月

復習

① 3次の行列式の性質 104～114 page.

A : 3×3

- $\det({}^tA) = \det(A)$ → … おまけ.

- A : 正則 $\Longleftrightarrow$ $\det(A) \neq 0$

- クラメール, 余因子展開

- A, B : 3×3

  $\det(AB) = \det A \cdot \det B$

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② 1次独立, 従属

③ テストの説明

$$A = \begin{pmatrix} 1 & 2 & 1 & -3 & 2 \\ 3 & 6 & 4 & 2 & -1 \\ 5 & 10 & 6 & -4 & 3 \\ 2 & 4 & 1 & -17 & 11 \end{pmatrix}$$

(kernel)

$$V_{1,0} = \{ \vec{v} \in \mathbb{R}^5 ; \; A\vec{v} = \vec{0} \} = \ker(A)$$

$$\begin{pmatrix} 1 & 2 & 1 & -3 & 2 \\ 3 & 6 & 4 & 2 & -1 \\ 5 & 10 & 6 & -4 & 3 \\ 2 & 4 & 1 & -17 & 11 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & -3 & 2 \\ 0 & 0 & 1 & 11 & -7 \\ 0 & 0 & 1 & 11 & -7 \\ 0 & 0 & -1 & -11 & 17 \end{pmatrix}$$

1re l × (−3) à 2e ligne

1re l × (−5) à 3e ligne

1re l × (−2) à 4e ligne

$$\begin{array}{rrrrrr} & 3 & 6 & 4 & 2 & -1 \\ - & 3 & 6 & 3 & -9 & 6 \\ \hline & 0 & 0 & 1 & 11 & -7 \end{array} \qquad \begin{array}{rrrrrr} & 5 & 10 & 6 & -4 & 3 \\ - & 5 & 10 & 5 & -15 & 10 \\ \hline & 0 & 0 & 1 & 11 & -7 \end{array}$$

$$\begin{array}{rrrrrr} & 2 & 4 & 1 & -17 & 11 \\ - & 2 & 4 & 2 & -6 & 4 \\ \hline & 0 & 0 & -1 & -11 & 7 \end{array} \longrightarrow \begin{pmatrix} 1 & 2 & 1 & -3 & 2 \\ 0 & 0 & 1 & 11 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2e l × (−1) à 1re ligne

$$\begin{array}{ccccc} \begin{pmatrix} 1 & 2 & 0 & -14 & 9 \\ 0 & 0 & 1 & 11 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\ x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \end{array}$$

$$\begin{array}{rrrrrr} & 1 & 2 & 1 & -3 & 2 \\ - & 0 & 0 & 1 & 11 & -7 \\ \hline & 1 & 2 & 0 & -14 & 9 \end{array}$$

$$x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_5\vec{a}_5$$

$\uparrow$

V_{11}

$$= x_1\vec{a}_1 + x_2(2\vec{a}_1) + x_3\vec{a}_3$$
$$+ x_4(-14\vec{a}_1 + 11\vec{a}_3)$$
$$+ x_5(9\vec{a}_1 - 7\vec{a}_3)$$
$$= (x_1 + 2x_2 - 14x_4 + 9x_5)\vec{a}_1$$
$$+ (x_3 + \underbrace{11}_{11} x_4 - 7x_5)\vec{a}_3$$

$$V_{10} = \{A\vec{v} \in \mathbb{R}^5 ; A\vec{v} = \vec{0}\} \subset \mathbb{R}^5$$

3次元

$$V_{11} = \{A\vec{v} ; \vec{v} \in \mathbb{R}^5\} \subset \mathbb{R}^4$$

$\uparrow$

2次元

$\vec{a}_1, \cdots, \in \mathbb{R}^4$

$$V_{11} = \{ A\vec{v} \ ; \ \vec{v} \in \mathbb{R}^5 \} \qquad \boxed{\begin{array}{c} A = (\vec{a}_1\ \vec{a}_2\ \vec{a}_3\ \vec{a}_4\ \vec{a}_5) \\ 4 \times 5 \end{array}}$$

$$\cap$$

$$\mathbb{R}^4$$

$$= \{ x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 + x_4\vec{a}_4 + x_5\vec{a}_5 \ ; \ x_1, \dots, x_5 \in \mathbb{R} \}$$

$$A = (\vec{a}_1 \cdots \vec{a}_5) \to \cdots \to \begin{pmatrix} 1 & 2 & 0 & -14 & 9 \\ 0 & 0 & 1 & 11 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\| $$

$$B = (\vec{b}_1 \cdots \vec{b}_5)$$

$$\left.\begin{array}{l} \vec{b}_2 = 2\vec{b}_1 \\ \vec{b}_4 = -14\vec{b}_1 + 11\vec{b}_3 \\ \vec{b}_5 = 9\vec{b}_1 - 7\vec{b}_3 \end{array}\right\} \xrightarrow{P^{-1}} \left\{\begin{array}{l} \vec{a}_2 = 2\vec{a}_1 \\ \vec{a}_4 = -14\vec{a}_1 + 11\vec{a}_3 \\ \vec{a}_5 = 9\vec{a}_1 - 7\vec{a}_3 \end{array}\right.$$

$$\underbrace{\boxed{P_k \cdots P_1}}_{=P} A = B$$

$PA = B$, P : 正則

$$\vec{a}_j = P^{-1}\vec{b}_j$$

$$\Updownarrow$$

$$P(\vec{a}_1 \cdots \vec{a}_5) = (\vec{b}_1 \cdots \vec{b}_5) \longrightarrow \boxed{P\vec{a}_j = \vec{b}_j}$$

$$A\vec{v} = \vec{0} \iff \begin{pmatrix} 1 & 2 & 0 & -14 & 9 \\ 0 & 0 & 1 & +11 & -7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \vec{v} = \vec{0}$$

$$\vec{v} = \begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix} \qquad x_2 = \alpha,\ x_4 = \beta,\ x_5 = \gamma$$

$$x_1 = -2\alpha + 14\beta - 9\gamma$$

$$\cancel{x_2 = -11\beta}$$

$$x_2 = \alpha$$

$$x_3 = -11\beta + 7\gamma$$

$$x_4 = \beta$$

$$x_5 = \gamma$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \alpha \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 14 \\ 0 \\ -11 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -9 \\ 0 \\ 7 \\ 0 \\ 1 \end{pmatrix}$$

$$V_{1,0} = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 14 \\ 0 \\ -11 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -9 \\ 0 \\ 7 \\ 0 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ -2 & 5 & -9 & c \end{array}\right) \xrightarrow[\text{3行に+}]{\text{1行×2して}} \left(\begin{array}{ccc|c} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ 0 & -3 & 5 & c+2a \end{array}\right)$$

$$\begin{array}{rcccc} & -2 & 5 & -9 & c \\ +) & 2 & -8 & 14 & 2a \\ \hline & & -3 & 5 & c+2a \end{array}$$

$$\xrightarrow{\text{2行を3行に+}} \left(\begin{array}{ccc|c} 1 & -4 & 7 & a \\ 0 & 3 & -5 & b \\ \boxed{0 & 0 & 0 & -} \end{array}\right)$$

$2a+b+c$

$2a+b+c \neq 0$

$0x + 0y + 0z = 2a+b+c$

は解なし

$2a+b+c=0$ のとき解あり

$$\left(\begin{array}{ccc|c} 1 & -4 & 7 & a \\ 0 & 1 & -\frac{5}{3} & \frac{b}{3} \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[\text{1行に+}]{\text{2行×4して}} \left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & a+\frac{4}{3}b \\ 0 & 1 & -\frac{5}{3} & \frac{b}{3} \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{array}{rcccc} & 1 & -4 & 7 & a \\ +) & 0 & 4 & -\frac{20}{3} & \frac{4}{3}b \\ \hline & 1 & 0 & \frac{1}{3} & a+\frac{4}{3}b \end{array}$$

$$\Updownarrow$$

$$\begin{cases} x + \frac{1}{3}z = a + \frac{4}{3}b \\ y - \frac{5}{3}z = \frac{b}{3} \end{cases}$$

$z = \alpha$ とおく

$$\begin{cases} x = -\frac{1}{3}\alpha + a + \frac{4}{3}b \\ y = \frac{5}{3}\alpha + \frac{b}{3} \\ z = \alpha \end{cases}$$

$A\vec{x} = \vec{b}$ は解をもつ $\iff \mathrm{rank}(A) = \mathrm{rank}(A|\vec{b})$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & -2 \\ 4 & -1 & -5 & -6 & a \\ 2 & 1 & -1 & 0 & 6 \end{array}\right) \xrightarrow[\substack{1行\times(-2)を\\3行に加え}]{\substack{1行\times(-4)を\\2行に加え}} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & -2 \\ 0 & -9 & -9 & -18 & a+8 \\ 0 & -3 & -3 & -6 & 10 \end{array}\right)$$

$$\begin{array}{rccccc} & 4 & -1 & -5 & -6 & a \\ -) & 4 & 8 & 4 & 12 & -8 \\ \hline & 0 & -9 & -9 & -18 & a+8 \end{array}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & -2 \\ 0 & 1 & 1 & 2 & -\frac{a+8}{9} \\ 0 & -3 & -3 & -6 & 10 \end{array}\right)$$

$$\begin{array}{rccccc} & 2 & 1 & -1 & 0 & 6 \\ -) & 2 & 4 & 2 & 6 & -4 \\ \hline & 0 & -3 & -3 & -6 & 10 \end{array}$$

$$\xrightarrow{\substack{2行\times3を\\3行に加え}} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & -2 \\ 0 & 1 & 1 & 2 & -\frac{a+8}{9} \\ 0 & 0 & 0 & 0 & \frac{22-a}{3} \end{array}\right)$$

$$\begin{array}{rccccc} & 0 & -3 & -3 & -6 & 10 \\ +) & 0 & 3 & 3 & 6 & -\frac{a+8}{3} \\ \hline & 0 & 0 & 0 & 0 & 10-\frac{a+8}{3} \\ & & & & & = \frac{22-a}{3} \end{array}$$

$a = 22$ の時解を持つ

[illegible]

[illegible]

$\underline{a = 22}$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 3 & -2 \\ 0 & 1 & 1 & 2 & -\frac{10}{3} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow[\substack{を1行に加え}]{2行\times(-2)} \begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & -1 & -1 & \frac{14}{3} \\ 0 & 1 & 1 & 2 & -\frac{10}{3} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

$$\begin{array}{rccccc} & 1 & 2 & 1 & 3 & -2 \\ -) & 0 & 2 & 2 & 4 & -\frac{20}{3} \\ \hline & 1 & 0 & -1 & -1 & \frac{14}{3} \end{array}$$

$$
\begin{cases}
x \quad - z - w = \frac{14}{3} \\
\quad y + z + 2w = -\frac{10}{3}
\end{cases}
$$

$z = \alpha$, $w = \beta$ [illegible]

$$
\begin{cases}
x = \alpha + \beta + \frac{14}{3} \\
y = -\alpha - 2\beta - \frac{10}{3}
\end{cases}
$$ [illegible]

$$\begin{vmatrix} 1 & 3 & 0 \\ 1 & 0 & 1 \\ -1 & 3 & 2 \end{vmatrix} = 1\times 0\times 2 + 1\times 3\times 0 + (-1)\times 3\times 1$$
$$- 1\times 3\times 1 - 1\times 3\times 2 - (-1)\times 0\times 0$$

$$\begin{vmatrix} 1 & 3 & 0 \\ 1 & 0 & 1 \\ -1 & 3 & 2 \end{vmatrix} = 0 + 0 - 3 - 3 - 6 + 0$$
$$= -12$$

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1)列×(−3)2

[illegible]

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & -3 & 1 \\ -1 & 6 & 2 \end{vmatrix}$$

$$\begin{array}{r} 3 \quad 0 \quad 3 \\ -)\ 3 \quad 3 \quad -3 \\ \hline 0 \quad -3 \quad 6 \end{array}$$

$$= 1\times \begin{vmatrix} -3 & 1 \\ 6 & 2 \end{vmatrix} = (-3)\times 2 - 6\times 1$$
$$= -12$$

$$\begin{vmatrix} \alpha & 0 & 0 \\ * & a & b \\ * & c & d \end{vmatrix} = \alpha \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} 3 & 10 & 5 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3\times1\times2 + 2\times3\times5 + 1\times10\times1 - 3\times3\times1 - 2\times10\times2 - 1\times1\times5$$

$$= 6 + 30 + 10 \qquad 46$$

$$- 9 - 40 - 5$$

$$= -8$$

$$= 3 \begin{vmatrix} 1 & 10 & 5 \\ \frac{2}{3} & 1 & 1 \\ \frac{1}{3} & 3 & 2 \end{vmatrix} = 3 \begin{vmatrix} 1 & 0 & 0 \\ \frac{2}{3} & -\frac{17}{3} & -\frac{7}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{vmatrix}$$

$$\begin{array}{rccc} & 10 & 1 & 3 \\ - & 10 & \frac{20}{3} & \frac{10}{3} \\ \hline & 0 & -\frac{17}{3} & \end{array} \qquad \begin{array}{rccc} & 5 & 1 & 2 \\ - & 5 & \frac{10}{3} & \frac{5}{3} \\ \hline & & -\frac{7}{3} & \end{array}$$

$$= 3 \times \begin{vmatrix} -\frac{17}{3} & -\frac{7}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{vmatrix}$$

$$\begin{vmatrix} 4 & -3 & 2 \\ -4 & 3 & 2 \\ 1 & 0 & 2 \end{vmatrix} \overset{1372\ 2251\ 2+}{=} \begin{vmatrix} 4 & -3 & 2 \\ 0 & 0 & 4 \\ 1 & 0 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 0 & 1 \\ -3 & 0 & 0 \\ 2 & 4 & 2 \end{vmatrix}$$

$$= - \begin{vmatrix} -3 & 0 & 0 \\ 4 & 0 & 1 \\ 2 & 4 & 2 \end{vmatrix}$$

$$= -(-3) \begin{vmatrix} 0 & 1 \\ 4 & 2 \end{vmatrix}$$

$$= 3 \times (0 \times 2 - 4 \times 1)$$

$$= -12$$

Ⅰ 各行について線形

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定理 4.4

$$\begin{vmatrix} a + b \\ c \\ d \end{vmatrix} = \begin{pmatrix} a \\ c \\ d \end{pmatrix} + \begin{vmatrix} b \\ c \\ d \end{vmatrix}$$

$$\begin{vmatrix} \lambda a \\ b \\ c \end{vmatrix} = \lambda \begin{vmatrix} a \\ b \\ c \end{vmatrix}$$

Ⅱ 2行が一致

例

$$\begin{vmatrix} a \\ a \\ b \end{vmatrix} = 0$$

Ⅳ

$$\begin{vmatrix} a \\ b \\ c \end{vmatrix} = - \begin{vmatrix} b \\ a \\ c \end{vmatrix}$$

Ⅴ 例

$$\begin{vmatrix} a \\ \lambda a + b \\ c \end{vmatrix} = \begin{vmatrix} a \\ b \\ c \end{vmatrix}$$

i 行の λ 倍を j 行に加えても
($i \neq j$)
行列式は不変

$A, B : 3 \times 3$

$$\det(AB) = \det(A)\det(B)$$

$$B = (\vec{b}_1 \;\; \vec{b}_2 \;\; \vec{b}_3) \qquad AB = (A\vec{b}_1 \;\; A\vec{b}_2 \;\; A\vec{b}_3)$$

$$\det(AB) = \det(A\vec{b}_1 \;\; A\vec{b}_2 \;\; A\vec{b}_3)$$

$$\vec{b}_1 = \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix} = b_{11}\vec{e}_1 + b_{21}\vec{e}_2 + b_{31}\vec{e}_3$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \det\left(A(b_{11}\vec{e}_1 + b_{21}\vec{e}_2 + b_{31}\vec{e}_3) \;\text{——}\; \right)$$

$$= b_{11}\det(A\vec{e}_1 \;\text{——}\;) + b_{21}\det(A\vec{e}_2 \;\text{—}\;)$$

$$+ b_{31}\det(A\vec{e}_3 \;\text{—}\;)$$

$$\vec{b}_2 = \begin{pmatrix} b_{12} \\ b_{22} \\ b_{32} \end{pmatrix} = b_{12}\vec{e}_1 + b_{22}\vec{e}_2 + b_{32}\vec{e}_3$$

$$\vec{b}_3 = \begin{pmatrix} b_{13} \\ b_{23} \\ b_{33} \end{pmatrix} = b_{13}\vec{e}_1 + b_{23}\vec{e}_2 + b_{33}\vec{e}_3$$

$f : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$$\boxed{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}$$

テキスト
25, 26 page.

3×3 行列

多重線形性 I, II

定理 $f(A) = \det(A) f(E_3)$

A : 3×3 行列 で成立.

行列式の定理 4.2 を用いる.

$f(B) = \det(AB)$ は 多重線形性 I, II
を満たす.

$$\det(AB) = \det(B) \underbrace{\det(AE_3)}_{\det(A)}$$

$\det({}^tA) = \det(A) \qquad A : 3 \times 3$

$A \to \cdots \to$ rank 0 $\quad A = O$

rank 1 $\quad \begin{pmatrix} 1 & * & * \\ & O & \end{pmatrix}, \begin{pmatrix} 0 & 1 & * \\ & O & \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 1 \\ & O & \end{pmatrix}$

rank 2 $\quad \begin{pmatrix} 1 & * & * \\ & 1 & * \\ & & \end{pmatrix} \begin{pmatrix} 1 & * & 0 \\ & & 1 \\ & & \end{pmatrix}$

${}^t(AB)$

$= {}^tB\,{}^tA$

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

rank 3 $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A \to \cdots \to \begin{pmatrix} \alpha & * & * \\ 0 & \beta & * \\ 0 & 0 & \gamma \end{pmatrix}$

$P_k \cdots P_2 P_1 A = \begin{pmatrix} \alpha & * & * \\ 0 & \beta & * \\ 0 & 0 & \gamma \end{pmatrix}$

${}^tA\,{}^tP_1 \cdots {}^tP_k = \begin{pmatrix} \alpha & 0 & 0 \\ * & \beta & 0 \\ * & * & \gamma \end{pmatrix}$

$$\det(AB) = \det(A)\det(B)$$

$A, B : 3 \times 3$ 行&列

$$\boxed{\det(P_1)\cdots\det(P_k)}\det(A) = \alpha\beta\gamma$$

$$\det({}^tA)\boxed{\det({}^tP_1)\cdots\det({}^tP_k)} = \alpha\beta\gamma$$

$$\det(\text{基本行列}) = \det({}^t\text{基本行列})$$

① $${}^t\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

② $${}^t\begin{pmatrix} 1 & 0 & \lambda \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda & 0 & 1 \end{pmatrix}$$

$$\det\begin{pmatrix} 1 & 0 & \lambda \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1, \quad \det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda & 0 & 1 \end{pmatrix} = 1$$

③ $${}^t\begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

A : 3×3 行列 定理 4.7.

$\det(A) \neq 0 \iff (A)$: 正則

$\left(\begin{array}{l} \iff A : \mathrm{rank}\ 3 \\ \iff (A\vec{x} = \vec{0} \implies \vec{x} = \vec{0}) \end{array} \right)$

$\Leftarrow$ A : 正則 と仮定

$$A \cdot A^{-1} = E_3.$$

$$\underset{\displaystyle \det(A)\cdot\det(A^{-1})}{\underset{\shortparallel}{\det(A\cdot A^{-1})}} = \det(E_3) = 1$$

$\rightsquigarrow \det(A) \neq 0$

$(\Rightarrow)$ $\det(A) \neq 0 \implies A : \mathrm{rank}\ 3$

対偶 $\mathrm{rank}(A) < 3$

$\implies \det(A) = 0.$

$$\underset{\displaystyle 0}{\underset{\shortparallel}{\boxed{\det(P_1)\cdots\det(P_k)}}}\ \det(A) = \underset{\displaystyle 0}{\underset{\shortparallel}{\underbrace{\alpha\beta\gamma}}}$$

$\longrightarrow \det(A) = 0$ $\qquad \mathrm{rank}(A) < 3$

$$\begin{pmatrix} 1 & k & 2 \\ 1 & 2 & k \\ 2 & k & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{0}$$ が 非自明解を持つ k を求めよ!

非自明解が存在 $\iff$ $\mathrm{rank}(A) <$ 変数の個数

$A\vec{x} = \vec{0}$ は

$\mathrm{rank}(A) < 3$ が 必要十分条件.

$$\Updownarrow$$

$$\det(A) = 0$$

問題

$$\begin{cases} x + y - z = 0 \\ 2x + 3y + cz = 0 \\ x + cy + 3z = 0 \end{cases}$$

は 非自明解が存在するのは いつか?

A: n次正方行列

$A\vec{x} = \vec{0}$ は 非自明解あり

$\iff$ $\det(A) = 0$

固有値問題

主成分分析

x_1	y_1
$\vdots$	$\vdots$
x_N	y_N

$$z = c_1 x_1 + c_2 x_2$$

$$V(z) = c_1^2 V(x) + 2 c_1 c_2 C(x, y) + c_2^2 V(y)$$

$c_1^2 + c_2^2 = 1$ の下で $V(z)$ を最大化.

$$C = \begin{pmatrix} V(x) & C(x, y) \\ C(x, y) & V(y) \end{pmatrix}$$ 分散共分散行列

$\det(\lambda E_2 - C) = 0$ を解く.

余因子展開

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\tilde{A}_{ij} = \underbrace{}_{(-1)^{i+j}} \det (A \text{ の } i \text{ 行}, j \text{ 列を除く})$$

$$\tilde{A}_{11} = (-1)^{1+1} \det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$$

$$\tilde{A}_{23} = (-1)^{5} \det \begin{pmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{pmatrix}$$

etc.

1列に関する展開

$$\det A = a_{11} \det \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$$

$$- a_{21} \det \begin{pmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{pmatrix}$$

$$+ a_{31} \det \begin{pmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{pmatrix}$$

$$\begin{vmatrix} 3 & 10 & 5 \\ 2 & 1 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= 3 \times \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} - 2 \times \begin{vmatrix} 10 & 5 \\ 3 & 2 \end{vmatrix} + 1 \times \begin{vmatrix} 10 & 5 \\ 1 & 1 \end{vmatrix}$$

$$\vec{x_1} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{x_2} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{x_3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$c_1 \vec{x_1} + c_2 \vec{x_2} + c_3 \vec{x_3} = \vec{0}$ [illegible].

$$= \begin{pmatrix} c_1 \\ 0 \\ c_1 \end{pmatrix} + \begin{pmatrix} c_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} c_3 \\ c_3 \\ c_3 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 + c_3 \\ c_3 \\ c_1 + c_3 \end{pmatrix}$$

$c_3 = 0$, $\quad c_1 + c_3 = 0 \longrightarrow c_1 = 0$

$$\underset{0}{\underset{\uparrow}{c_1}} + c_2 + \underset{0}{\underset{\uparrow}{c_3}} = 0 \longrightarrow c_2 = 0$$

$\underline{\vec{x_1}, \vec{x_2}, \vec{x_3}}$ [illegible] 自明な [illegible]

[illegible] 1次独立

$\vec{x_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\vec{x_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\vec{x_3} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\vec{x_1}, \vec{x_2}, \vec{x_3}$: 1次従属

$$\vec{x_3} = \vec{x_1} + \vec{x_2}$$

$$\Longleftrightarrow \quad \vec{x_1} + \vec{x_2} - \vec{x_3} = \vec{0}$$

$\downarrow$

非自明な1次関係

$$0\vec{x_1} + 0\vec{x_2} + 0\vec{x_3} = \vec{0}$$

自明な1次関係.

$$\boxed{c_1\vec{x_1} + c_2\vec{x_2}} = \vec{0} \quad \Longrightarrow \quad c_1 = c_2 = 0$$

$$\parallel$$

$$\begin{pmatrix} c_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$\vec{x_1}$ と $\vec{x_2}$ には自明な1次関係しかない

$\rightarrow$ $\vec{x_1}, \vec{x_2}$: 線形独立

1次

$V_{10} = \{ \vec{v} \in \mathbb{R}^5 ;\ A\vec{v} = \vec{0} \}$

$A = \begin{pmatrix} & & \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x_1 \\ \vdots \\ x_5 \end{pmatrix} = \alpha \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 14 \\ 0 \\ -11 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -4 \\ 0 \\ 7 \\ 0 \\ 1 \end{pmatrix}$$

$$c_1 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 14 \\ 0 \\ -11 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} -4 \\ 0 \\ 7 \\ 0 \\ 1 \end{pmatrix} = \vec{0}$$

$$= \begin{pmatrix} * \\ c_1 \\ * \\ c_2 \\ c_3 \end{pmatrix} \rightsquigarrow \underline{c_1 = c_2 = c_3 = 0}$$

3本のベクトルは 自明な線形関係しか ない

<u>V_{10} は 3次元</u>

$$V_{11} = \{ x_1\vec{a_1} + \cdots + x_5\vec{a_5} \ ; \ x_1, \cdots, x_5 \in \mathbb{R} \}$$

$$= \{ x_1\vec{a_1} + x_3\vec{a_3} \ ; \ x_1, x_3 \in \mathbb{R} \}$$

$\vec{a_1}, \vec{a_3}$ は 線形独立.

$$(\vec{a_1} \cdots \vec{a_5}) \cdots\longrightarrow \begin{pmatrix} 1 & 2 & 0 & * & * \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & & \end{pmatrix}$$

$$\begin{array}{ccc} \| & & \\ \vec{e_1} & & \vec{e_2} \end{array}$$

$$\boxed{P\vec{a_j} = \vec{e_j}}$$

$$c_1\vec{a_1} + c_3\vec{a_3} = \vec{0}$$

$$P(c_1\vec{a_1} + c_3\vec{a_3}) = P\cdot\vec{0} = \vec{0}$$

$$\|$$

$$c_1 P\vec{a_1} + c_3 P\vec{a_3}$$

$$\|$$

$$c_1\vec{e_1} + c_2\vec{e_2} = c_1\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + c_2\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$= \begin{pmatrix}c_1\\c_2\\0\\0\end{pmatrix} = \vec{0}$$

$$\longrightarrow \ c_1 = c_2 = 0$$

V_{11} は 2次元

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$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix} \rightarrow \cdots \rightarrow$$

이거 풀기 생략함.

$$\boxed{\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -6 \\ 0 \end{pmatrix}}$$