X, Y 集合　$f: X \longrightarrow Y$ 写像.

1) $f: X \longrightarrow Y$ が単射とは.

例1) $\vec{a}, \vec{b} \in \mathbb{K}^n$, $\vec{a} \nparallel \vec{b}$

$$f: \mathbb{K}^2 \longrightarrow \mathbb{K}^n$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto x\vec{a} + y\vec{b}$$

は単射

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = f\left(\begin{pmatrix} x' \\ y' \end{pmatrix}\right) \iff x\vec{a} + y\vec{b} = x'\vec{a} + y'\vec{b}$$

$$\iff (x - x')\vec{a} + (y - y')\vec{b} = \vec{0}$$

$$\iff x = x', y = y' \iff \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$\vec{a} \parallel \vec{b}$ のときは?

2) $f: X \longrightarrow Y$ が全射とは.

<u>例</u> $\vec{a}, \vec{b} \in \mathbb{R}^2$

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto x\vec{a} + y\vec{b}$$

$\vec{a} \nparallel \vec{b}$ のとき f は全射.

$$\forall \begin{pmatrix} s \\ t \end{pmatrix} \in \mathbb{R}^2 \ \exists \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \ \Big| \ x\vec{a} + y\vec{b} = \begin{pmatrix} s \\ t \end{pmatrix}$$

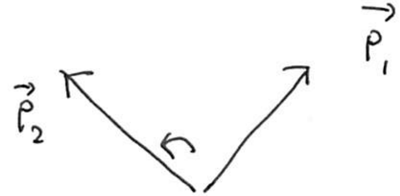
実際 $\begin{cases} a_1 x + b_1 y = s \\ a_2 x + b_2 y = t \end{cases}$

は $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$ なので $x = \frac{1}{D} \begin{vmatrix} s & b_1 \\ t & b_2 \end{vmatrix}$, $y = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ が解.

$\vec{a} \parallel \vec{b}$ のときは?

L01 Ⅱ

(1) $\vec{p}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $\vec{p}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$



座標変換.

$$\begin{pmatrix}x\\y\end{pmatrix} = \xi \vec{p}_1 + \eta \vec{p}_2$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}\xi-\eta\\ \xi+\eta\end{pmatrix}$$

を考える.

$$x+y = \frac{1}{\sqrt{2}}\cdot 2\xi = \sqrt{2}\,\xi \quad , \quad xy = \frac{1}{2}(\xi^2-\eta^2)$$

$$x-y = \frac{1}{\sqrt{2}}\cdot 2\eta = \sqrt{2}\,\eta$$

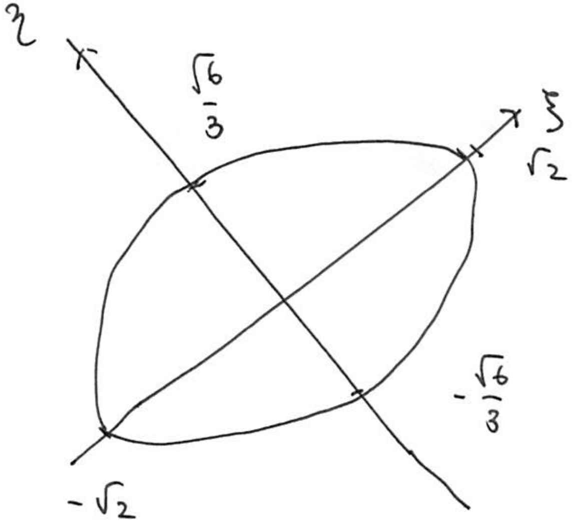
なので

$$z = x^2 - xy + y^2 = (x+y)^2 - 3xy.$$

$$= 2\xi^2 - 3\cdot\frac{1}{2}(\xi^2-\eta^2)$$

$$= \frac{1}{2}\xi^2 + \frac{3}{2}\eta^2$$

$z=1$ の等高線は右図の楕円である。



Ⅱ (2) $z = xy - 3x + 3y - 1 = 0$ の等高線は

$$z = (x+3)(y-3) + 8 = 0$$

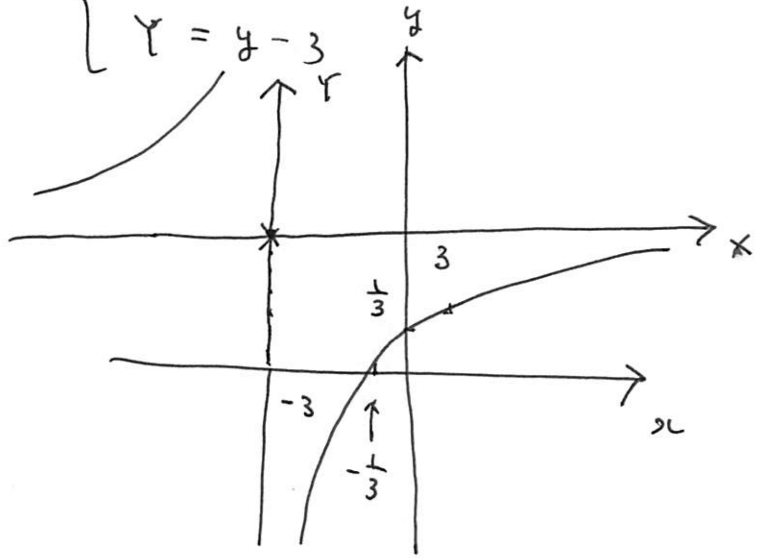
すなわち $(x+3)(y-3) = -8$

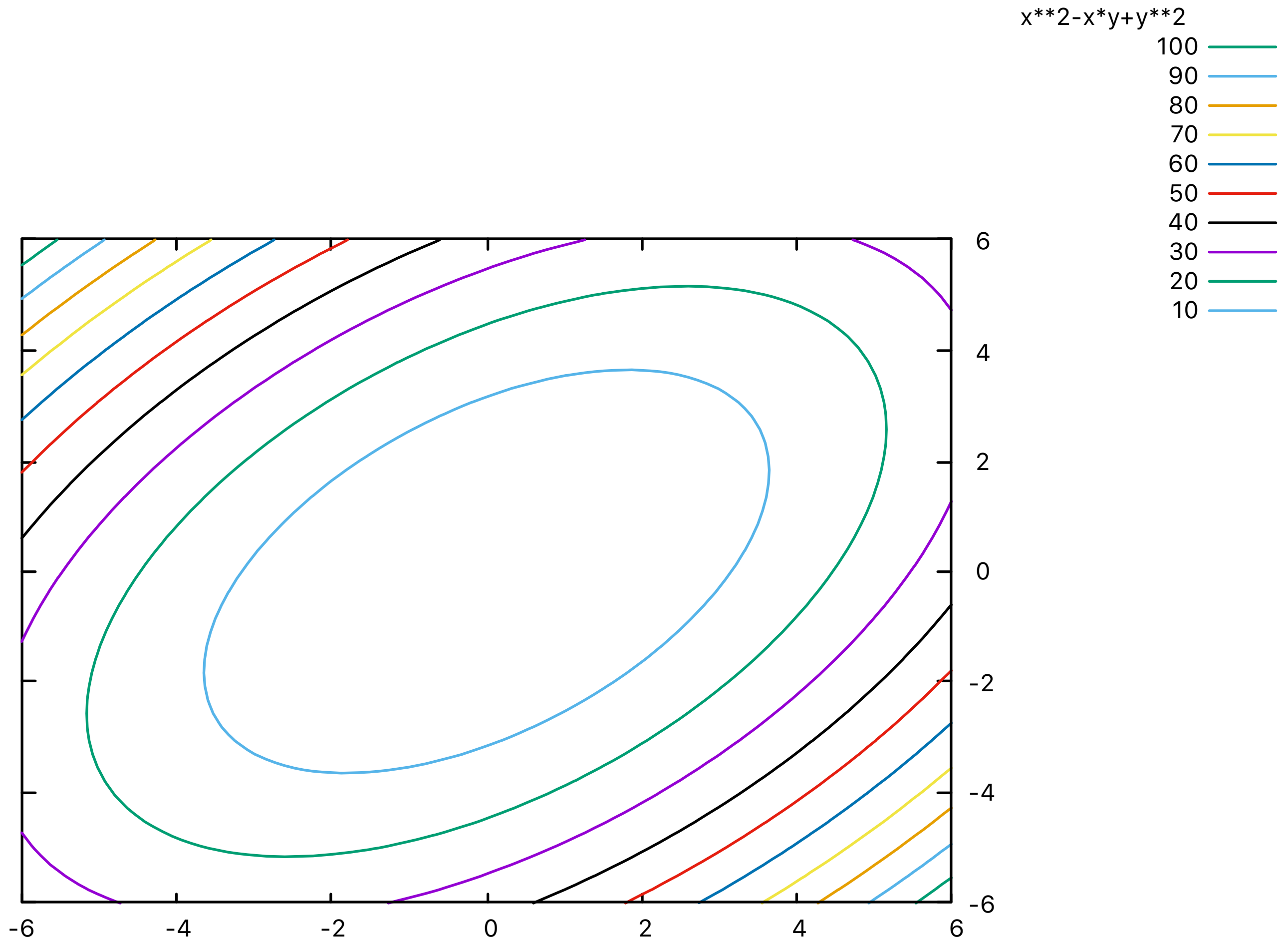
は右の座標の平行移動で

$$XY = -8$$

平行移動

$$\begin{cases} X = x+3 \\ Y = y-3 \end{cases}$$




x**2-x*y+y**2
100
90
80
70
60
50
40
30
20
10
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

III

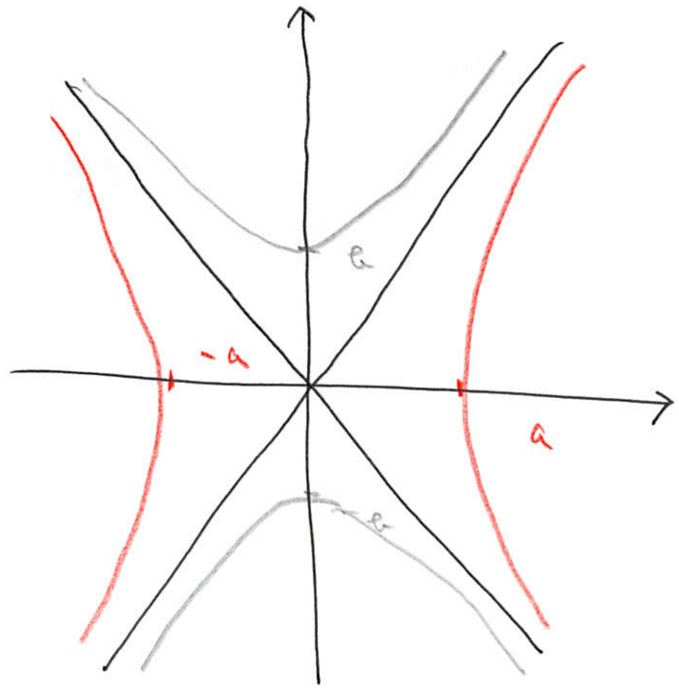
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$$

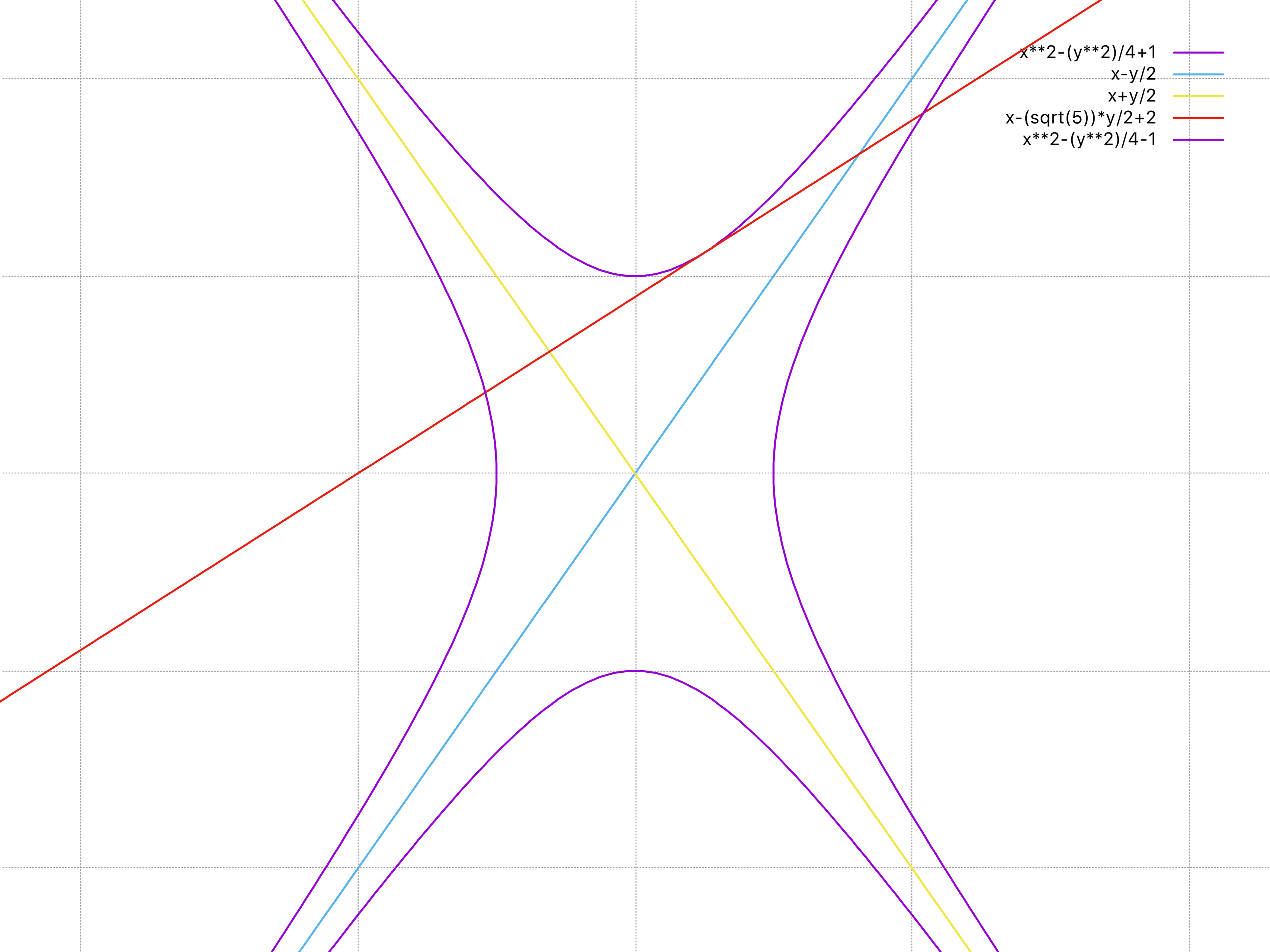
$+1$

-1

$a, b > 0$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \quad \longleftrightarrow \quad y = \pm \frac{b}{a} y$$

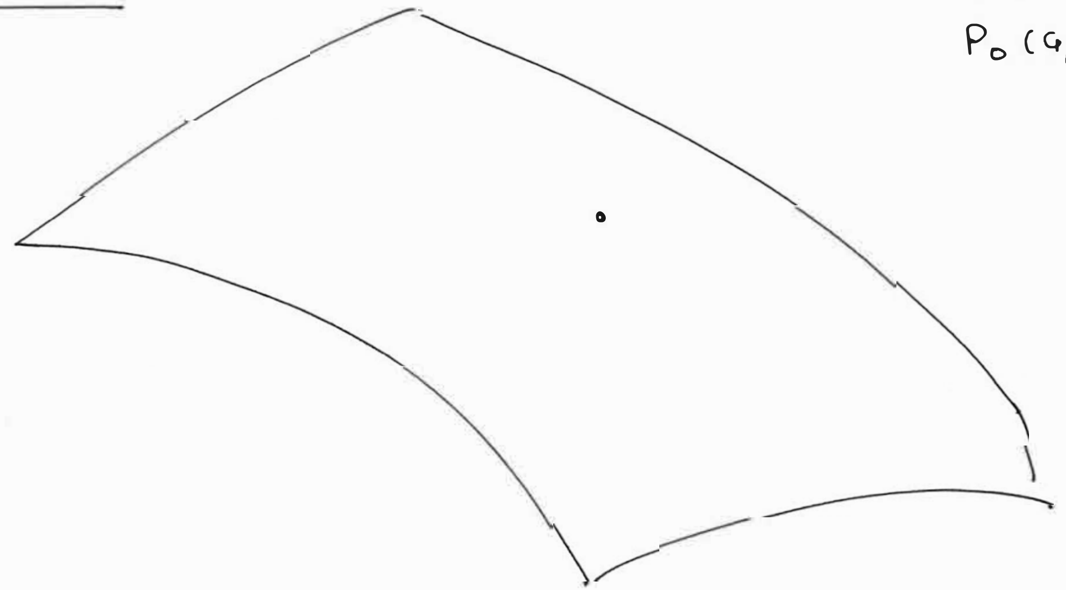



x**2-(y**2)/4+1
x-y/2
x+y/2
x-(sqrt(5))*y/2+2
x**2-(y**2)/4-1

$\mathbb{R}^3$中の等量線の法面ベクトル

$g(x, y, z) = 0$

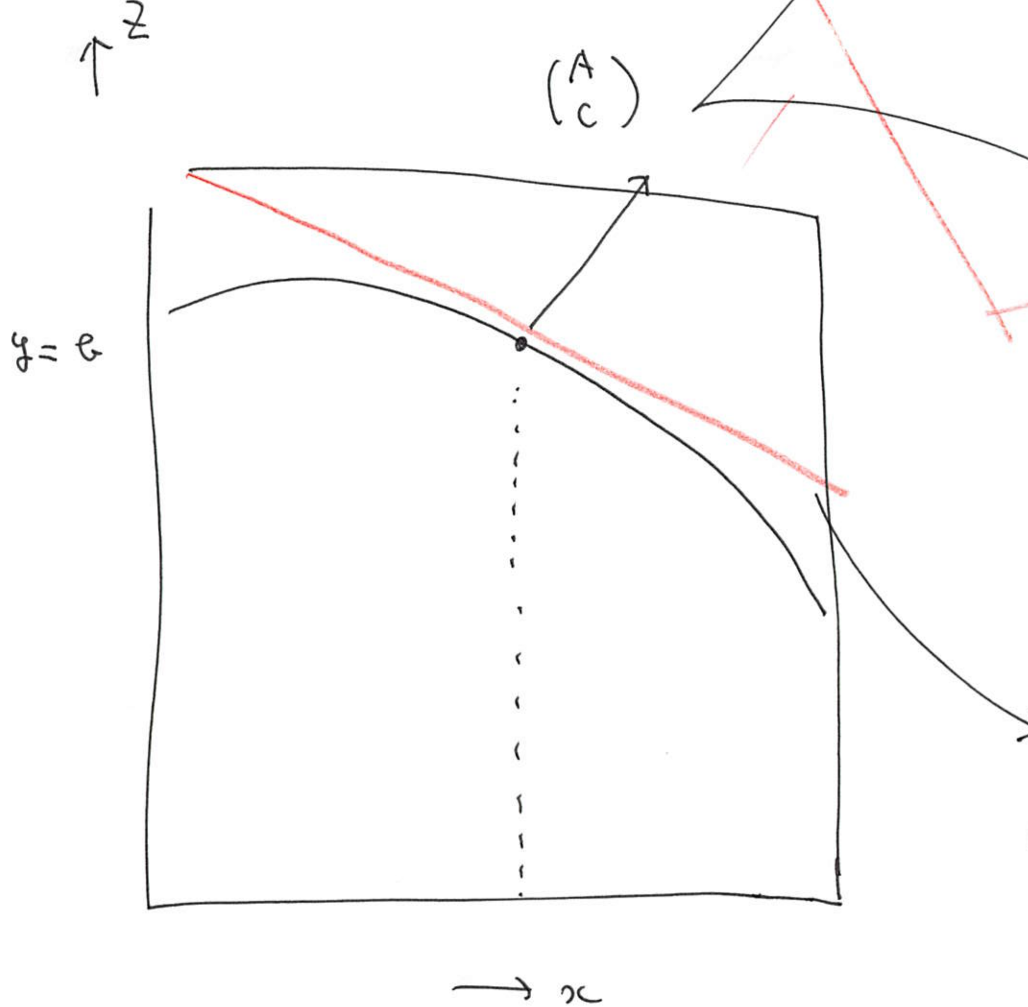
$P_0(a, b, c)$

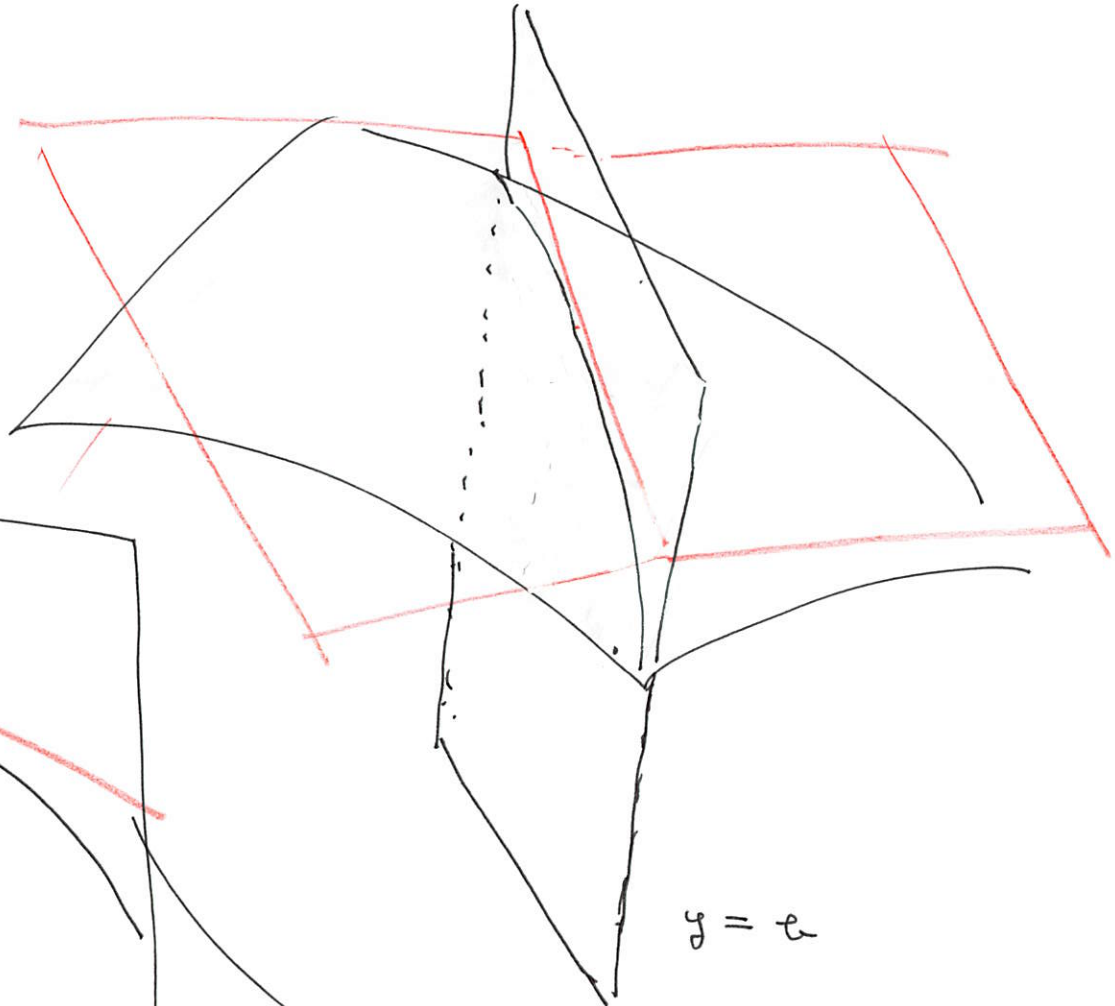


接線の方程式は？

$$A(x-a) + B(y-b) + C(z-c) = 0$$

とする。





$$A(x-a) + C(z-c) = 0$$

$$\begin{pmatrix} A \\ C \end{pmatrix} \parallel \begin{pmatrix} g_x(P_0) \\ g_z(P_0) \end{pmatrix}$$

同様に

$$\begin{pmatrix} B \\ C \end{pmatrix} \parallel \begin{pmatrix} g_y(P_0) \\ g_z(P_0) \end{pmatrix}$$

$$\rightsquigarrow \begin{pmatrix} A \\ B \\ C \end{pmatrix} \parallel \begin{pmatrix} g_x(P_0) \\ g_y(P_0) \\ g_z(P_0) \end{pmatrix}$$

$$g_x(P_0)(x-a) + g_y(P_0)(y-b) + g_z(P_0)(z-c) = 0$$

$$\left(\nabla(g)(P_0), \begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix} \right) = 0$$