Search

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The Basic Model of Sequential Search

- Discrete Time Model
- Continuous Time Model

The Set Up of Discrete Time Model

- The decision maker samples sequentially from the distribution F. I.e., he faces a sequence $\{x_i\}$ of i.i.d. random variables with cumulative distribution function F
- Each observation costs c.
- He can stop search at any time n.
- Let y_n be the prize received if search is stopped after n samples.
  For search with recall, $y_n = \max\{x_1, x_2, ..., x_n\}$.
  For search without recall, $y_n = x_n$
- Payoff for the decision maker stopping after n samples is denoted Y_n where
  $$Y_n = y_n - nc$$
  If the decision maker discount the the sampling time with discount factor $\delta < 1$, then the payoff is
  $$Y_n = \delta^{n-1} y_n - \sum_{i=1}^{n} \delta^{i-1} c = \delta^{n-1} y_n - \frac{1-\delta^n}{1-\delta} \cdot c.$$

The Problem

- A **stopping rule** prescribes a rule after what sequences of observations to stop the sampling.
- A **stopping time** resulting from a stopping rule is the integer n after which the sampling will stop, if that stopping rule is invoked. *The stopping time is a random variable* (because samples are random variables).
- A stopping rule, S, determines a random stopping time $N(S)$.
- The problem the decision maker is facing is to find and use a stopping rule, S, that maximizes its expected payoff,

$$E[Y_{N(S)}].$$

- The solution to this problem (a stopping rule), is called an **optimal stopping rule**.

An Optimal Stopping Rule is a Reservation Value Rule

- A **reservation value rule** is a stopping rule which prescribes to stop sampling after n observations if and only if $y_n \geq y$. The level y is called the reservation value.
- It turns out that under certain conditions, a optimal stopping rule is a reservation value rule.

Proposition
If $E[x_i^2]$ is finite, then there exists an optimal stopping rule. It is a reservation value rule.

See Maurice DeGroot (1970, chapter 13.9) for the proof.

Some Properties of the Reservation Value Rule (I)

- Let $N(y)$ be the stopping time associated with the reservation value rule with reservation value y. Since probability of $N(y)$ being i is $F(y)^{i-1}(1-F(y))$, we can calculate the expected value of $N(y)$ as follows;

$$E[N(y)] = \sum_{i=1}^{\infty} iF(y)^{i-1}(1-F(y)) = 1/(1-F(y)). \qquad (1)$$

- Let $V(y)$ be the expected value of reservation value rule with reservation value y. I.e., $V(y) \equiv E[Y_{N(y)}]$. This can be calculated as follows;

$$\begin{aligned} V(y) = E[Y_{N(y)}] &= E[y_n - N(y)c] = E[x|x \geq y] - cE[N(y)] \\ &= \int_y^{\infty} x \frac{dF(x)}{1-F(y)} - \frac{c}{1-F(y)}. \end{aligned} \qquad (2)$$

Some Properties of the Reservation Value Rule (II)

Proposition

If F has a connected support, then the optimal policy has a reservation value y^* such that $y^* = V(y^*)$.

- The optimal reservation value y^* solves the equation,

$$c = \int_{y^*}^{\infty} (x - y^*) dF(x) \tag{3}$$

Comparative statics results;
(1) If the search cost c is higher, then the reservation value y^* becomes lower.
(2) If cumulative distribution function for samples G is a mean preserving spread of F, the reservation value under G is higher than that under F. (G is mean preserving spread of F if (i) they have the same mean and (ii) $\int_{-\infty}^{y} G(x) - F(x) dx \geq 0$ for all y)

Homework I

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- Prove comparative statics results in the previous slide.
- Derive expression for $V(y)$ (equivalent of expression (2)) if decision maker discounts time with discount factor $\delta < 1$.
- Using above result, perform a comparative static of change in δ on the optimal reservation value.