I (1) $f_x = \dfrac{2x}{x^2+y^2+1}$, $f_y = \dfrac{2y}{x^2+y^2+1}$

(2) $f_x = 1 \cdot e^{x-y} + x \cdot e^{x-y} = (x+1)e^{x-y}$

$f_y = e^{x-y} \cdot (-1) = -e^{x-y}$

(3) $f_x = 3(x-2y)^2 \cdot 1 = 3(x-2y)^2$

$f_y = 3(x-2y)^2 \cdot (-2) = -6(x-2y)^2$

II

$$\begin{cases} f_x = 2x - y - 2 = 0 \\ f_y = -x + 2y + 3 = 0 \end{cases} \Leftrightarrow \begin{cases} 2x - y = 2 \\ -x + 2y = -3 \end{cases}$$

これをクラメールの公式で解くと

$$x = \frac{\begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \frac{1}{3}$$

$$y = \frac{\begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \frac{-4}{3} = -\frac{4}{3}$$

III

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & -\cos\alpha\sin\beta - \cos\alpha\sin\beta \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos\beta \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}$$