基本 I $\vec{a}, \vec{b}, \vec{c}, \cdots \in \mathbb{K}^2$

基本は列ベクトル

$$(\vec{a}\ \vec{b})\begin{pmatrix} x \\ y \end{pmatrix} = x\vec{a} + y\vec{b}$$

$$(\vec{a}\ \vec{b}\ \vec{c})\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\vec{a} + y\vec{b} + z\vec{c}$$

$$\vdots$$

II

$$A(\vec{x}\ \vec{y}) = (A\vec{x}\ A\vec{y})$$

$$A(\vec{x}\ \vec{y}\ \vec{z}) = (A\vec{x}\ A\vec{y}\ A\vec{z})$$

$$\vdots$$

Aが2列ならば $\vec{x}, \vec{y}, \vec{z} \in \mathbb{K}^2$

Aが3列 ―― $\in \mathbb{K}^3$

$\vdots$

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1 $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, $\vec{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$

$$\begin{cases} a_1 x + b_1 y = \beta_1 \\ a_2 x + b_2 y = \beta_2 \end{cases} \iff x\vec{a} + y\vec{b} = \vec{\beta} \iff (\vec{a}\ \vec{b})\begin{pmatrix} x \\ y \end{pmatrix} = \vec{\beta}$$

$$\begin{cases} a_1 x + b_1 y + c_1 z = \beta_1 \\ a_2 x + b_2 y + c_2 z = \beta_2 \end{cases} \iff x\vec{a} + y\vec{b} + z\vec{c} = \vec{\beta} \iff (\vec{a}\ \vec{b}\ \vec{c})\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{\beta}$$

2 $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \quad \vec{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$

$$\begin{cases} a_1 x + b_1 y = \beta_1 \\ a_2 x + b_2 y = \beta_2 \\ a_3 x + b_3 y = \beta_3 \end{cases} \Longleftrightarrow x\vec{a} + y\vec{b} = \vec{\beta} \Longleftrightarrow (\vec{a}\ \vec{b})\begin{pmatrix} x \\ y \end{pmatrix} = \vec{\beta}$$

$$\begin{cases} a_1 x + b_1 y + c_1 z = \beta_1 \\ a_2 x + b_2 y + c_2 z = \beta_2 \\ a_3 x + b_3 y + c_3 z = \beta_3 \end{cases} \Longleftrightarrow x\vec{a} + y\vec{b} + z\vec{c} = \vec{\beta}$$

$$\Longleftrightarrow (\vec{a}\ \vec{b}\ \vec{c})\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{\beta}$$

線形写像

$\vec{a}, \vec{b}, \vec{c} \in \mathbb{K}^4$

$\vec{p}, \vec{q}, \vec{r} \in \mathbb{K}^2$

$$F : \mathbb{K}^2 \longrightarrow \mathbb{K}^4$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto x\vec{a} + y\vec{b} = (\vec{a}\ \vec{b})\begin{pmatrix} x \\ y \end{pmatrix}$$

は $F(\lambda\vec{v} + \mu\vec{w}) = \lambda F(\vec{v}) + \mu F(\vec{w})$ を満たす. ← 示せるか.

$$G : \mathbb{K}^2 \longrightarrow \mathbb{K}^2$$
$$\begin{pmatrix} s \\ t \end{pmatrix} \longmapsto s\vec{p} + t\vec{q} = (\vec{p}\ \vec{q})\begin{pmatrix} s \\ t \end{pmatrix}$$

$$\begin{tikzcd} \mathbb{K}^2 \arrow[r, "G"] \arrow[rr, bend left, "F \circ G"] & \mathbb{K}^2 \arrow[r, "F"] & \mathbb{K}^4 \end{tikzcd}$$

$$F \circ G\left(\begin{pmatrix} s \\ t \end{pmatrix}\right) = F\left(G\begin{pmatrix} s \\ t \end{pmatrix}\right) = F(s\vec{p} + t\vec{q})$$
$$= sF(\vec{p}) + tF(\vec{q})$$
$$= \left(F(\vec{p})\ \ F(\vec{q})\right)\begin{pmatrix} s \\ t \end{pmatrix} = \left((\vec{a}\ \vec{b})(\vec{p}\ \vec{q})\right)\begin{pmatrix} s \\ t \end{pmatrix}$$

ここで $\left(F(\vec{p})\ \ F(\vec{q})\right) = \left((\vec{a}\ \vec{b})\vec{p}\ \ \ (\vec{a}\ \vec{b})\vec{q}\right)$
$$= (\vec{a}\ \vec{b})(\vec{p}\ \vec{q})$$

$$H : \mathbb{K}^3 \longrightarrow \mathbb{K}^2$$

$$\begin{pmatrix} s \\ t \\ u \end{pmatrix} \longmapsto s\vec{p} + t\vec{q} + u\vec{r} = (\vec{p}\ \vec{q}\ \vec{r})\begin{pmatrix} s \\ t \\ u \end{pmatrix}$$

を用いて

$$(F \circ H)\begin{pmatrix} s \\ t \\ u \end{pmatrix} = \cdots$$

を計算できるか？