

連立一次方程式の同値変形、消去法問題の解答

(1)

$$x_1, x_2, x_3, y_1, y_2, c \in \mathbb{R} \quad \text{DIII}$$

$$\begin{cases} x_1 + x_2 + x_3 = y_1 \\ 2x_1 - x_2 + 2x_3 = y_2 \\ 3x_1 + x_2 - x_3 = y_3 \end{cases}$$

左辺の係数行列 $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{pmatrix}$ を表す。

(消去法) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & -2 & -4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$

$$\begin{cases} x_1 + x_2 + x_3 = y_1 & (i) \\ 2x_1 - x_2 + 2x_3 = y_2 & (ii) \\ 3x_1 + x_2 - x_3 = y_3 & (iii) \end{cases}$$

$$\begin{cases} x_1 + x_2 + x_3 = y_1 \\ 3x_1 - x_2 = y_2 + y_1 & (ii)_1 \\ -2x_1 - x_2 = y_3 - y_1 & (iii)_1 \end{cases}$$

$$\begin{aligned} (i)_1 &:= (i) \\ (ii)_1 &:= (ii) + (i) \times (-2) \\ (iii)_1 &:= (iii) + (i) \times (-3) \end{aligned}$$

$$\begin{cases} x_1 + x_2 + x_3 = y_1 \\ x_2 = \frac{1}{3}(y_2 + y_1) \\ x_3 = \frac{1}{3}(y_3 - y_1) \end{cases}$$

$$\begin{aligned} (i)_2 &:= (i)_1 \\ (ii)_2 &:= (ii)_1 \times \left(-\frac{1}{3}\right) \\ (iii)_2 &:= (iii)_1 \end{aligned}$$

$$\begin{cases} x_1 + x_2 + x_3 = y_1 \\ x_2 = \frac{1}{3}(y_2 + y_1) \\ x_3 = \frac{1}{3}(y_3 - y_1) \end{cases}$$

$$\begin{aligned} (i)_3 &:= (i)_2 + (ii)_2 \times (-1) \\ (ii)_3 &:= (ii)_2 \\ (iii)_3 &:= (iii)_2 + (ii)_2 \times 2 \end{aligned}$$

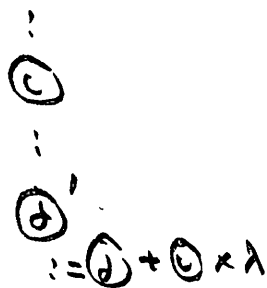
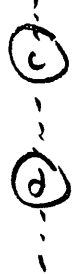
$$\begin{cases} x_1 + x_2 + x_3 = y_1 \\ x_2 = \frac{1}{3}(y_2 + y_1) \\ x_3 = \frac{1}{3}(y_3 - y_1) \end{cases}$$

$$\begin{aligned} (i)_4 &:= (i)_3 \\ (ii)_4 &:= (ii)_3 \\ (iii)_4 &:= (iii)_3 \times \left(-\frac{1}{4}\right) \end{aligned}$$

\Leftrightarrow

基本変形と行基本変形

(I) $(c \neq d)$



$$d' \text{行} = c \text{行} + \lambda \text{行}$$

$$d' = d + c \times \lambda$$

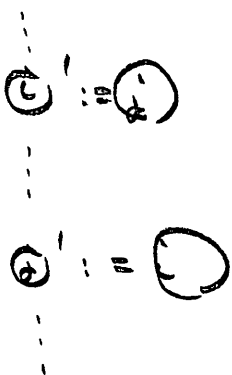
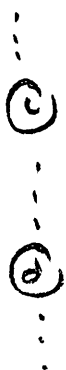
(II) $\lambda \neq 0$



$$c' \text{行} = \lambda \text{行}$$

$$c' = c \times \lambda$$

(III) $c \neq d$



$$c' \text{行} = d \text{行}$$

$$c' \leftrightarrow d$$

④

LA_L01_0410 連立方程式の基本変形 sympyを使う

演習問題解答

Python3 上のSympy を用いると簡単に答えが出ます。

```
>>> from sympy import *
>>> A=Matrix([[1,1,1,0,0],[2,-1,2,0,1],[3,1,-1,0,1]])
>>> A
Matrix([
[1, 1, 1, 1, 0, 0],
[2, -1, 2, 0, 1, 0],
[3, 1, -1, 0, 0, 1]])
>>> A.rref()
(Matrix([
[1, 0, 0, -1/12, 1/6, 1/4],
[0, 1, 0, 2/3, -1/3, 0],
[0, 0, 1, 5/12, 1/6, -1/4]]), (0, 1, 2))
```