$\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$ は線形独立か？
1次

$$c_1\begin{pmatrix}1\\2\\1\end{pmatrix} + c_2\begin{pmatrix}1\\0\\-1\end{pmatrix} + c_3\begin{pmatrix}0\\0\\1\end{pmatrix} = \vec{0}$$

(1次関係があるとする.)

$$= \begin{pmatrix}c_1 + c_2\\ 2c_1\\ c_1 - c_2 + c_3\end{pmatrix}$$

第2成分 $2c_1 = 0$ だから $c_1 = 0$

第1成分 $c_1 + c_2 \underset{\substack{\uparrow\\ c_1 = 0}}{=} c_2 = 0$

$c_1 = c_2 = 0$

第3成分 $0 = c_1 - c_2 + c_3 = c_3 \longrightarrow c_3 = 0$

$\Longrightarrow c_1 = c_2 = c_3 = 0$

([illegible])

$\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$ は線形独立
1次.

$$\begin{pmatrix}1&1&0\\2&0&0\\1&-1&1\end{pmatrix} \xrightarrow[\substack{1行\times(-1)\\を3行に}]{\substack{1行(-2)\\を2行に}} \begin{pmatrix}1&1&0\\0&-2&0\\0&-2&1\end{pmatrix} \to \begin{pmatrix}1&1&0\\0&-2&0\\0&0&1\end{pmatrix}$$

$$\begin{array}{rrrr} & 2 & 0 & 0\\ -) & 2 & 2 & 0\\ \hline & 0 & -2 & 0\end{array} \qquad \begin{array}{rrrr} & 1 & -1 & 1\\ -) & 1 & 1 & 0\\ \hline & 0 & -2 & 1\end{array} \qquad \begin{array}{rrrr} & 0 & -2 & 1\\ -) & 0 & -2 & 0\\ \hline & & & 1\end{array}$$

rank 3

$\vec{a}_1, \cdots, \vec{a}_k \in \mathbb{R}^n$ が 1次独立

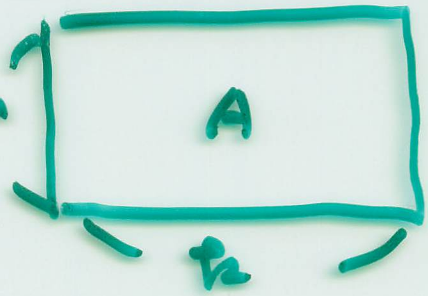
$\Longleftrightarrow$ $\boxed{c_1 \vec{a}_1 + c_2 \vec{a}_2 + \cdots + c_k \vec{a}_k} = \vec{0}$

$\Longrightarrow$ $c_1 = c_2 = \cdots = c_k = 0$

(自明な線形関係しかない)

$A = (\vec{a}_1 \cdots \vec{a}_k)$

$n \times k$ ぎょうれつ



$= A \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix}$

$\Longleftrightarrow$ $A\vec{x} = \vec{0}$ は自明解しかない

$\Longleftrightarrow$ $\mathrm{rank}(A) = k$

$\leftarrow$ 変数の個数,

れつベクトルの個数

A : n次正方行列 とする

A : rank n $\Longleftrightarrow$ $\det(A) \neq 0$

$$\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \times (-2) \times 1 = -2$$

三角行列

1行 × (−2) を 2行に +

1行 × (−3) を 3行に +

1列について余因子展開

$$= 1 \times \begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix} - 0 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & 0 \\ -2 & 0 \end{vmatrix}$$

---

(4) $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & -1 & -3 \\ 3 & 1 & -1 \\ 2 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & -3 \\ 0 & 4 & 8 \\ 0 & 3 & 6 \end{pmatrix}$$

$$\begin{array}{rrr} 3 & 1 & -1 \\ -)\ 3 & -3 & -9 \\ \hline 0 & 4 & 8 \end{array} \qquad \begin{array}{rrr} 2 & 1 & 0 \\ -)\ 2 & -2 & -6 \\ \hline 0 & 3 & 6 \end{array}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & -3 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$\mathrm{rank}(A) = 2$

$\Longrightarrow$ $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix}$ は

[illegible]

$$\begin{vmatrix} 1 & -1 & -3 \\ 3 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 \\ 0 & 4 & 8 \\ 0 & 3 & 6 \end{vmatrix} = 4\times 3 \begin{vmatrix} 1 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= 0$$

[illegible] $\times(-3)$ [illegible]

[illegible] $\times(-2)$ [illegible]

[illegible] $\begin{vmatrix} a \\ b \\ b \end{vmatrix} = 0$ [illegible]

$$\det\begin{pmatrix} \lambda a \\ b \\ c \end{pmatrix} = \lambda \det\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\det\begin{pmatrix} a \\ \lambda b \\ c \end{pmatrix} = \lambda \det\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\begin{array}{ccc} 0 & k-4 & -2k-1 \\ 0 & k-4 & -k+4 \\ \hline 0 & 0 & -k-5 \end{array}$$

-2

$k = -5$ and 2

rank$(A) = 2$

$k \neq -5$ and 2

rank$(A) = 3$

$k = 2$ & $k = -5$ and 2

rank$(A) = 2$

otherwise

rank$(A) = 3$

$$\cancel{\begin{pmatrix} 1 & k & 2 \\ 2 & ? & 0 \\ 1 & -1 & 1 \end{pmatrix}}$$

$$A = \boxed{\begin{pmatrix} 1 & k & 2 \\ 1 & 2 & k \\ 2 & k & -1 \end{pmatrix}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{0}$$

[illegible] k는?

① $\mathrm{rank}(A) \le 2$ [illegible]

② $\det(A) = 0$

---

①
$$\begin{pmatrix} 1 & k & 2 \\ 1 & 2 & k \\ 2 & k & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & k & 2 \\ 0 & \boxed{2-k} & k-2 \\ 0 & \boxed{k-4} & -2k-1 \end{pmatrix} \rightarrow$$

$$\begin{array}{rrr} 1 & 2 & k \\ -)\ 1 & k & 2 \\ \hline 0 & 2-k & k-2 \end{array} \qquad \begin{array}{rrr} 2 & k & -1 \\ -)\ 2 & 4 & 2k \\ \hline 0 & k-4 & -1-2k \end{array}$$

case 1 $k = 2$ [illegible]

$$\begin{pmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & -2 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 2 \\ 0 & -2 & -5 \\ 0 & 0 & 0 \end{pmatrix}$$

$\mathrm{rank}(A) = 2$

case 2 $k \ne 2$ [illegible]

$\xrightarrow{\text{[illegible]}}$ $\frac{1}{2-k}$ [illegible]

$$\begin{pmatrix} 1 & k & 2 \\ 0 & 1 & -1 \\ 0 & k-4 & -2k-1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & k & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -k-5 \end{pmatrix}$$

[illegible] $-(k-4)$ [illegible]

[illegible]

$$\det\begin{pmatrix} 1 & k & 2 \\ 1 & 2 & k \\ 2 & k & -1 \end{pmatrix} = \det\begin{pmatrix} 1 & k & 2 \\ 0 & 2-k & k-2 \\ 0 & k-4 & -2k-1 \end{pmatrix}$$

$$= 1 \times \begin{vmatrix} 2-k & k-2 \\ k-4 & -2k-1 \end{vmatrix} = (2-k)\begin{vmatrix} 1 & -1 \\ k-4 & -2k-1 \end{vmatrix}$$

1列に関する余因子展開

$$= (2-k)\left\{(-2k-1) - (-1)(k-4)\right\}$$

$$= (2-k)(-k-5)$$

$$= (k-2)(k+5) = 0$$

0が [illegible]

$$\boxed{k = 2 \text{ または } k = -5}$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 2 & 3 & c \\ 1 & c & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

1に [illegible]

Dが [illegible] ?

$$\rightarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 5 & c+2 \\ 0 & c+1 & 4 \end{pmatrix}$$

$$\begin{array}{rrr} 2 & 3 & c \\ -)\ 2 & -2 & -2 \\ \hline 0 & 5 & c+2 \end{array}$$

$$\begin{array}{rrr} 1 & c & 3 \\ -)\ 1 & -1 & -1 \\ \hline 0 & c+1 & 4 \end{array}$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 5 & c+2 \\ 0 & c+1 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & \frac{c+2}{5} \\ 0 & c+1 & 4 \end{pmatrix}$$

2행 $\times(c+1)$ [illegible]

$$\longrightarrow \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & \frac{c+2}{5} \\ 0 & 0 & \boxed{-\frac{(c+1)(c+2)}{5} + 4} \end{pmatrix}$$

$$\begin{matrix} 0 & c+1 & 4 \\ 0 & c+1 & \frac{(c+2)(c+1)}{5} \end{matrix}$$

$$= \frac{-(c^2+3c+2)+20}{5}$$

$$= -\frac{c^2+3c-18}{5}$$

$$= -\frac{(c+6)(c-3)}{5}$$

① $c = -6$ 또는 3 이면

rank 2

② otherwise

rank 3

$$\det\begin{pmatrix} 1 & -1 & -1 \\ 2 & 3 & c \\ 1 & c & 3 \end{pmatrix} = \det\begin{pmatrix} 1 & -1 & -1 \\ 0 & 5 & c+2 \\ 0 & c+1 & 4 \end{pmatrix}$$

$$= 1 \times \begin{vmatrix} 5 & c+2 \\ c+1 & 4 \end{vmatrix} = 20 - (c+1)(c+2)$$

$$= -(c^2+3c-18)$$

$$x = \frac{\left| \vec{b} \ \ \vec{a_2} \ \ \vec{a_3} \right|}{|A|}$$

$$y = \frac{\left| \vec{a_1} \ \ \vec{b} \ \ \vec{a_3} \right|}{|A|}$$

$$z = \frac{\left| \vec{a_1} \ \ \vec{a_2} \ \ \vec{b} \right|}{|A|}$$

クラメールの公式.

$$\boxed{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boxed{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}$$

$$\| \quad A = (\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3) \qquad\qquad \| \quad \vec{b}$$

$$\boxed{x\vec{a}_1 + y\vec{a}_2 + z\vec{a}_3 = \vec{b}}$$

$\det(A) \neq 0$ とする → 解は一意 (な)

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\vec{b} \text{ が解.}$$

$$|\vec{b} \ \vec{a}_2 \ \vec{a}_3| = |(x\vec{a}_1 + y\vec{a}_2 + z\vec{a}_3) \ \vec{a}_2 \ \vec{a}_3|$$

1列に関する線形性

$$= x|\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3| + y\underbrace{\boxed{|\vec{a}_2 \ \vec{a}_2 \ \vec{a}_3|}}_{} \overset{0}{\|}$$

$$+ z\boxed{|\vec{a}_3 \ \vec{a}_2 \ \vec{a}_3|} \ \overset{\|}{0}$$

$$|\vec{b} \ \vec{a}_2 \ \vec{a}_3| = x|\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3| \quad \neq 0$$

$$x = \frac{|\vec{b} \ \vec{a}_2 \ \vec{a}_3|}{|\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3|}$$

~~偶置換 奇回~~

偶順列，奇順列

(1 2 3 4)

(3 2 4 1)

$(3\ 2\ 4\ 1) \to (1\ 2\ 4\ 3) \to (1\ 2\ 3\ 4)$

2回

$(3\ 2\ 1\ 4) \to (1\ 2\ 3\ 4)$

$(3\ 1\ 2\ 4) \to (1\ 3\ 2\ 4) \to (1\ 2\ 3\ 4)$

4回

偶数回でできる.

$\mathrm{sign}(3\ 2\ 4\ 1) = 1$

sign

$(1\ 3\ 2\ 4) \to (1\ 2\ 3\ 4)$ odd

$\mathrm{sign}(1\ 3\ 2\ 4) = -1$

置換 σ : odd -1

even 1

$$A = \begin{pmatrix} a_{11} & \cdots & a_{14} \\ \vdots & & \vdots \\ a_{41} & \cdots & a_{44} \end{pmatrix}$$

a_{ij}

i행 j열

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$\det(A) = \sum_{\sigma} \operatorname{sgn}(\sigma)\, a_{\sigma(1)1}\, a_{\sigma(2)2}\, a_{\sigma(3)3}\, a_{\sigma(4)4}$$

4! 순열

$$\sigma = (3\ 2\ 4\ 1)$$

$$\sigma(1) = 3,\ \sigma(2) = 2,\ \sigma(3) = 4,\ \sigma(4) = 1$$