Part 02 b ベクトル積. $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$\vec{b} \times \vec{c} = \begin{pmatrix} \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \\ -\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} \\ \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \end{pmatrix}$$

$$|\vec{a}\ \vec{b}\ \vec{c}| = (\vec{a}, \vec{b} \times \vec{c})$$

↑ 各自 check.

$$0 = |\vec{b}\ \vec{b}\ \vec{c}| = (\vec{b}, \vec{b} \times \vec{c})$$

$$0 = |\vec{c}\ \vec{b}\ \vec{c}| = (\vec{c}, \vec{b} \times \vec{c})$$

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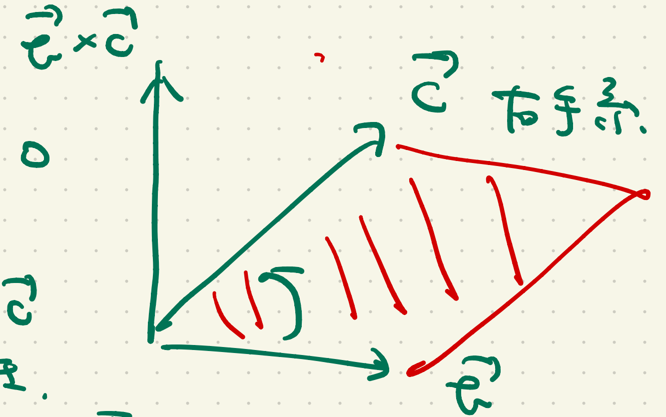
$|\vec{a}\ \vec{b}\ \vec{c}| = (\vec{a}, \vec{b}\times\vec{c})$

$|\vec{b}\times\vec{c}\ \ \vec{b}\ \ \vec{c}| = \|\vec{b}\times\vec{c}\|^2 > 0$

$\uparrow$

$\vec{b} \nparallel \vec{c}$ のとき.

$$\boxed{\begin{array}{c} \vec{b} \nparallel \vec{c} \text{ のとき} \\ \Updownarrow \\ \vec{b}\times\vec{c} \neq \vec{0} \end{array}}$$



連続.

$|\vec{a}\ \vec{b}\ \vec{c}| > 0 \Rightarrow |\vec{a}(t)\ \vec{b}(t)\ \vec{c}(t)| > 0 \quad 0 \le t \le 1$

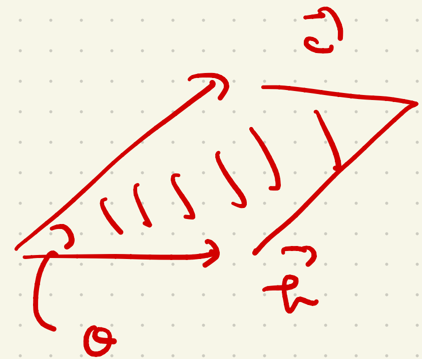
$\vec{a}(0) = \vec{a},\ \vec{b}(0) = \vec{b},\ \vec{c}(0) = \vec{c}$

$\vec{a}(1) = \vec{e}_1,\ \vec{b}(1) = \vec{e}_2,\ \vec{c}(1) = \vec{e}_3$

$$S = \|\vec{e}\| \cdot \|\vec{c}\| \cdot \sin\theta = \|\vec{e}\| \cdot \|\vec{c}\| \sqrt{1 - \left(\frac{(\vec{e},\vec{c})}{\|\vec{e}\| \cdot \|\vec{c}\|}\right)^2}$$

平行四边形の
面積

$$= \sqrt{\|\vec{e}\|^2 \|\vec{c}\|^2 - (\vec{e},\vec{c})^2}$$



$$S^2 = (e_1^2 + e_2^2 + e_3^2)(c_1^2 + c_2^2 + c_3^2) - (e_1 c_1 + e_2 c_2 + e_3 c_3)^2$$

名同

$$= \begin{vmatrix} e_2 & c_2 \\ e_3 & c_3 \end{vmatrix}^2 + \begin{vmatrix} e_1 & c_1 \\ e_3 & c_3 \end{vmatrix}^2 + \begin{vmatrix} e_1 & c_1 \\ e_2 & c_2 \end{vmatrix}^2 = \|\vec{e} \times \vec{c}\|^2$$

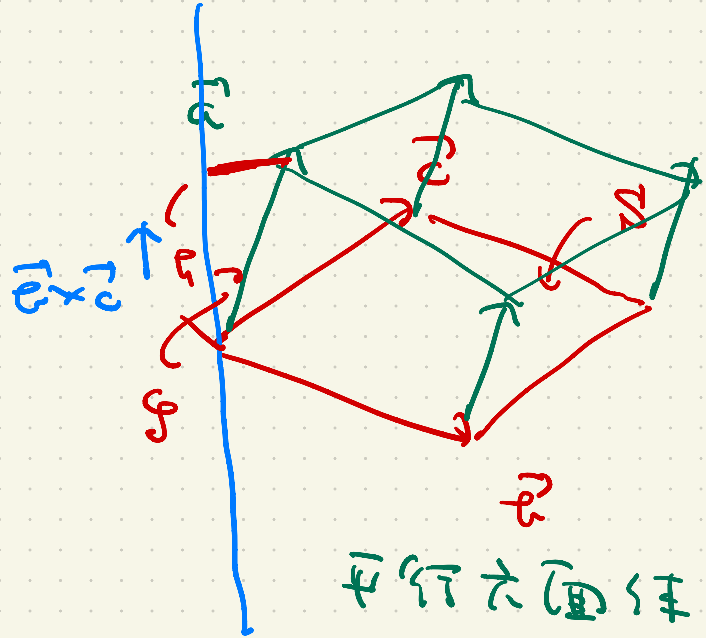
$$h = \left| \, \|\vec{a}\| \cos\varphi \, \right|$$

$$= \left| \, \|\vec{a}\| \cdot \frac{(\vec{a}, \vec{b}\times\vec{c})}{\|\vec{a}\| \cdot \|\vec{b}\times\vec{c}\|} \right|$$

$$= \left| \frac{(\vec{a}, \vec{b}\times\vec{c})}{\boxed{\|\vec{b}\times\vec{c}\|}} \right| \to S$$

$$hS = \left| (\vec{a}, \vec{b}\times\vec{c}) \right|$$

$$= \left| \, \left| \vec{a} \; \vec{b} \; \vec{c} \right| \, \right|$$



平行六面体.

$$V = \mathrm{abs}\left(\left| \vec{a} \; \vec{b} \; \vec{c} \right| \right)$$

abs.