The Effects of Economic Integration on Trade Volumes under International Oligopoly

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Abstract
We examine under what conditions the formation of free trade agreement (FTA) with common tariff will enhance both trade volumes within FTA region and between FTA and non-FTA region. Specifically, we examine it under the Hotelling’s model of spatial competition with quality dimension in order to capture typical features in automotive industry. If tariff reduction after the formation of FTA is large enough, both trade volumes inside and outside FTA region increases with the formation of FTA.

1. Introduction

It is widely accepted that the regional trade agreements (RTA) such as “customs union” (CU) and “free trade agreements” (FTA) enhance trade within region, especially in case of CU by diverting trade from outside the region to inside. In fact, casual observations of automotive industry, which is a major industry in EU, clearly indicate that trade volume (of passenger cars) within EU has increased rapidly since the formation of EU. However, import of passenger cars into EU from non-EU

1 We thank Koji Ishibashi for helpful discussions. Corresponding author’s e-mail address is yshirai@econ.keio.ac.jp. The paper is still preliminary so comments are very welcome.
countries also increased since the formation of EU as well. Since EU is basically a customs union for the automotive industry, it is somewhat difficult to understand the rise in its imports.

Because automotive industry is a typical oligopoly industry, one may argue that strategic complementarity induces that within region competition after the formation of EU also leads to lower price in imported cars which in turn leads to increase in import. This must accompany the increase in the size of automotive market within EU. However, the automotive market is a satiated market such that it seems difficult to account for increase in inside and outside region trade volumes by relatively small increase in total sale within EU. The purpose of our paper is to provide one possible explanation for the formation of FTA with common external tariff (CU) can lead to increase in both internal and external trade volume in a satiated oligopoly market.

The key for opening up the possibility of external trade volume to increase after formation of FTA is that the common tariff rate under FTA must be sufficiently lower than tariff rate adopted before the formation of FTA\(^2\). Abolition of tariff within region enhance trade within region, which is a trade liberalization effect, and with lower external tariff outside region the firms outside FTA region can also increase their export to the FTA region. The firms outside FTA can increase their sales to the FTA region by taking away the domestic share of firm within FTA region.

We carry out our examination of above possibility by setting up two country version of standard Hotelling’s spatial competition model with quality dimension. The reason why we adopt Hotelling’s model is that it captures the feature of satiated market where the size of market is constant. Often in such a market, firms not only compete in price but also in the quality of product they produce. To our knowledge, there is no analysis of the FTA with Hotelling’s model.

Automotive industry in EU market is subject to rules of origin (ROO) which is originally set up to prevent the circumvention of outside firms to export into EU through the lowest tariff country in EU. As it is pointed out by Ishikawa, Mukunoki, \(^2\) Whether the formation of EU in fact has lowered the external tariff or not is arguable. However, harmonization of technical regulation such as the EC Whole Vehicle Type-Approval can also deemed as reduction in effective tariff for the firms outside EU since they no longer have to tailor their passenger cars for country specific regulatory codes when exporting them to EU countries.
and Mizoguchi (2005), ROO make it possible for firms outside EU to segment the EU markets country by country, where as EU firms not subject to ROO face arbitrage in selling their final goods product within EU markets. This arbitrage condition imposed on within FTA firms is called market integration effect. We also identify this effect in the context of Hotelling’s spatial competition model.

In the Hotelling’s model, firm sets their price and quality levels to maximize profit, which as a result compete over the share of market with constant size. In this model, the existence of tariff gives advantage to firm inside FTA region in acquiring market share, which we call a tariff advantage effect. On the other hand market integration effect makes a firm inside FTA to shrink its share in small size market and enlarge its share in large size market. We find that these two effects work independently. Moreover, we find that quality competition between firms inside and outside FTA enforces the tariff advantage effect on firm inside FTA. Identification of these effects are contributions of our paper in the literature analyzing the role ROO in FTA.

The remainder of the paper is organized as follows. In section 2, we present the two countries version of Hotelling’s spatial competition model and define trade liberalization effect and market integration effect under the formation of FTA. Section 3 equilibria in Bertland price competition between duopoly with given level of qualities are characterized. Tariff advantage and quality advantage in market share acquisition for firm inside FTA will be identified. Here the basic effect of FTA formation on trade volumes within FTA regions and between FTA and non-FTA region is given. Lastly, in section 4, equilibria with quality competition is examined. It will be clarified that tariff advantage can enforce quality advantage under certain condition. We show what extra condition is needed for within FTA trade to increase under quality competition.

2. The Model

We consider the formation of an FTA between countries 1 and 2. Firm I (the inside firm) and firm O (the outside firm) respectively produce one final goods. Firm I produces goods I in country 1 and firm O produces goods O in the third country outside the FTA. The two firms supply their products to both countries 1 and 2 and in a

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3 They examined the role of ROO under FTA in a standard differentiated goods model with Bertland duopoly.
manner of Hotelling’s spatial competition with quality dimension of product
differentiation. Before FTA formation, country i (i=1, 2) sets a non-discriminatory tariff
on both goods, which is denoted by $t_i$, which is positive and exogenously given. After
FTA formation, both countries set zero tariffs on imports from the partner, but they
maintain their own tariff on imports form the non-members. We concentrate on the
case where both firms have the same production technology.

Following the Hotelling’s spatial competition model, in each country, there is a
continuum of consumers uniformly distributed on $[0, 1]$ interval. The population
(mass) of consumers in country 1 is normalized to one and the population of
consumers in country 2 is set to $\alpha$. If $\alpha$ is larger (smaller) than one, then the market
size is larger (smaller) in country 2. Firm $I$ and $O$ supply goods at point 0 and 1 of the
intervals respectively. Firm $J \in \{I, O\}$ supplies product at quality $q'$ with price
$p'_i$ in country $i$ (i=1, 2). A consumer in country $i$ located at $x \in [0, 1]$ receives net
utility $v + q' - p'_i - \tau$ if he/she buys goods from firm $I$ and $v + q^o - \tau + \alpha \tau$.
if he/she buys goods from firm $O$. A parameter $v>0$ captures the willingness to pay for
the product with minimum quality and $\tau>0$ captures the unit cost of transportation or
distance between the ideal product the consumer likes to consume and actual product
supplied.

Each consumer buys from the firm that offers higher net utility, as long as it is positive.
We shall assume that $v$ is large enough so that all consumers buy a product in
equilibrium, which captures the feature that markets are satiated. Hence, the demand
functions for firm $I$ in country 1 and 2 are given by

$$x^I_i(p'^I_i, p^O_i, q'^I_i, q^O) = \frac{1}{2\tau} \left( p^O_i - p'^I_i + q'^I_i - q^O + \tau \right),$$

$$x^I_2(p'^2_2, p^O_2, q'^2_i, q^O) = \frac{1}{2\tau} \left( p^O_2 - p'^2_2 + q'^2_i - q^O + \alpha \tau \right).$$

The demand functions for firm $O$ in country 1 and 2 are given by

$$x^O_i(p'^I_i, p^O_i, q'^I_i, q^O) = 1 - x^I_i(p'^I_i, p^O_i, q'^I_i, q^O) = \frac{1}{2\tau} \left( p^I_i - p'^I_i + q^O - q'^I_i + \tau \right).$$
\[ x^0_2(p^1_2, p^0_2, q^1, q^0) = \alpha - x^I_2(p^1_2, p^0_2, q^1, q^0) = \frac{1}{2\tau} (p^1_2 - p^0_2 + q^1 - q^0 + \alpha\tau). \]

After FTA formation, firm 1 meets ROOs but firm 0 cannot meet it. Thus, good 1 is always qualified for FTA preferential treatments (i.e., duty free access in the FTA) but good 0 is subject to the external tariffs when being traded between countries 1 and 2. The cost function for producing \( x \) units of goods with quality \( q \) is given by \( c \cdot x + F(q) \), in which \( c \) is a constant marginal cost and \( F(q) \) a fixed cost. We assume that \( F(q) > 0, F'(q) > 0 \) and \( F''(q) > 0 \). We also assume that the production technology is identical across firms. The total profit of each firm are given by

\[
\begin{align*}
\pi^I &= (p^I_1 - c)x^I_c(p^I_1, p^0_1, q^I, q^0) + (p^I_2 - c - \gamma t_2)x^I_c(p^I_2, p^0_2, q^I, q^0) - F(q^I) \\
\pi^O &= \sum_{i=1,2} (p^O_i - c - t_i)x^O_i(p^O_i, p^0_i, q^I, q^0) - F(q^O)
\end{align*}
\]

where \( \gamma \) is the parameter which takes \( \gamma = 0 \) if an FTA is formed, and it takes \( \gamma = 1 \) if an FTA is not formed.

There are many competitive arbitragers in countries 1 and 2 who engage in parallel imports or re-imports by purchasing goods in the low price market and selling them in the high price market. There are no additional costs in those arbitrage activities. Since good 1 is freely traded after the FTA formation, any price differential of good 1 within the FTA leads to arbitrage activities. Consequently, firm 1 is forced to set the uniform price in the two markets. That is, the markets for good 1 are completely integrated.

In the presence of ROOs, however, arbitrage activities must bear the external tariff when good 0 is traded between countries 1 and 2. That is, arbitrage activities cannot eliminate the price differential made by firm 0. Thus, the markets for good 0 are incompletely integrated or even segmented. ROOs basically play a role to preserve the differential tariffs. When the markets for good 0 are incompletely integrated, firm 0 sets the price differential such that all arbitrage activities are actually exhausted. When the markets for good 0 are segmented, firm 0 freely sets the prices among the markets. We consider the case where tariffs prevent price arbitrages and markets for both goods are segmented.

Therefore, FTA formation with ROOs leads to two effects. On one hand, an FTA
reduces trade costs of firm I. We call this effect the trade liberalization effect. On the other hand, the FTA induces arbitrage activities and eliminates the price differential made by firm I. We call this effect the market integration effect.

3. FTA formation and its effect on trade volumes under given quality levels

The sequence of events taken place in this model is as follows. Given tariff rates, both firms choose their quality levels in the first stage. In the second stage, both firms choose their price levels in each market given a pair of quality levels chosen at the first stage. We start our analysis from characterization of equilibrium of the second stage, in which basic results of the paper will be given.

3.1 Pre-FTA equilibrium

We first derive the pre-FTA equilibrium under given quality levels $q^I$ and $q^O$. Each firm chooses its price in order to maximize its profit. The first order conditions of profit maximization in this case are:

\begin{align}
\frac{\partial \pi}{\partial p^I} &= p^0_I - 2p^I_I + q^I - q^O + \tau + c = 0, \\
\frac{\partial \pi}{\partial p^I} &= p^O_I - 2p^I_I + q^I - q^O + \alpha \tau + t_2 = 0, \\
\frac{\partial \pi}{\partial p^O} &= p^I_I - 2p^I_O + q^O - q^I + \tau + t_1 + c = 0, \\
\frac{\partial \pi}{\partial p^O} &= p^O_O - 2p^O_O + q^O - q^I + \alpha \tau + t_2 + c = 0.
\end{align}

By solving above equations, the equilibrium prices are given by

\begin{align}
p^I_I &= \tau + \frac{1}{3} \{3c + t_1 + (q^I - q^O)\}, \\
p^O_I &= \tau + \frac{1}{3} \{3c + t_1 + (q^O - q^I)\}.
\end{align}
Moreover, the equilibrium quantities supplied in each market are given by

\[
\begin{align*}
(5-a) & \quad x^I_1 = \frac{1}{2} + \frac{1}{6\tau} \{ t_1 + (q^I - q^O) \}, \\
(5-b) & \quad x^O_1 = \frac{1}{2} - \frac{1}{6\tau} \{ t_1 + (q^I - q^O) \}, \\
(5-c) & \quad x^I_2 = \frac{\alpha}{2} + \frac{1}{6\tau} (q^I - q^O), \\
(5-d) & \quad x^O_2 = \frac{\alpha}{2} - \frac{1}{6\tau} (q^I - q^O).
\end{align*}
\]

The second term of (5-a), which is the equilibrium quantity of firm I in country 1, shows the market share advantage of firm I over firm O, which consists of two components; the tariff advantage component \( t_1 \) and the quality advantage component \( q^I - q^O \). Since the fundamental production technology and demand structure is symmetric among firms I and O, the basic shares of market for firm I and O are 1/2 and \( \alpha /2 \) respectively for country 1 and 2. The tariff advantage and quality advantage of each firm gives extra share of market in addition to these "basic shares of markets".

### 3.2 Post-FTA equilibrium without market integration

As a benchmark, we next examine post-FTA equilibrium when the market integration effect is absent. Here we examine the custom union equivalent of FTA in which tariff rate outside FTA is set to a common rate \( t \). The equilibrium prices and quantities supplied are given by

\[
\begin{align*}
(6-a) & \quad p^I_1 = \tau + \frac{1}{3} (3c + t + (q^I - q^O)),
\end{align*}
\]
Proposition 1
FTA formation without market integration increases trade volume within FTA and it increases trade volume between FTA region and non FTA region if and only if $t < t_1/2$.

Proof: It is straightforward to see the trade volume within FTA increases with the formation of FTA. Trade volume within FTA is $x_t^I$ which increases by a tariff advantage component after the formation of FTA. The trade volume between FTA region and non-FTA region is given by $x_t^O + x_t^I$. It is $(1+\alpha)/2 - (t_1 + 2(q^I - q^O))/6\tau$ before FTA is formed and $(1+\alpha)/2 - (t + 2(q^I - q^O))/6\tau$ after FTA is formed. The latter is larger if and only if $t < t_1/2$.

Since FTA formation yields trade liberalization effect on firm $I$ which is a reduction of tariff for firm $I$ in exporting its product to country 2, the export of firm $I$ to country 2 increases. This can be identified as the term $t$ in a right-hand-side bracket of equation (7-c). This also means that share of firm $O$ in country 2 decreases. Thus in order to have total exports of firm $O$ to FTA region increase, the firm $O$’s export to country 1 must increase more than compensating the loss of its market share in country 1. This can be achieved only when tariff rate of FTA region must decrease large enough. Both necessity and sufficiency for such a situation is given by the condition $t < t_1/2$. The
reason why this condition is necessary and sufficient is as follows. First of all, before FTA formation, there is no tariff advantage for firm \( I \) in country 2. But with formation of FTA, tariff advantage of firm \( I \) in country 2 arises and firm \( O \) looses its market share in country 2 by the amount \( t/6\tau \). In order to compensate this loss of market share in country 2, firm \( O \) should gain its market share in country 1 through overall reduction in tariff rate. Firm \( O \)'s gain in market share in country 1 from reduction of tariff from \( t \) to \( (t_1 - t)/6\tau \) which should be larger than \( t/6\tau \) in order to more than compensate the loss of its market share in country 2. This yields the necessary and sufficient conditions for the overall increase in the exports of firm \( O \) to the FTA region after the formation of FTA.

In next subsection, we examine how the market integration effect changes the condition required for the increase in trade volumes.

### 3.3 Post-FTA equilibrium with market integration

We now examine the post-FTA equilibrium in the presence of market integration. Let \( p' (= p'_1 = p'_2) \) denote the uniform price set by firm \( I \). The first order conditions of profit maximization are:

\[
\begin{align*}
\text{(8-a)} & \quad \frac{\partial \pi^I}{\partial p'} = p'_1^O + p'_2^O - 4 p' + 2(q^I - q^O) + (1 + \alpha)\tau + 2c = 0, \\
\text{(8-b)} & \quad \frac{\partial \pi^O}{\partial p'_1} = p' - 2p'_1^O + q^O - q^I + \tau + t + c = 0, \\
\text{(8-c)} & \quad \frac{\partial \pi^O}{\partial p'_2} = p' - 2p'_2^O + q^O - q^I + \alpha\tau + t + c = 0.
\end{align*}
\]

The equilibrium prices and quantities are given by

\[
\begin{align*}
\text{(9-a)} & \quad p' = \frac{1 + \alpha}{2} \tau + \frac{1}{3} (3c + t + (q^I - q^O)), \\
\text{(9-b)} & \quad p'_1^O = \frac{3 + \alpha}{4} \tau + \frac{1}{3} (3c + 2t + (q^O - q^I)), \\
\text{(9-c)} & \quad p'_2^O = \frac{1 + 3\alpha}{4} \tau + \frac{1}{3} (3c + 2t + (q^O - q^I)).
\end{align*}
\]
We have a following proposition on trade volumes.

**Proposition 2**
*FTA formation with market integration increases trade volume within FTA if and only if*  
\[ t > (3/4)(1 - \alpha) \]  
*and it increases trade volume between FTA region and non FTA region if and only if*  
\[ t < t_1/2. \]

**Proof:** The sufficient and necessary condition for the within FTA trade volume to increase can be straightforwardly derived by comparison of equations (5-c) and (10-c). The sufficient and necessary condition for the increase in trade volume between FTA region and non FTA region can be derived in the same manner as we did in Proposition 1. 

The market integration effects appear in the changes in basic market shares of firm $I$ and $O$. After the formation of FTA with market integration, basic market shares for firm $I$ in country 1 and country 2 change from $1/2$ and $\alpha/2$ to $(5 - \alpha)/8$ and $(5\alpha - 1)/8$ respectively. If the market size of country 2 is small, i.e. if $\alpha$ is small, the basic market share of firm $I$ in country 1 becomes larger and that of country 2 becomes smaller after the formation of FTA. This result can be interpreted as follows. If market size of country 2 is small, firm $I$'s price before FTA is basically lower in country 2 than in country 1. In this case, the market integration effect, which is just an arbitrage condition for firm $I$’s product, forces firm $I$ to set lower price in country 1 and higher price in country 2 after the formation of FTA with market integration compared to the case where there is no presence of market integration. The lower price in country 1 enlarges basic market share of firm $I$ in country 1 and the lower price in country 2 shrinks basic market share of firm $I$ in country 2. However, total basic market share of firm $I$, which is a sum of basic market shares of firm $I$ in country 1 and 2, remains to be
(1+\alpha)/2 even after the formation of FTA with market integration. The decrease in basic market share in one market is compensated by the increase in the other market.

Above interpretation facilitates us to understand the difference and similarity between proposition 1 and 2. The extra condition \( t > (3\tau/4)(1-\alpha) \), in another words extra tariff advantage, is needed to assure the increase in within FTA regional trade after the formation of FTA with market integration, since firm I's basic market share in country 2 becomes smaller after the FTA formation with market integration if market size of country 2 is small. On the other hand, total basic market share of firm O in FTA region remains constant even after the FTA formation with market integration, the condition required for between FTA and non-FTA region remains same as in proposition 1 where there is no market integration.

So far, we have kept quality levels constant and focused on “the tariff advantage” to see what conditions are required for the increase in trade volumes after the formation of FTA. As is often the case with in satiated market, firms often compete in quality in such a market. In this case, the quality advantage component \( q' - q^o \) of trade volumes are also affected by the formation of FTA. We examine how the conditions required for the increase in trade volumes after FTA formation under quality competition.

4. Quality Competition

We now examine the effect of quality competition on trade volumes. Expecting the result of the second stage game, firm I and firm O choose their quality levels.

4.1 Pre-FTA equilibrium conditions

Given the equilibrium in the second stage game as described in subsection 3.1, firm I and O’s profit functions under pre-FTA regime are

\[
\pi^I(q',q^o) = \frac{\tau}{2} \left(1 + \frac{q' - q^o + t_i}{3\tau}\right)^2 + \frac{\tau}{2} \left(\alpha + \frac{q' - q^o + t_i}{3\tau}\right) - F(q')
\]
\[
\pi^0(q^I, q^O) = \frac{\tau}{2} \left( 1 + \frac{q^I - q^0}{3} - t_I \right)^2 + \frac{\tau}{2} \left( \alpha + \frac{q^O - q^I}{3} \right)^2 - F(q^O).
\]

Two firms choose their quality levels in order to maximize their profits. The first order conditions of their profit maximization are:

(11-a) \[ \frac{\partial \pi^I}{\partial q^I} = \left[ \frac{1 + \alpha}{3} + \frac{2(q^I - q^0) + t_I}{9\tau} \right] - F'(q^I) = 0, \]

(11-b) \[ \frac{\partial \pi^O}{\partial q^0} = \left[ \frac{1 + \alpha}{3} - \frac{2(q^I - q^0) + t_I}{9\tau} \right] - F'(q^O) = 0. \]

We need to have a second order condition which is \( F''(q) > 2/9\tau \). However, we need extra condition, a condition for the stability of equilibrium, for second order derivative of fixed cost function, which we examine in the following paragraph.

The equation (11-a) is a reaction function of firm \( I \) whose slope in the \((q^I, q^O)\) space can be calculated as

(12-a) \[ \frac{dq^0}{dq^I} = \frac{2/9\tau - F''(q^I)}{(2/9\tau)}. \]

Similarly, the equation (11-b) is a reaction function of firm \( O \) and its slope is given by

(12-b) \[ \frac{dq^0}{dq^I} = \frac{2/9\tau}{(2/9\tau - F''(q^O))}. \]

In order to have a stability of the equilibrium, we must have the slope of the reaction function of firm \( I \) to be steeper than that of firm \( O \). This can be assured by the condition,

(13) \[ F''(q) > 4/9\tau, \]

which also assures the second order condition. A pre-FTA equilibrium pair of quality levels is a solution to the system of equations (11-a) and (11-b).
4.2 Conditions for post-FTA equilibrium without market integration

Given the equilibrium of the second stage game described in section 3.2, firm I and O’s profit functions under post-FTA without market integration regime are

\[
\pi^I(q^I, q^O) = \frac{\tau}{2} \left( 1 + \frac{q^I - q^O + t}{3\tau} \right)^2 + \frac{\tau}{2} \left( \alpha + \frac{q^I - q^O + t}{3\tau} \right)^2 - F(q^I),
\]

\[
\pi^O(q^I, q^O) = \frac{\tau}{2} \left( 1 + \frac{q^O - q^I - t}{3\tau} \right)^2 + \frac{\tau}{2} \left( \alpha + \frac{q^O - q^I - t}{3\tau} \right)^2 - F(q^O).
\]

Two firms choose their quality levels to maximize their profits. The first order conditions of their profit maximizations are,

(14-a) \[ \frac{\partial \pi^I}{\partial q^I} = \left[ \frac{1 + \alpha}{3} + \frac{2(q^I - q^O) + 2t}{9\tau} \right] - F'(q^I) = 0, \]

(14-b) \[ \frac{\partial \pi^O}{\partial q^O} = \left[ \frac{1 + \alpha}{3} - \frac{2(q^I - q^O) + 2t}{9\tau} \right] - F'(q^O) = 0. \]

As in pre-FTA equilibrium, the condition (13) is sufficient for the second order condition and a stability of equilibrium to be satisfied, since the slope of reaction functions (14-a) and (14-b) on \((q^I, q^O)\) space are given again by (12-a) and (12-b) respectively. An equilibrium pair of quality levels in Post-FTA without market integration is a solution to the system of equations (14-a) and (14-b).

4.3 Conditions for post-FTA equilibrium with market integration

Given the equilibrium of second stage game described in section 3.3, firm I and O’s profit functions under post-FTA with market integration regime are
Two firms choose their quality levels to maximize their profits. The first order conditions of their profit maximizations turn out to be the same ones as in post-FTA with market integration which are (14-a) and (14-b). Hence an equilibrium pair of quality levels in post-FTA with market integration becomes the same as in that without market integration.

4.4 Results under quality competition

Given conditions for determining equilibrium quality levels for each regime, we can compare the equilibrium quality levels and corresponding trade volumes. We first examine the property of firm I’s quality advantage component, and state the results on effect of FTA formation on trade volumes under quality competition.

Proposition 3
Under condition (13), in all equilibrium with a positive tariff, the equilibrium quality level of firm I is higher than that of firm O.

Proof: See Appendix.

Proposition 3 states that as long as there is a tariff advantage for firm I, there will also be a positive quality advantage exists for firm I regardless of whether FTA is formed or not. Imposition of tariff on firm O is a virtually increase in marginal cost for firm O. Thus price cost margin of firm O is smaller than that of firm I when there is a positive tariff. This means that marginal revenue of increasing quality level is higher for firm I than firm O so that firm I sets higher quality level than firm O in equilibrium. Hence, tariff advantage is complemented with quality advantage effect when there is a quality competition.

The rest of the section is devoted to examine how this quality advantage effect is
affected by FTA formation. In order to examine the effect of FTA formation on trade volumes, we now consider a specific form of fixed cost function $F(q)$ in order to further take a look at the effect of FTA formation on trade volume under quality competition. Let the fixed cost function be

\[
F(q) = \beta q^2 / 2.
\]

The condition (13) imposes the condition $\beta > 4/9\tau$.

With this specification, solving the system of equations (11-a) and (11-b) the pre-FTA equilibrium quality difference between firm I and O can be expressed as,

\[
q^I - q^O = \frac{2t_1}{9\tau\beta - 4} > 0.
\]

In pre-FTA equilibrium with quality competition, quantity of firm I and O supplying in each market can be calculated as follows,

\[
\begin{align*}
\text{(17-a)} & \quad x^I_t = \frac{1}{2} + \frac{t_1}{6\tau} \left( \frac{9\tau\beta - 2}{9\tau\beta - 4} \right), \\
\text{(17-b)} & \quad x^O_t = \frac{1}{2} - \frac{t_1}{6\tau} \left( \frac{9\tau\beta - 2}{9\tau\beta - 4} \right).
\end{align*}
\]

Hence, total trade volume between firm O and countries 1 and 2 is given by

\[
\text{(18)} \quad x^O_t + x^O_t = \frac{1 + \alpha}{2} - \frac{t_1}{6\tau} \left( \frac{9\tau\beta}{9\tau\beta - 4} \right).
\]

Solving for the system of equations (14-a) and (14-b) the post-FTA equilibrium quality difference between firm I and O can be expressed as,

\[
q^I - q^O = \frac{4t_1}{9\tau\beta - 4} > 0.
\]

In post-FTA equilibrium without market integration, the equilibrium quantities supplied by firm I and O can be calculated as,
Similarly, in post-FTA equilibrium with market integration, the equilibrium quantities supplied by firm $I$ and $O$ can be calculated as,

\begin{align*}
(20-a) & \quad x_1^I = \frac{1}{2} + \frac{t}{6\tau} \left( \frac{9\tau \beta}{9\tau \beta - 4} \right), \quad x_1^O = \frac{1}{2} - \frac{t}{6\tau} \left( \frac{9\tau \beta}{9\tau \beta - 4} \right) \\
(20-b) & \quad x_2^I = \frac{\alpha}{2} + \frac{t}{6\tau} \left( \frac{9\tau \beta}{9\tau \beta - 4} \right), \quad x_2^O = \frac{\alpha}{2} - \frac{t}{6\tau} \left( \frac{9\tau \beta}{9\tau \beta - 4} \right).
\end{align*}

Note that the second terms of right-hand-side of corresponding equations for (19) and (21) are exactly the same. Hence, total trade volume between firm $O$ and countries 1 and 2 in both for post-FTA equilibrium with and without market integration is given by

\begin{equation}
(22) \quad x_1^O + x_2^O = \frac{1+\alpha}{2} - \frac{t}{3\tau} \left( \frac{9\tau \beta}{9\tau \beta - 4} \right).
\end{equation}

We have a following proposition regarding effects of FTA formation on the quality difference between firm $I$ and $O$, and on the trade volumes.

Proposition 4
Under the specification of fixed cost function (15) with a stability condition $\beta > 4/9\tau$,

(a) the trade volume within FTA region increases after the formation of FTA if $t > 2t_1/9\tau \beta$ in case of FTA without market integration, and if $t > \frac{2t_1}{9\tau_\beta} + \frac{9\tau \beta - 4 - \alpha}{3\tau \beta} \quad$ in case of FTA with market integration,

(b) the trade volume between FTA region and non-FTA region increases after FTA formation if $t < t_1/2$, and

(c) the quality advantage of firm $I$, $q^I - q^O$, decreases if $t < t_1/2$. 

\[\text{(a) } \quad x_1^O + x_2^O = \frac{1+\alpha}{2} - \frac{t}{3\tau} \left( \frac{9\tau \beta}{9\tau \beta - 4} \right).\]
Proof: Part (a) is straightforwardly derived by comparing equations (17-b), (20-b) and (21-b). Part (b) is straightforwardly derived by comparing equations (18) and (22). Part (c) is straightforwardly derived by comparing equations (16) and (19).

Let us first discuss part (c) of the proposition 4 to gain insights on the effect of FTA formation on trade volume. The condition required for part (c) is exactly same as the condition for total supply of firm \( O \) to FTA region increase in case of no quality competition (proposition 1 and 2). In order for firm \( O \) to be able to expand its production, its price-cost margin should become wider. The condition \( t < t_i / 2 \) is exactly a condition needed for the price-cost margin to increase. If price cost margin for firm \( O \) increases, the marginal revenue from increasing quality for firm \( O \) increases so that firm \( O \) raise its quality level. The opposite is happening under the condition \( t < t_i / 2 \) for firm I. Hence the decrease in quality advantage for firm I occurs.

The decrease in quality advantage for firm I enables for firm \( O \) to expand its supply. Thus under the condition that enables for firm \( O \) to increase the price cost margin, quality competition enforces the increase in total supply of firm \( O \) by the formation of FTA. This is why the condition in part (b) of proposition 4 is exactly same as in propositions 1 and 2.

The condition required for the within FTA trade volumes (firm I’s supply to country 2) to increase becomes more stringent compared to the case where there is no quality competition. This is because that with a possibility that price cost margin of firm \( O \) to increase after the formation of FTA, there must be some tariff advantage for firm I to be left in order to assure its market share in country 2. It turned out that lower bound for the common tariff after FTA formation is \( 2t_i / 9\pi \beta \) in case of FTA without market integration, which is smaller than \( t_i / 2 \) under the stability condition \( \beta > 4/9\pi \). Hence only when common tariff rate after the formation of FTA is between \( t_i / 2 \) and \( 2t_i / 9\pi \beta \) both trade volumes within and outside FTA region increases after the FTA formation. The market integration effect works in a similar manner as in case without quality competition.

Appendix
Proof of Proposition 3
The proof proceeds as follows. First, (a) we show that equilibrium level of qualities chosen by both firms become identical if tariff rate is zero. Then (b) we show that equilibrium quality level of firm I increases and that of firm O decreases as tariff increases starting from any positive tariff. These facts implies that equilibrium quality level of firm I is higher than that of firm O for a given positive tariff.

Part (a): Equilibrium pairs of quality levels are determined by (11-a) and (11-b) in case of pre-FTA, and (14-a) and (14-b) in cases of both post-FTA with and without market integration. In all case it can be easily seen that equilibrium quality levels becomes identical if tariff is zero.

Part (b): Now totally differentiating the system of equations (11-a) and (11-b), we get

\[
\begin{bmatrix}
1 - \frac{9\pi}{2} F^*(q^I) & -1 \\
-1 & 1 - \frac{9\pi}{2} F^*(q^O)
\end{bmatrix}
\begin{bmatrix}
dq^I \\
dq^O
\end{bmatrix}
= \begin{bmatrix}
-\frac{dt}{2} \\
\frac{dt}{2}
\end{bmatrix}
\]

The determinant of coefficient matrix of the left hand side of equation (A-1) can be calculated as

\[
\Delta = (1 - \frac{9\pi}{2} F^*(q^I))(1 - \frac{9\pi}{2} F^*(q^O)) - 1
\]

which is positive under condition (13). We readily solve for (A-1) to obtain

\[
dq^I / dt = \frac{9\pi F^*(q^O)}{4\Delta} > 0
\]

and

\[
dq^O / dt = -\frac{9\pi F^*(q^I)}{4\Delta} < 0.
\]

Hence in pre-FTA case, for a given positive tariff \( t_i \), is \( q^I \) higher than \( q^O \). In case of post-FTA, the total differentiation of the system of equations (14-a) and (14-b) becomes similar to (A-1) where the left hand side of (A-1) is \((- dt, \ dt)^T\). Thus under condition (13), we obtain

\[
dq^I / dt = \frac{9\pi F^*(q^O)}{2\Delta} > 0
\]

and

\[
dq^O / dt = -\frac{9\pi F^*(q^I)}{2\Delta} < 0.
\]

These proves the proposition 3. □

References