Economic Growth and Business Cycles with Commitment Problem in Credit Market

Yoshimasa Shirai*
Keio University

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Abstract

Commitment problem in credit market and its effects on economic growth are discussed. Completions of investment projects increase capital stock of the economy. These projects require credits which are financed by finacial intermediaries. A simplified credit model of Dewatripont and Maskin is used to describe the financing process, in which the commitment problem or the "soft budget constraint" problem arises. However, in dynamic general equilibrium setup with endougenous determination of value and cost of projects, there arise multiple equilibria in the project financing model, namely refinancing equilirium and no-refinancing equilibrium. The former leads the economy to the stationary state with smaller capital stock level than the latter. Both the elimination of refinancing equilibrium and the possibility of "Animal Spirits Cycles" equilibrium are also discussed.

Keywords: Economic Growth, Commitment Problem, Termination and Refinancing of Projects, Multiple Equilibria, Animal Spirits Cycles

JEL: G3, E1.

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1 Introduction

Financial intermediaries and governments often face the decision over refinancing or termination of ongoing projects when they are not performing well. The purpose of the paper is to point out that there is a complementarity effect in refinancing and termination decisions among creditors. Furthurmore, we investigate its implication to economic growth and show the possiblity of "Animal Spirits" cycles.

In order to make our points clear, we simplify a project financing game proposed by Dewatripont and Maskin(1995) into the one with a feature of commitment problem over termination or refinancing of an already started project. This project financing game is incorporated into dynamic general equilibrium models whose outcome determines the payoffs of the project financing game played between entrepreneurs and creditors.

Our credit provision game is a degenrated version of the moral hazard problem. Each entrepreneur has a potential project. The completion of a project results in a unit of capital stock which yields marketable return from that time on and a non-marketable private benefit to the entrepreneur. In order to complete the project, the entrepreneur must hire a certain amount of labor (or utilize a certain amount of final products as an input). However, each entrepreneur needs to finance this cost. The creditors first faces the decision whether to finance the entrepreneur or not. If the entrepreneur is financed, then he decides to exert low effort or high effort to complete the project. Exerting high effort costs him higher disutility than exerting low effort. If he exerts high effort, the project is completed with probability one. If he exerts low effort, it is completed with probability zero[1] . In the latter case, the entrepreneur can complete the project by hiring another certain amount of labor or final products (it will be assumed that this amount is more than the initial requirement of labor or final porducts). Hence, he will ask the creditor for a refinancing. Finally, the creditor faces a decision whether to refinance or terminate the project by not refinancing it.

[1] If we assume that exrting high effort completes the project with some probability less than one and that exerting low effort completes the project with some positive probability (the former is higher than the latter), then it becomes a usual moral hazard problem in which the creditor can not distinguish whether the failure of the project is due to unluck or low effort on the entrepreneur's side. In our case where probabilities of success and failure are one or zero, the creditor can detect with certainty that the results of the project are due to the entrepreneur's according effort levels. There are many existing works considering the effect of moral hazard or agency problem in financial markets to the aggregate economic activities. See, Bernanke and Gertler(1989), King and Levine (1993), Kiyotaki and Moore(1997), Kahn and Ravikumar(1997). Unlike this literature our focus is on commitment problem in financial markets and its effect on aggregate economic activities.

This creates a commitment problem over termination or refinancing of the project with an initial failure. If the entrepreneur believe that the creditor will reject the refinancing request (terminate the project), then the entrepreneur will exert high effort and complete the project without using up furthur resources. On the other hand, if the entrepreneur believe that the creditor will accept the refinancing request, then the entrepreneur will exert low effort and ask for a refinancing of the project.

The main point we would like to make in this paper is that if payoffs of the credit provision game are endogenously determined in an economic model, then there is a possibility for multiple equilibria. Typical equilibria are the one with all project being completed with refinancing and the other with those being completed without refinancing. In the former equilibrium, creditors' commitment in not refinancing is credible and in the latter it is not credible. The expectation that creditors do commit themselves not to refinance makes enterpreneurs refrain from exerting low effort, which in turn makes more entrepreneurs being financed with a given amount of credit available in the economy. This lowers the value of each project due to the decreasing returns to scale in accumulated number of projects. Hence, on creditors' side, the loss from terminating the project when they are asked for refinancing becomes smaller compared to the cost of refinancing. In this event, there actually is an incentive for creditors not to refinance. Opposite things happen in case of refinancing equilibrium. The expectation of the entrepreneurs over the creditors refinancing decision plays a key role.

If payoffs of the credit provision game are exogenously given, then the sub-game perfect equilibrium of the game becomes unique. Dewatripont and Maskin(1995) and Qian and Roland(1998) display credit provision games where refinancing equilibrium is unique. Their main point was that credit decentralization provides creditors an incentive to commit not to refinance and that it helps the economy to establish more efficient outcome. If our economy is in refinancing equilibrium, their arguments becomes valid. However, what we would like to point out is that even in the centralized credit system, no-refinancing equlibrium can also be established.

By supposing that the completion of each project adds a unit of capital stock of the economy, our project financing game is incoporated into the dynamic general equilibrium models. As a result we can see the aggregate economic consequence of the project financing game. The result is that the economy accumulate larger capital stock in refinanicing equilibrium than in no-refinancing equilibrium. In refinancing equilibrium, extra resources are needed to compelete the project compared to no-refinancing equilibrium. We first present the model of economic growth with labour being the only fundamental factor of production. The same result can be derived in the model with neo-classical growth model where final product is required to accumulate the capital (This version of the model is presented in appendix).

With our framework, we can interpret the fundamental source of the "Animal Spirits" cycles formalized by Howitt and McAfee(1992). In their model, the source of animal spirits cycles is an exogenously given increasing returns to scale matching technology. Thus, they are assuming a technological mechanism that creates complementarity effect in economic activities. This creates dynamic multiple equilibrium paths. With the shifts in expectation over the equilibrium paths, the actual paths of the economy shifts accordingly. What we provided in this paper is a mechanism that causes the complementarity effect due to commitment problem in credit market which naturally is closely related to entrepreneurs investment activities.

The paper proceeds as follows. We present a project financing game in section 2. Multiple equilibria in a project financing game will be characterized. In section 3, the model of economic growth with labor being the fundamental factor of production is presented. The result of section 2 is used in describing the capital accumulation process of this model. Section 4 discusses the elimination of refinancing equilibirum and the possiblity of Animal Spirits cyles equilibrium. Section 5 concludes. In Appendix, proof of propositions and a neo-classical version of economic growth model is presented.

2 A Project Financing Game

Consider a following game at the moment of time t between an entrepreneur and a creditor. In order to start the project, the entrepreneur must hire some units of labor. The entrepreneur needs to finance this start-up labor costs by borrowing liquidity from the creditor. If the creditor rejects the provision of credit, the game ends and both party get payoff of zero. Once the entrepreneur acquires enough credit to hire labor, he can exert either high effort or low effort in completing the project. If he exerts high effort, the project only requires $\underline{a}_I$ units of labor which is positive and constant. If he exerts low effort, the project requires $\overline{a}_I$ units of labor which is larger than $\underline{a}_I$ and constant. When the project is completed, it yields an expected discount sum of future returns at time t which we denote by v_t. In addition to this, it yields a private benefit B to the entrepreneur which is a positive and constant value in utility units. The entrepreneur suffers a private cost of E if he exerts high effort and e if he exerts low effort. Both E and e are constants and measured in utility units where we assume $E > e$. Assume that the creditor has all bargaining power over the share of all marketable value. Then what the creditor should do is to provide the least amount of credit and absorb all marketable value created by the completed project which is v_t. In doing so, the creditor should initially provide the credit of amount $w_t \underline{a}_I$ where w_t is a real wage at time t. If the entrepreneur exerts high effort, the project is completed and the creditor receives a payoff of $v_t - w_t \underline{a}_I$ while the entrepreneur receives a payoff of $B - E$. We assume $B > E$. If the entrepreneur exerts low effort, the project is still not completed and the entrepreneur will ask the creditor for a refinancing of amount $w_t(\overline{a}_I - \underline{a}_I)$. If the creditor terminates this project, the entrepreneur gets payoff of $-e$ and the creditor gets a payoff of $-w_t \underline{a}_I$. On the other hand, if the creditor refinances the project, the project is completed and the entrepreneur and the creditor get payoffs $B - e$ and $v_t - w_t \overline{a}_I$ respectively. The sequence of this game is shown in Figure 1.

[Figure 1 around here]

There are potentially many entrepreneurs in this economy. They want to enter at any moment of time t if the projects they have were to realize. On the other hand, the creditors make financing decisions in order to maximize expected net benefit from financing these projects. We assume that both creditors and entrepreneurs are risk nutral. Let us consider the possible outcome of this game. Before the analysis of the game, let us assume that v_t, the value of the completed project at time t, is a decreasing function with respect to the accumulated number of completed projects at time t.

First, consider the sub-game where the creditor has already provided the credit of amount $w_t \underline{a}_I$ and the entrepreneur has exerted low effort. At this sub-game, the creditor terminates the project if a condition $v_t - w_t \overline{a}_I \leq -w_t \underline{a}_I$ is satisfied (we call this a no-refinancing condition). If this is the case, then the entrepreneur will not exert low-effort, because $B - E > -e$. Given entrepreneurs exerting high effort after initial credit provision, the creditor will provide $w_t \underline{a}_I$ of credit at the beginning as long as $v_t > w_t \underline{a}_I$. Thus, the number of entry at the moment t will be determined by the condition $v_t = w_t \underline{a}_I$. Substituting this into the no-refinancing condition, it can be rewritten as $\overline{a}_I \geq 2\underline{a}_I$. As long as this condition is satisfied, we have a "no-refinancing equilibrium" in which all entrepreneurs exerting high-effort.

However, there is another possibility. If $v_t - w_t \overline{a}_I > -w_t \underline{a}_I$ (we call this a refinancing condition), then the creditor cannot commit himself to terminate the project when he is asked for a refinancing. In this case, the entrepreneur will exert low effort after acquiring the initial credit provision because $B - e > B - E$. Foreseeing this, the creditor provides initial credit to entrepreneurs only if $v_t > w_t \overline{a}_I$. Thus, the number of entry at time t is determined by the condition $v_t = w_t \overline{a}_I$. Substituting this condition into the refinancing condition, we obtain $0 > -w_t \underline{a}_I$ which is always satisfied. Hence, we have a "refinancing equilibrium" in which all entrepreneurs are exerting low-effort.

The readers may now easily anticipate that there is one more equilibrium in this game. It is a mixed strategy equilibrium in which the creditors take a mixed strategy over termination or refinancing after the initial provision of credit to the entrepreneur, and the entrepreneurs taking mix strategy over exerting high-effort and low-effort. We ignore this equilibrium, because it is not a strict equilibrium as in the cases of no-refinancing and refinancing equilibria.

The result of this section can be summarized as follows:

Assumption 1 $\overline{a}_I > 2\underline{a}_I$.

Proposition 1 *Suppose v_t is a decreasing function with respect to the number of entries. Under assumption 1, both no-refinancing and refinancing equilibria exist. In both equilibria, termination of projects will not happen on the equilibrium path. In no-refinancing equilibrium, $\underline{a}_I$ units of labor are used in completing each project and the condition $v_t = w_t \underline{a}_I I$ holds. In refinancing equilibrium $\overline{a}_I$ units of labor are used in realizing each project and the condition $v_t = w_t \overline{a}_I$ holds.*

We can modify this project financing game into that where final goods instead of labor are used to complete the project. With this modification, the cost of project simply becomes a_I. One can easily check that proposition 1 holds in this case as well.

3 Dynamic Aggregate Economy

In this section, we introduce a model which explains the determination of variables w_t and v_t given in the game presented in the previous section. We will see the implication of the project financing game presented in the previous section to the growth path of the economy.

There are consumers, entrepreneurs, final goods producers and creditors (financial intermediaries). The consumers are identical and each of them is endowed with one unit of labor. The population of consumers is L which is constant over time. Each consumer has an inter-temporal utility function

$$\int_t^\infty \log c_\tau \cdot e^{-\rho\tau} d\tau,$$

where c_t is a consumption level of final goods at time t and ρ is their discount rate. We set the final goods as numeraire. The wage rate and instantaneous interest rate at time t are denoted by w_t and r_t respectively. The real holdings of asset at time t is denoted by s_t. Hence, $s_t L$ equals an aggregate liability of creditors. The savings of the consumer at time t is denoted by $\dot{s}_t \equiv ds_t/dt$. The consumers' problem is to maximize above utility subject to the following budget constraint;

$$c_t + \dot{s}_t = w_t + r_t s_t \tag{1}$$

given initial holdings of real asset s_t.

The first order condition of above maximization problem is summarized as,

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{2}$$

Final goods producers have the Cobb-Douglas production function of the form,

$$X_t = K_t^\alpha N_t^{1-\alpha}, \tag{3}$$

where K_t and N_t are capital stock and labor force employed for final goods production at time t. The profit maximization yields following conditions;

$$w_t = (1-\alpha) K_t^\alpha N_t^{-\alpha} = (1-\alpha) c_t L / N_t, \tag{4}$$

$$\pi_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} = \alpha c_t L / K_t, \tag{5}$$

where π_t is a users cost of capital stock at time t. The last terms are derived from the market clearing condition for the final goods;

$$X_t = c_t L. \tag{6}$$

We assume that capital stock depreciates at rate δ. The value of a unit of capital stock in terms of final goods at time t, is defined as,

$$v_t = \int_t^\infty \pi_\tau \exp\left[-\int_t^\tau r_\theta + \delta d\theta \right] d\tau. \tag{7}$$

Differentiating (7) with respect to time we obtain a relation,

$$\frac{\dot{v}_t + \pi_t}{v_t} = r_t + \delta, \tag{8}$$

which is a gross rate of return for a unit of capital stock.

A newly completed project adds one unit of capital stock to the economy (or rented to the final goods producers). However, each potential entrepreneur needs a_I units of labor to complete the project and thus must finance the paymet for hiring that labor. This process is described in the previous section. Creditors finance each project as long as its value exceeds the amount of credit $w_t a_I$. Hence, entry condtion for potential entrepreneur being financed is given by

$$v_t = w_t a_I, \tag{9}$$

where $a_I = \overline{a}_I$ in refinancing equilibrium and $a_I = \underline{a}_I$ in no-refinancing equilibrium.

We can now define the labor market clearing condition as follows;

$$L = \begin{cases} N_t + a_I \cdot (\dot{K}_t + \delta K_t). & \text{if } w_t a_I = v_t \\ N_t & \text{if } w_t a_I > v_t \end{cases} \tag{10}$$

The left hand side is a labor suuply. The right hand side is a labor demand which consists of the labor demand of final goods sector N_t and the labor demand from gross investment (or labor required for the completion of the projects) at time t.

Finally, market clearing condition for the credit market is given by,

$$s_t L = v_t K_t. \tag{11}$$

The left hand side is a supply of aggregate credit and the right hand side is a demand for credit (capital stock) both in terms of final goods unit. Differentiating this with resepct to time, we furthur obtain,

$$\dot{s}_t L = \dot{v}_t K_t + \dot{K}_t v_t. \tag{12}$$

Given stock variables s_t and K_t which satisfy (11), the system of equations (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), and (12), determine the endogenous flow variables, c_t, $\dot{c}_t$, $\dot{s}_t$, w_t, π_t, r_t, v_t, $\dot{v}_t$ N_t, X_t and $\dot{K}_t$.

We are now at the stage to reduce the number of endogenous variables into two and construct a phase diagram for such a reduced system of the economy. For this purpose, define $V_t \equiv v_t/c_t$. Differentiating this with respect to time and substituting into it, (2), (5), (3), (6), and (8), we obtain a differential equation for V_t,

$$\dot{V}_t = (\rho + \delta)V_t - \frac{\alpha}{K_t}. \tag{13}$$

Hereby, we have eliminated variables c_t, v_t, $\dot{c}_t$, $\dot{v}_t$, X_t, π_t, and r_t.

By rearranging (10), we can determine the net investment level $\dot{K}_t$ as follows;

$$\dot{K}_t = \begin{cases} -\delta K_t & \text{if} \quad v_t < w_t a_I \\ (L - N_t)/a_I - \delta K_t & \text{if} \quad v_t = w_t a_I. \end{cases}$$

By using (4), (3), and the definition of V_t, we can furthur rewrite above as,

$$\dot{K}_t = \max\left\{-\delta K_t, \frac{L}{a_I} - \frac{(1-\alpha)}{V_t} - \delta K_t\right\}. \tag{14}$$

In deriving (14), we eliminated variables w_t and N_t. So far we have not used equations (1) and (12) in reducing the system of equations into (13) and (14). However, equation (1) and (12) together with (3), (4), (5), and (8), implies (6). Hence there is no need for using equations (1) and (12) in reducing the system of equations into (13) and (14).

The dynamics of the economy is determined by the system of differential equations (13) and (14) where a_I is $\underline{a}_I$ in no-refinancing equilibrium and $\overline{a}_I$ in refinancing equilibrium. The phase diagram for the variables V_t and K_t is shown in Figure 2.

Above the locus $\dot{V}_t = 0$, the value of V_t increases and vice versa. Below the locus $\dot{K}_t = 0$, the capital stock increases and vice versa. The curve CD is the path of termination equilibrium. This leads to the stationary state which is indicated at point E_2 in the diagram. On the other hand, the curve AB is the path of refinancing equilibrium. The stationary state in this case is indicated at point E_1.

The result is summarized as follows;

Proposition 2 *Under assumption 1, both the refinancing equilibrium and no-refinancing equilibrium paths exist. The refinancing equilibrium path leads the economy to the stationary state with its capital stock level,* $\frac{\alpha L}{[\delta+\rho(1-\alpha)]\overline{a}_I}$. *The no-refinancing equilibrium path leads the economy to the stationary state with its capital stock level,* $\frac{\alpha L}{[\delta+\rho(1-\alpha)]\underline{a}_I}$.

[Figure 2 around here]

The no-refinancing equilibrium path leads economy to the stationary state with larger amount of capital stock than that of the refinancing equilibrium path. This result simply comes from the fact that in the refinancing equilibrium the newly entering entrepreneurs are wasting extra labor $(\dot{K}_t+\delta K_t)\cdot(\overline{a}_I-\underline{a}_I)$ at each moment of time t. Thereby, the capital accumulation process is slower in refinancing equilibrium path than that of termination equilibrium. Hence, the accumuation of capital stock stops at the smaller level in refinancing equilibrium. The smaller level of capital stock implies the smaller amount of final goods produced with given amount of labor force, which gives less utility to consumers.

The emergence of multiple dynamic equilibria is a consequence of the project financing game described in the previous section. The expectation held by the potential entrepreneurs has a key role in this game. Once entrepreneurs believe that there will be no refinancing, they will exert hight effort and all new projects only require $w_t \underline{a}_I$ of credit for each. With the increase in capital stock, the stream of instantaneous returns to each unit of capital stock declines and thus v_t also decreases down to the level of $w_t \underline{a}_I$. This low value of v_t makes each creditor to posess an incentive for terminating the project when he is asked for a refinance. Under assumption 1, it is worth forfeiting the initial provision of credit $w_t \underline{a}_I$ than to widen the loss up to $w_t(\overline{a}_I - \underline{a}_I)$ by refinancing the project. Hence, the belief held by entrepreneurs is re-confirmed (or self-fulfilled). On the other hand, if entrepreneurs believe that there will be refinancing when they exert only low effort, they will exert low effort. In this case, the overall amount of credit provided to each entrepreneur becomes $w_t \overline{a}_I$ and thus entries are restricted to the level where the value of a unit of capital stock only decreases to the level $w_t \overline{a}_I$. This high value of v_t makes creditors to posess an incentive for refinancing. In fact, it is worth refinancing because the net benefit of refinancing is $v_t - w_t \overline{a}_I = 0$ while the net benefit of termination is $-w_t \underline{a}_I$. Again, the initial belief held by potential entrepreneurs is re-confirmed.

The aggregate dynamic economy presented in this section is just one example among other possiblie models. One can straightfowardly introduce our project financing game into a growth model with expanding variety of intermediate inputs as in Grossman and Helpman(1991, Ch. 3). With slight modification of the project financing game, one can also introduce it into more standard growth model in which final goods are used in capital accumulation. This version of the model is displayed in appendix B.

4 Discussions

a. Elimination of Refinancing Equilibrium

The refinancing equilibrium leads the economy to a stationary state with smaller amount of capital stock compared to that in the no-refinancing equilibrium. Thus, the final goods production levle is greater in the stationary state of no-refinancing equilibrium than that of refinancing equilibrium. The expectation held by newly entering entrepreneurs played a key role in the establishment of each equilibrium. To eliminate the refinancing equilibrium, we must credibly convince entrepreneurs to expect that there will be no refinancing.

Any measure which provides creditors an incentive not to refinance the projects will convince entrepreneurs that there will be no refinancing. Dewatripont and Maskin(1995) argues that decentralization of creditors serves as a commitment device for creditors not to refinance inefficient projects. Their argument is valid if the size of each project is large. However, if it is small, a creditor (in a decentralized system) has enough amount of liquidity to refinance the project with low effort level. Thus in a context of capital accumulation via completions of small size projects, their argument may not apply.

There is another way to provide creditors an incentive not to refinance. Suppose the economy is in the refinancing equilibrium. The government can collect tax in a lump-sum manner and compensate the creditors' losses in not refinancing. Then, the creditors can commit themselves to terminate such projects. This changes the expectation held by entrepreneurs so that they exert high effort, and hence the economy turns into the termination equilibrium. There will be no need for the government to cover the losses made by creditors through not refinancing the projects with low effort, since there are no entrepreneurs exerting low effort.

However, it should be reminded that above policy is effective due to our assumption on informational structure. In our model, it is assumed that the information about the entrepreneurs exerting high or low effort levels is revealed to the creditors *perfectly* and *immediately* after the moment where initial credit provision is made. If information about entrepreneurs exerting low effort levels is not revealed immediately or imperfectly, a simple provision of termination incentive to creditors not only eliminate refinancing equilibrium but also with termination of some projects with high effort levels. This loss may outweigh the loss coming from refinancing equilibrium. We have set the assumption of immediate revelation of information about the effort levels in order to high-light the effects of commitment problem in credit market and incorporate them into general equilibrium set up in a simple manner. In reality, one should be careful in the use of the government policy proposed here.

b. Animal Spirits Cycles

As in Howitt and McAfee (1992), there is a possibility for Animal Spirits cycles in our model. The reason for this is that we have multiple equilibrium paths for the same initial conditions (accumulated number of projects or capital stock levels). The expectation held by the agents in economy plays a key role. The rest of the section presents its workings with an example of the capital stock accumulation model presented in the previous section. This is a continuous time version of Animal Sprits cycles with standard economic growth model.

Suppose now that there are exogenous signal which all agents can see. With the change in signal, all agents change their existing expectation about creditors' refinancing behavior to the other. Assume that the change of signal happens with rate of ξ for any moment of time which is constant over time. Under this setting, we have two different expected value for a project. The one is that where the current expectation about project financing game is no-refinancing, which we denote by v_t^N. The other is that where the current expectation is refinancing, which we denote by v_t^R. When the economy is in the former situation, we say that it is in "N-phase". For the latter case, we say that the economy is in"R-phase".

We proceed with the model of capital accumulation given in section 3. For a given capital stock level at time t, the consumption levels, labor input level for final goods production, rate of return for capital, wage rate, and rate of increase in capital stock are different according to the current expectation held by agents. We put superscript R and N to all these variables when the economy is in R-phase and N-phase respectively.

With above modifications, the optimality condition for the consumer's problem becomes (instead of equation (2)),

$$\begin{aligned}\frac{\dot{c}_t^N}{c_t^N} &= (r_t^N-\rho)+\xi\cdot\left(\frac{c_t^N}{c_t^R}-1\right) \quad \text{and}\\ \frac{\dot{c}_t^R}{c_t^R} &= (r_t^R-\rho)+\xi\cdot\left(\frac{c_t^R}{c_t^N}-1\right).\end{aligned}$$

See appendix for the derivation of above conditions. Moreover, instead of (8), we have following two relationships for the value of projects;

$$(\delta+r_t^N)v_t^N = \pi_t^N+\xi(v_t^R-v_t^N)+\dot{v}_t^N \tag{15}$$

$$(\delta+r_t^R)v_t^R = \pi_t^R+\xi(v_t^N-v_t^R)+\dot{v}_t^R, \tag{16}$$

where r_t^N and r_t^R are rates of return to capital stock when the economy is in N-phase and R-phase at time t respectively. Furthermore, define variables $V_t^N\equiv v_t^T/c_t^N$ and $V_t^R\equiv v_t^R/c_t^R$. With a procedure similar to that in deriving equation (13), we have obtain two differential equations for the movement of V_t^N and V_t^R;

$$\frac{\dot{V}_t^N}{V_t^N} = (\rho+\delta)-\frac{\alpha}{K_tV_t^N}-\xi\left(\frac{v_t^R}{v_t^N}+\frac{c_t^N}{c_t^R}-2\right) \tag{17}$$

$$\frac{\dot{V}_t^R}{V_t^R} = (\rho+\delta) - \frac{\alpha}{K_t V_t^R} - \xi\left(\frac{\upsilon_t^N}{\upsilon_t^R} + \frac{c_t^R}{c_t^N} - 2\right). \tag{18}$$

On the other hand, the equations corresponding to (14) can be written as,

$$\dot{K}_t^N = \max\left\{-\delta K_t,\ \frac{L}{\underline{a}_I} - \frac{(1-\alpha)}{V_t^N} - \delta K_t\right\} \tag{19}$$

$$\dot{K}_t^R = \max\left\{-\delta K_t,\ \frac{L}{\overline{a}_I} - \frac{(1-\alpha)}{V_t^R} - \delta K_t\right\}. \tag{20}$$

If the economy is in N-phase (R-phase), then its dynamics is described by a pair of differential equations (17) and (19) ((18) and (20)). We can draw schedules for $\dot{K}_t^N = 0$ and $\dot{K}_t^R = 0$ in a diagram by setting vertical axis to V_t^N and V_t^R, and horizontal axis to K_t as in Figure 4. These schedules are upward sloping and the schedule of $\dot{K}_t^N = 0$ is positioned on the lower right of the schelue of $\dot{K}_t^R = 0$. Above these schedules, capital stock increases and below these it decreases.

Now, let us move on to draw schedules for $\dot{V}_t^N = 0$ and $\dot{V}_t^R = 0$. We have following lemma for such schedules;

Lemma 1 *The loci of $\dot{V}_t^N = 0$ and $\dot{V}_t^R = 0$ are identical and represented by pair of (V_t^s, K_t) where $s \in \{N, R\}$, which satisfy following equation;*

$$0 = (\rho+\delta)V_t^s - \frac{\alpha}{K_t}.$$

Proof: See Appendix A.

Clearly, the schedule for $\dot{V}_t^s = 0$ are identical to the schedule of $\dot{V}_t = 0$ in the previous section. Hence, above (below) the schedule $\dot{V}_t^N = \dot{V}_t^R = 0$, the economy move upward (downward). As it is shown in figure 3, the economy moves along the path AB toward the point E^R if it is in R-phase. If the economy is in T-phase, it moves along the path CD toward the point E^N. With stochastic arrival of change in signal, the economy jumps vertically between the paths AB and CD. In the long run, the capital stock level of the economy moves up and down between points F and G. Note that the paths AB and CD are the same as those in figure 3. In N-phase and R-phase, the economy moves along the same paths as those of pure termination and refinancing equilibrium paths. The result of this subsection can be summarize as,

Proposition 3 *In animal spirits cycles equilibrium, the economy moves along the refinancing equilibrium path in R-phase and no-refinancing equilibrium path in N-phase. In* *the long run, the economy stchastically cycles between the capital stock levels* $\frac{\alpha L}{[\delta+\rho(1-\alpha)]\overline{a}_I}$ *and* $\frac{\alpha L}{[\delta+\rho(1-\alpha)]\underline{a}_I}$.

[Figure 3. around here.]

5 Concluding Remarks

In this paper, I have introduced commitment problem in financing the set up costs required for new projects which contribute to capital accumulation. Unlike, Dewatripont and Maskin(1995) and Qian and Roland(1997), the payoffs for the agents are determined endogenously through the aggregate economic models in my paper. This creates the complementarity effect to the incentives for creditors in committing themselves to terminate or refinance the projects implemented with low effort level. Hence, it leads to the existence of multiple equilibrium growth paths. The basic idea of explaining this phenomenon is that when returns from credit provision is low, there are no incentives for creditors to finance costly projects which even lowers the returns from credit provision through faster capital accumulation in the economy. The opposite things happen when returns from credit provision is high.

It is well known that some form of increasing returns or technical externalities create multiple equilibria in growth models. However, in my model, no such features are introduced. The source of multiplicity in my model is the assumption of commitment problem in financing new projects. However, it should be noted that not only this but also the assumption that the amount of credit required for refinancing is more than the double amount of initial credit provision (Assumption 1) is crucial for this result. Without this assumption, there will be no incentive for creditors to terminate costly projects and will make refinancing equilibrium unique. In this respect, the government policy to eliminate refinancing equilibrium is most effective or in need when Assumption 1 is violated.

Moreover, I have put the assumption that effort levels of entrepreneurs are known to credit providers immediately after initial credit provisions are made. With this assumption, we got away from the agency problems in credit provision. If this assumption is violated, agency problems introduced in Bernanke and Gertler (1989) arises. However, my conjecture is that it will only add agency costs associated with credit provision to our analysis and my basic idea about complementarity effect in refinancing decision is still viable.

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Appendix

A. Derivation of Equations for Animal Spirits Cycles Model and Proof of Lemma 1.

The representative consumer's utility maximization problem is given as follows,

$$\begin{aligned} W^N(s_t) \quad &\equiv \quad \max_{\{c_\tau^N\}_{\tau=t}^\infty} E_{t_1}\left[\int_t^{t_1} \log c_\tau^N \cdot e^{-\rho(\tau-t)} d\tau + W^R(s_{t_1}) e^{-\rho(t_1-t)}\right] \\ &\quad \text{s.t. } c_t^N + \dot{s}_t^N = w_t^N L + r_t^N s_t \\ &\quad \text{given } s_t, \end{aligned}$$

and

$$\begin{aligned} W^R(s_t) \quad &\equiv \quad \max_{\{c_\tau^R\}_{\tau=t}^\infty} E_{t_1}\left[\int_t^{t_1} \log c_\tau^R \cdot e^{-\rho(\tau-t)} d\tau + W^N(s_{t_1}) e^{-\rho(t_1-t)}\right] \\ &\quad \text{s.t. } c_t^R + \dot{s}_t^R = w_t^R L + r_t^R s_t \\ &\quad \text{given } s_t, \end{aligned}$$

where $W^N(s_t)$ and $W^R(s_t)$ are the consumer's value functions when the economy is in N-phase and R-phase respectively at time t and with real asset of amount a_t at time t. Expectation is taken over t_1; the first change in signal from the current phase to the other after time t. This change in signal is subject to the Poisson process with its arrival rate ξ. Let $\{c_\tau^{N*}\}_{\tau=t}^\infty$ and $\{c_\tau^{R*}\}_{\tau=t}^\infty$ be the optimal streams of consumption level at R-phase and N-phase. Then, above value functions can be rewritten as follows.

$$\begin{aligned} \rho W^N(s_t) \quad &= \quad \log c_t^{T*} + \xi \cdot (W^R(s_t) - W^N(s_t)) + W^{N\prime}(s_t)\dot{s}_t^N \\ &\quad \text{where } \dot{s}_t^N = w_t^N L + r_t^N s_t - c_t^{N*}, \end{aligned}$$

and

$$\begin{aligned} \rho W^R(s_t) \quad &= \quad \log c_t^{R*} + \xi \cdot (W^N(s_t) - W^R(s_t)) + W^{R\prime}(s_t)\dot{s}_t^R \\ &\quad \text{where } \dot{s}_t^R = w_t^R L + r_t^R s_t - c_t^{R*}. \end{aligned}$$

The opitmality condition requires,

$$1/c_t^{N*} = W^{N\prime}(s_t) \text{ and } 1/c_t^{R*} = W^{R\prime}(s_t).$$

Differentiating these with respect to time, we have,

$$\begin{aligned} \frac{\dot{c}_t^{N*}}{c_t^{T*}} \quad &= \quad -W^{N\prime\prime}(s_t)\dot{s}_t^N \quad \text{and} \\ \frac{\dot{c}_t^{R*}}{c_t^{R*}} \quad &= \quad -W^{R\prime\prime}(s_t)\dot{s}_t^R. \end{aligned}$$

Moreover, differentiating value functions with respect to state variable s_t, we obtain,

$$\begin{aligned}\rho W^{N\prime}(s_t) &= \xi \cdot (W^{R\prime}(s_t) - W^{N\prime}(s_t)) + W^{N\prime}(s_t) r_t^N + W^{N\prime\prime}(s_t)\dot{s}_t^N \quad \text{and} \\ \rho W^{R\prime}(s_t) &= \xi \cdot (W^{N\prime}(s_t) - W^{R\prime}(s_t)) + W^{R\prime}(s_t) r_t^R + W^{R\prime\prime}(s_t)\dot{s}_t^R.\end{aligned}$$

From all these relationships, we end up getting the conditions for the rate of change in optimal streams of consumption at each phase;

$$\frac{\dot{c}_t^{N*}}{c_t^{N*}} = (r_t^N - \rho) + \xi \cdot \left(\frac{c_t^{N*}}{c_t^{R*}} - 1\right) \quad \text{and} \tag{21}$$

$$\frac{\dot{c}_t^{R*}}{c_t^{R*}} = (r_t^R - \rho) + \xi \cdot \left(\frac{c_t^{R*}}{c_t^{N*}} - 1\right). \tag{22}$$

Turning to the definition of value of projects at N and R-phases, they are defined as follows;

$$\begin{aligned}v_t^N &\equiv E_{t_1}\left[\int_t^{t_1} \pi_t^N \exp[-\int_t^\tau r_z^N + \delta dz] d\tau + v_{t_1}^R \exp[-\int_t^{t_1} r_z^N + \delta dz]\right] \quad \text{and} \\ v_t^R &\equiv E_{t_1}\left[\int_t^{t_1} \pi_t^R \exp[-\int_t^\tau r_z^R + \delta dz] d\tau + v_{t_1}^N \exp[-\int_t^{t_1} r_z^R + \delta dz]\right].\end{aligned}$$

These values of projects can be rewritten as,

$$\begin{aligned}(r_t^N + \delta) v_t^N &= \pi_t^N + \xi \cdot (v_t^R - v_t^N) + \dot{v}_t^N \quad \text{and} \\ (r_t^R + \delta) v_t^R &= \pi_t^R + \xi \cdot (v_t^N - v_t^R) + \dot{v}_t^R.\end{aligned}$$

Subsitituting the cost minimization conditions for the final product firms, $\pi_t^s = \alpha K_t^{\alpha-1}(N_t^s)^{1-\alpha}$ $= \alpha c_t^s / K_t$ where $s \in \{N, R\}$, and equations (21) and (22) into above, we finally obtain following;

$$\begin{aligned}\frac{\dot{V}_t^N}{V_t^N} &= (\rho + \delta) - \frac{\alpha}{K_t V_t^N} - \xi \cdot \left(\frac{v_t^R}{v_t^N} + \frac{c_t^N}{c_t^R} - 2\right) \quad \text{and} \\ \frac{\dot{V}_t^R}{V_t^R} &= (\rho + \delta) - \frac{\alpha}{K_t V_t^R} - \xi \cdot \left(\frac{v_t^N}{v_t^R} + \frac{c_t^R}{c_t^N} - 2\right),\end{aligned}$$

where $V_t^N \equiv v_t^N / c_t^N$ and $V_t^R \equiv v_t^R / c_t^R$.

Now we intend to see the loci of $\dot{V}_t^N = 0$ and $\dot{V}_t^R = 0$. On these loci, the pairs (K_t, V_t^N) and (K_t, V_t^R) respectively should satisfy following conditions:

$$\xi \cdot \left(\frac{v_t^R}{v_t^N} + \frac{c_t^N}{c_t^R} - 2\right) = (\rho + \delta) - \frac{\alpha}{K_t V_t^N} \quad \text{and} \tag{23}$$

$$\xi \cdot \left(\frac{v_t^N}{v_t^R} + \frac{c_t^R}{c_t^N} - 2\right) = (\rho + \delta) - \frac{\alpha}{K_t V_t^R}. \tag{24}$$

Our aim here is to show that these two loci are identical. For this purpose, we first have to proove the next claim.

Claim 1

$$\left(\frac{v_t^R}{v_t^N}+\frac{c_t^N}{c_t^R}-2\right)\geq(\leq)\left(\frac{v_t^N}{v_t^R}+\frac{c_t^R}{c_t^N}-2\right)\quad \textit{if and only if}\quad V_t^N\leq(\geq)V_t^R.$$

Moreover, $V_t^N=V_t^R$ *if and only if*

$$\left(\frac{v_t^R}{v_t^N}+\frac{c_t^N}{c_t^R}-2\right)=\left(\frac{v_t^N}{v_t^R}+\frac{c_t^R}{c_t^N}-2\right)=0.$$

Proof:

$$\begin{aligned}\left(\frac{v_t^R}{v_t^N}+\frac{c_t^N}{c_t^R}-2\right)-\left(\frac{v_t^N}{v_t^R}+\frac{c_t^R}{c_t^N}-2\right) &= (v_t^Rc_t^R+v_t^Nc_t^N)\left(\frac{1}{v_t^Nc_t^R}-\frac{1}{v_t^Rc_t^N}\right)\\ &= \frac{(v_t^Rc_t^R+v_t^Nc_t^N)}{v_t^Nv_t^R}(V_t^R-V_t^N)\end{aligned}$$

where the last expression $(V_t^R-V_t^N)$ is derived with the definitions of V_t^R and V_t^N. Thus, the sign of left hand side of above equations coincides with the sign of the expression $(V_t^R-V_t^N)$. Q.E.D.

With this claim, we can prove the lemma 1 in section 4. *Proof of Lemma 1.*: Suppose that the locus of $\dot{V}_t^N=0$ is above that of $\dot{V}_t^R=0$ on the space where vertical and horizontal axis are set to V_t^s and K_t respectively, where $s\in\{N,R\}$. I.e., for a given K_t suppose that V_t^N is larger than V_t^R where (K_t,V_t^N) and (K_t,V_t^R) satisfy equations (23) and (24). Then by claim 1, we know that the left hand side of (23) is smaller than that of (24). However, the right hand side of (23) is larger than that of (24). Hence we have a contradiction. Similarly, we obtain contradiction if the locus of $\dot{V}_t^N=0$ is below that of $\dot{V}_t^R=0$. Thus, the only possible case is that $V_t^N=V_t^R$. In this case, by lemma 1, equations (23) and (24) reduce to

$$0=(\rho+\delta)-\frac{\alpha}{K_tV_t^s}$$

where $s\in\{N,R\}$. Q.E.D.

B. A Growth Model with Final Products as an Input for Capital Production

Here, we slightly modify the project financing game. Each project require a_I units of final goods in order to produce a unit of capital stock. With high (low) effort exerted by an entrepreneur, $a_I = \underline{a}_I (= \overline{a}_I)$. It is straight forward to obtain Proposition 1 again. However, in this case, the entry conditions for termination and refinancing equilibria become $v_t = \underline{a}_I$ and $v_t = \overline{a}_I$ respectively.

All the features of the model is same as that of section 3, except that a_I units of final goods are required for the production of a unit of capital. There are identical consumers with their population L. The budget constraint and the first order condition for a representative consumer is given by (1) and (2). Instantaneous gross return from a unit of a capital is given by (8). Asset market clearing condition is given by (11). The production function is given given by (3) and thus, the profit maximization conditions are given by

$$w_t = (1-\alpha)X_t/N_t \tag{4'}$$

$$\pi_t = \alpha X_t/K_t. \tag{5'}$$

The differences between the model in this appendix and that of section 3 are seen in the labor market clearing condition and the final goods market clearing condition. The labor market clearing condition is given by,

$$N_t = L.$$

Using all these conditions, the budgent contraint of the consumers (aggregated) can be written as,

$$X_t - c_t L + = \delta K_t + v_t \dot{K}_t$$

which is just an identity between savings and gross investment.

The final goods market equilibrium condition is given by,

$$X_t = \begin{cases} c_t L & \text{if} \quad v_t < a_I \\ c_t L + a_I(\dot{K}_t + \delta K_t) & \text{if} \quad v_t = a_I. \end{cases}$$

This can be rewritten as,

$$\dot{K}_t = \begin{cases} -\delta K_t & \text{if} \quad v_t < a_I \\ \frac{K^\alpha L^{1-\alpha} - cL}{a_I} - \delta K_t & \text{if} \quad v_t = a_I. \end{cases}$$

Define $k_t \equiv K_t/L$. Deviding both sides of above equation by L, we obtain

$$\dot{k}_t = \max\left\{\frac{k_t^\alpha - c_t}{a_I}, 0\right\} - \delta k_t.$$

We assume that there is no possibility for a negative savings at any moment of time. This assumption can be expressed as a feasibility condition between c_t and k_t;

$$k_t^\alpha \geq c_t.$$

Under this restriction, we always have

$$\dot{k}_t = (k_t^\alpha - c_t)/a_I - \delta k_t \tag{25}$$

and the condition $v_t = a_I$ satisfied. Furthurmore, substituting (8) and (5′) into (2), we obtain the equation,

$$\frac{\dot{c}_t}{c_t} = \frac{\alpha k_t^{\alpha-1}}{a_I} - \delta - \rho. \tag{26}$$

The system of differential equations (25) and (26) describes the dynamics of the economy where $a_I = \underline{a}_I$ in no-refinancing equilibrium and $a_I = \overline{a}_I$ in refinancing equilibrium.

Taking c_t for vertical axis and k_t for horizontal axis, we can draw a phase diagram for c_t and k_t. The locus of $\dot{c}_t = 0$ is vertical line which crosses horizontal axis at $[\alpha/a_I(\delta+\rho)]^{\frac{1}{1-\alpha}}$. See figure 4. At the left (right) of this vertical line, c_t increases (decreases) over time. Moreover, the locus of $\dot{k}_t = 0$, a curve starting from the origin, is increasing at $k_t < [\alpha/a_I\delta]^{\frac{1}{1-\alpha}}$ and decreasing at $k_t > [\alpha/a_I\delta]^{\frac{1}{1-\alpha}}$. It crosses horizontal axis again at $k_t = [1/a_I\delta]^{\frac{1}{1-\alpha}}$. Above (below) this locus, k_t decreases (increases) over time. No-refinancing equilibrium path leads to a stationary state E^T which yeilds more per capita consumption and captial stock level than the stationary state of refinancing equilibrium path E^R.

[Figure 4 around here]

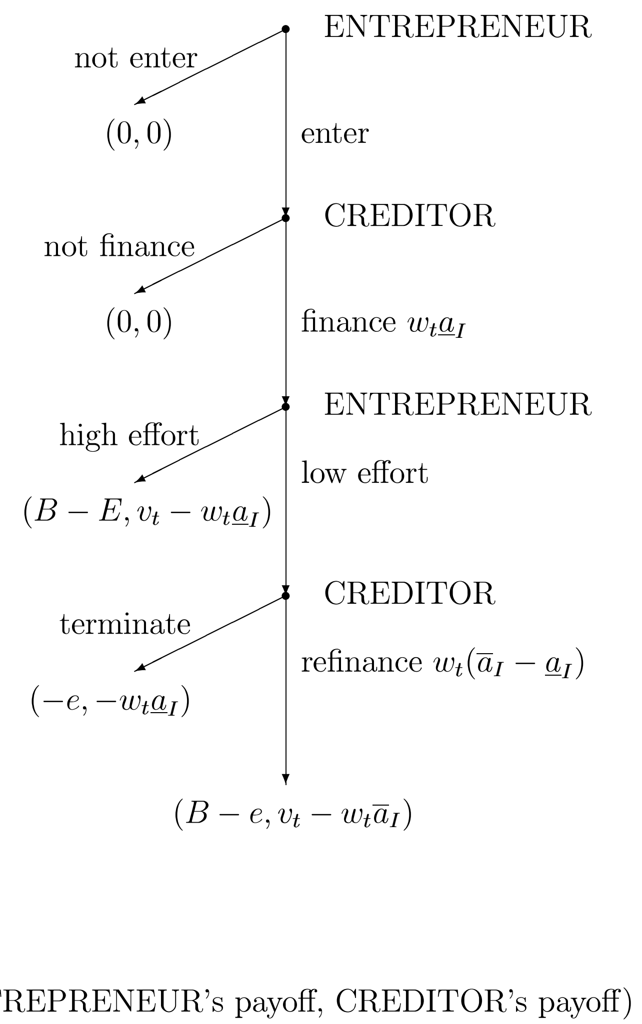

Figure 1. Entry and Financing Decision Process at a Moment t.

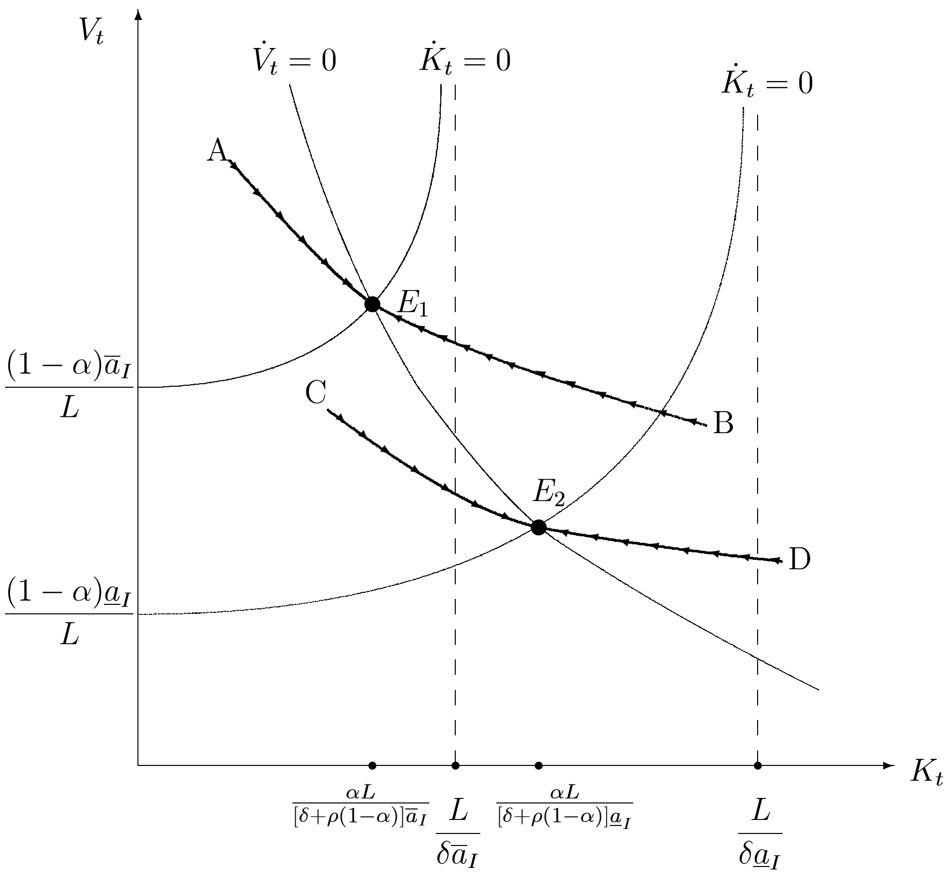

Figure 2 Equilibrium Paths for the Model with Capital Accumulation

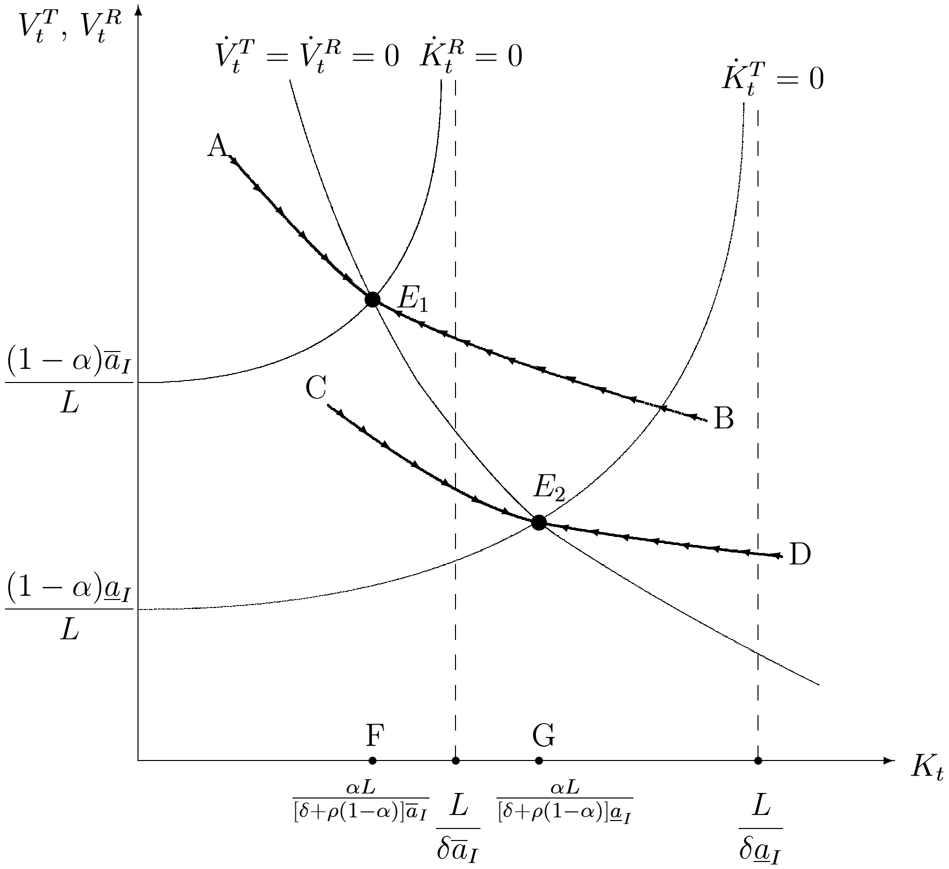

Figure 3 Animal Spirits Cycles

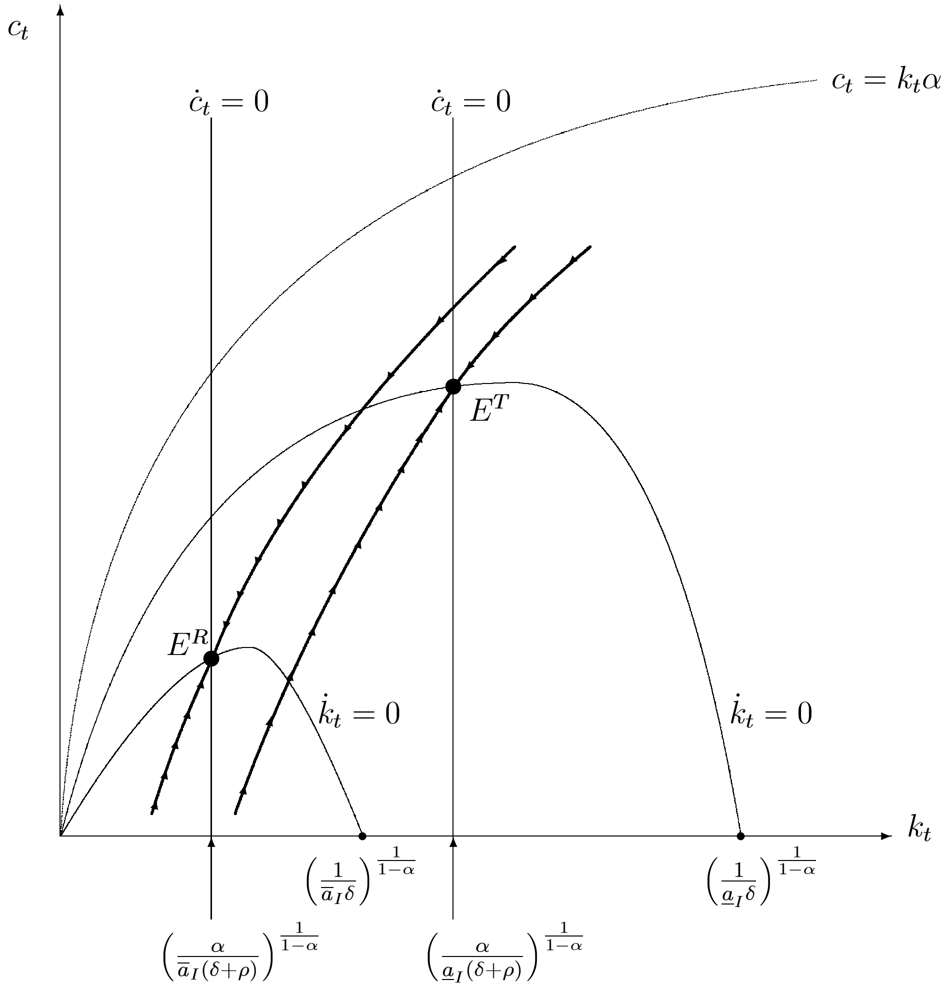

Figure 4. Phase Diagram of an Economy with Final Goods as Inputs for Capital Stock Production.