## 2019 年 12 月 18 日演習問題解答

以下の積分の値を求めましょう.
**(1)** $\int_0^1 e^{-2t}dt$,　**(2)** $\int_0^{\frac{\pi}{2}} \sin t dt$,　**(3)** $\int_0^{\frac{\pi}{2}} \sin 2t dt$,　**(4)** $\int_1^2 \frac{1}{x^3}dx$,　**(5)** $\int_1^8 x^{\frac{1}{3}}dx$,　**(6)** $\int_1^2 (x-1)^4 dx$,
**(7)** $\int_1^2 \frac{1}{2x+1}dx$

**(1)** $\left(e^{-2t}\right)' = -\frac{1}{2}e^{-2t}$ から $\left(-\frac{1}{2}e^{-2t}\right)' = e^{-2t}$ となるので

$$\int_0^1 e^{-2t}dt = \left[-\frac{1}{2}e^{-2t}\right]_0^1 = -\frac{1}{2}\left(e^{-2}-1\right) = \frac{e^2-1}{2e^2}$$

**(2)**

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin t dt &= \int_0^{\frac{\pi}{2}} (-\cos t)' dt \\ &= [-\cos t]_0^{\frac{\pi}{2}} = -(0-1) = 1\end{aligned}$$

**(3)** $(\cos 2t)' = -2\sin 2t$ から $\left(-\frac{1}{2}\cos 2t\right)' = \sin 2t$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \sin 2t dt &= \left[-\frac{1}{2}\cos 2t\right]_0^{\frac{\pi}{2}} \\ &= -\frac{1}{2}(\cos\pi - \cos 0) = -\frac{1}{2}(-1-1) = 1\end{aligned}$$

**(4)** $\left(\frac{1}{x^2}\right)' = -\frac{2}{x^3}$ から $\left(-\frac{1}{2}\cdot\frac{1}{x^2}\right)' = \frac{1}{x^3}$

$$\int_1^2 \frac{1}{x^3}dx = \left[-\frac{1}{2}\cdot\frac{1}{x^2}\right]_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right) = \frac{3}{8}$$

**(5)** $\left(x^{\frac{4}{3}}\right)' = x^{\frac{1}{3}}$ から $\left(\frac{3}{4}x^{\frac{4}{3}}\right)' = x^{\frac{1}{3}}$

$$\int_1^8 x^{\frac{1}{3}}dx = \left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8 = \frac{3}{4}(8\cdot 2-1) = \frac{45}{4}$$

**(6)** $\left\{(x-1)^5\right\}' = 5(x-1)^4$ から $\left\{\frac{1}{5}(x-1)^5\right\}' = (x-1)^4$

$$\int_1^2 (x-1)^4 dx = \left[\frac{1}{5}(x-1)^5\right]_1^2 = \frac{1}{5}$$

**(7)** $\{\log(2x+1)\}' = 2\cdot\frac{1}{2x+1}$ から $\left\{\frac{1}{2}\log(2x+1)\right\}' = \frac{1}{2x+1}$

$$\int_1^2 \frac{1}{2x+1}dx = \left[\frac{1}{2}\log(2x+1)\right]_1^2 = \frac{1}{2}(\log 5 - \log 3) = \frac{1}{2}\log\frac{5}{3}$$