $\theta_t^i q(\theta_t^i)$ is the rate for which an unemployed worker meets a vacant job.

Noting that condition (5) holds over time (i.e., it holds also in terms of time derivative), and using equations (3), (5), (6), (7), and (8), we obtain an expression for wage w_t^i ,

$$w_t^i = \beta(p^i + \theta_t^i c) + (1 - \beta)z \text{ for } i \in \{B, R\}. \quad (9)$$

Furthermore, differentiating (3) with respect to time and substituting (3), (6) and (9) into it, we obtain the law of motion for θ_t^i ,

$$\dot{\theta}_t^i = -\frac{q(\theta_t^i)^2}{cq'(\theta_t^i)} \left[(r + \lambda + \mu) \frac{c}{q(\theta_t^i)} - p^i + (1 - \beta)z + \beta(p^i + c\theta_t^i) - \mu \frac{c}{q(\theta_t^i)} \right] \text{ for } i \in \{B, R\}. \quad (10)$$

Finally, the flow of unemployment can be defined as,

$$\dot{u}_t^i = \lambda(1 - u_t^i) - \theta_t^i q(\theta_t^i) u_t^i \text{ for } i \in \{B, R\}. \quad (11)$$

The equilibrium path of the economy is described by the system of differential equations (10) and (11) in terms of variables θ_t^i and u_t^i .

It can be readily seen from (10) that θ_t^i that satisfies $\dot{\theta}_t^i = 0$ is uniquely determined.

Let θ^{i*} be the value of θ_t^i which makes the right hand side bracket of (10) equals to zero.

We claim that $\theta^{B*} > \theta^{R*}$. It can be seen as follows. Set the right hand side bracket of (10)

equal to zero and equating them, we get

$$(r + \lambda + 2\mu)\frac{c}{q(\theta^{B*})} - (1 - \beta)p^B + \beta c\theta^{B*} = (r + \lambda + 2\mu)\frac{c}{q(\theta^{R*})} - (1 - \beta)p^R + \beta c\theta^{R*},$$

which can be rearranged as

$$(1 - \beta)(p^B - p^R) + \beta c(\theta^{R*} - \theta^{B*}) - (r + \lambda + 2\mu)c \left[\frac{q(\theta^{R*}) - q(\theta^{B*})}{q(\theta^{R*})q(\theta^{B*})} \right] = 0.$$

Since $q' < 0$, above equation implies that $\theta^{B*} > \theta^{R*}$ is necessary.

From above observations, we can draw a phase diagram over the labor market tightness θ_t^i and unemployment u_t^i as in Figure 1. Equilibrium path traces $\dot{\theta}_t^B = 0$ line toward the point E^B when the economy is in boom and $\dot{\theta}_t^R = 0$ line toward the point E^R when it is in recession. The labor market tightness variable θ_t jumps vertically up and down as the state changes from recession to boom and from boom to recession, respectively. Unemployment decreases during boom and increases during recession between the points F and G.

The equilibrium path can be also shown in a diagram over the vacancy-unemployment space as in Figure 2. The path traces in the same manner as it is described above. When boom hits the economy, vacancy jumps up and gradually both vacancy and unemployment decreases along $\dot{\theta}_t^B = 0$ line toward the point E^B . When recession hits the economy, vacancy drops down and both vacancy and unemployment increases over time along $\dot{\theta}_t^R = 0$

line toward the point E^R . Thus, the path shows anticlockwise movement around the curve \dot{u}_t^i which is the Beveridge curve.

References

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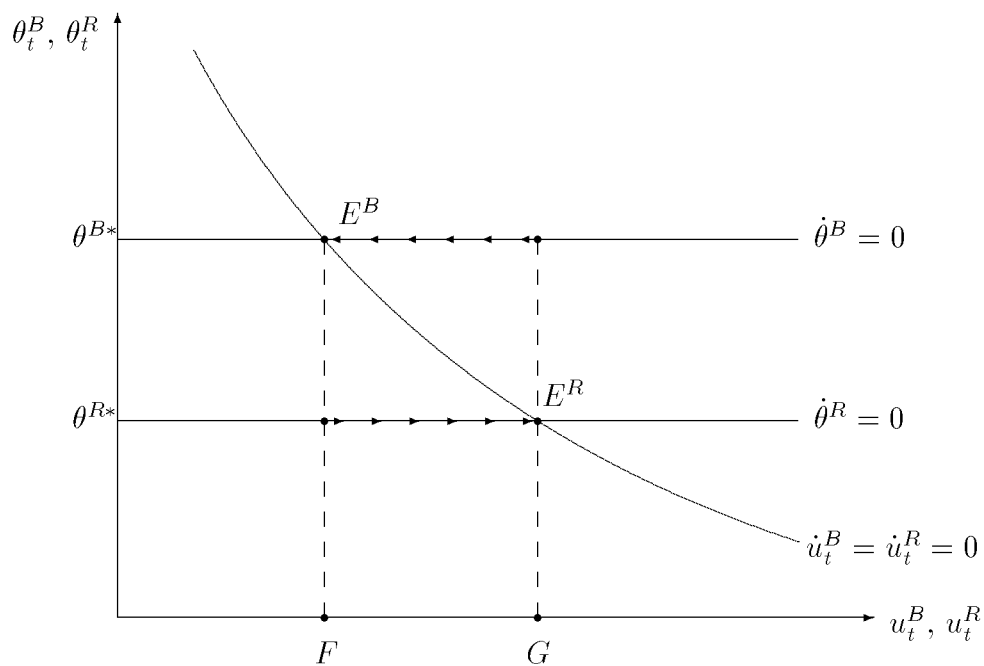


Figure 1: Equilibrium Path in Labor Market Tightness and Unemployment Space

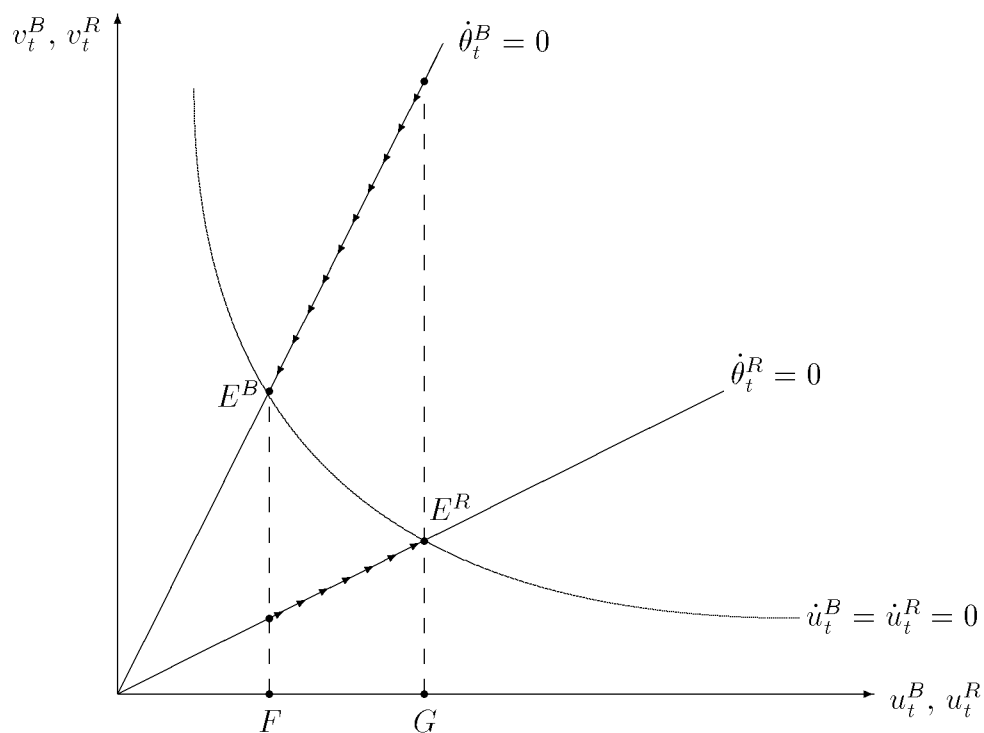


Figure 2: Equilibrium Path in Vacancy-Unemployment Space