$$L = \sqrt{xy} + \lambda(I - px - qy)$$

$$u(x,y) = \sqrt{xy}$$
$$g(x,y) = I - px - qy$$

$$L_x = \frac{1}{2}\cdot\frac{\sqrt{y}}{\sqrt{x}} + \lambda(-p)$$

$$L_y = \frac{1}{2}\cdot\frac{\sqrt{x}}{\sqrt{y}} + \lambda(-q)$$

$$L_{xx} = -\frac{1}{4}\cdot\frac{\sqrt{y}}{x\sqrt{x}}, \quad L_{xy} = \frac{1}{4}\cdot\frac{1}{\sqrt{xy}}, \quad L_{yy} = -\frac{1}{4}\cdot\frac{\sqrt{x}}{y\sqrt{y}}$$

$$\begin{vmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{vmatrix} = \begin{vmatrix} 0 & -p & -q \\ -p & -\frac{1}{4}\cdot\frac{\sqrt{y}}{x\sqrt{x}} & \frac{1}{4}\cdot\frac{1}{\sqrt{xy}} \\ -q & \frac{1}{4}\cdot\frac{\sqrt{x}}{y\sqrt{y}} & -\frac{1}{4}\cdot\frac{\sqrt{x}}{y\sqrt{y}} \end{vmatrix}$$

$$= \frac{1}{4}\cdot\frac{\sqrt{y}}{x\sqrt{x}}\cdot q^2 + \frac{1}{2}\cdot\frac{1}{\sqrt{xy}}\cdot pq + \frac{1}{4}\frac{\sqrt{x}}{y\sqrt{y}}\cdot p^2 > 0$$

から $(x,y) = \left(\frac{I}{2p}, \frac{I}{2q}\right)$ で極大.