2018/01/15

(1) $\int_0^{\pi} \sin t \, dt = \left[ -\cos t \right]_0^{\pi}$

$= (-\cos \pi) - (-\cos 0)$

$= (-(-1)) - (-1) = 2$

$(\sin t)' = \cos t$

$(\cos t)' = -\sin t$

$\hookrightarrow (-\cos t)' = \sin t.$

(2) $\int_0^{\frac{\pi}{2}} t \cos t \, dt = \int_0^{\frac{\pi}{2}} t (\sin t)' \, dt$

$= \left[ t \sin t \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 1 \cdot \sin t \, dt$

$= \frac{\pi}{2} \cdot 1 - 0 \cdot 0 - \left[ -\cos t \right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2} - ((-0) - (-1)) = \frac{\pi}{2} - 1$

(3) $\displaystyle\int_1^e x^2 \log x\,dx = \int_1^e \left(\frac{x^3}{3}\right)' \log x\,dx.$

$\left(\dfrac{x^3}{3}\right)' = x^2$

$(\log x)'$

$$= \left[\frac{x^3}{3}\cdot \log x\right]_1^e - \int_1^e \frac{x^3}{3}\cdot\frac{1}{x}\,dx$$

$$= \frac{e^3}{3}\cdot 1 - \frac{1}{3}\int_1^e x^2\,dx$$

$$= \frac{e^3}{3} - \frac{1}{3}\left[\frac{x^3}{3}\right]_1^e = \cdots = \frac{2e^3+1}{9}$$

(4) $\displaystyle\int_0^1 t e^{-2t}\,dt = \int_0^1 t\cdot\left(-\frac{1}{2}e^{-2t}\right)' dt$

$(e^{-2t})' = -2e^{-2t} \rightsquigarrow \left(-\dfrac{1}{2}e^{-2t}\right)' = e^{-2t}$

$$= -\frac{1}{2}\left[t\cdot e^{-2t}\right]_0^1 + \frac{1}{2}\int_0^1 1\cdot e^{-2t}\,dt$$

$$= -\frac{1}{2}\cdot 1\cdot e^{-2} + \frac{1}{2}\left[-\frac{1}{2}e^{-2t}\right]_0^1$$

$$= -\frac{1}{2}e^{-2} - \frac{1}{4}(e^{-2}-1) = -\frac{3}{4e^2} + \frac{1}{4} = \frac{e^2-3}{4e^2}$$

(5) $\int_0^1 x(x+1)^4 \, dx = \int_0^1 (x+1-1)(x+1)^4 \, dx$

$$= \int_0^1 (x+1)^5 \, dx - \int_0^1 (x+1)^4 \, dx$$

$$= \cdots$$

$$\int_0^1 x \cdot \left( \frac{(x+1)^5}{5} \right)' dx$$

$$= \left[ x \cdot \frac{(x+1)^5}{5} \right]_0^1 - \int_0^1 1 \cdot \frac{(x+1)^5}{5} \, dx$$

$$= 1 \cdot \frac{2^5}{5} - \frac{1}{5}\left[ \frac{(x+1)^6}{6} \right]_0^1 = \frac{32}{5} - \frac{1}{5} \cdot \frac{1}{6}(2^6 - 1) = \cdots = \frac{43}{10}.$$

(6) $\displaystyle\int_0^1 \frac{dt}{3t+2} = \frac{1}{3}\int_0^1 \frac{dt}{3t+2}\cdot \underbrace{3}_{=\,\varphi'(t)}\, dt = \frac{1}{3}\int_2^5 \frac{1}{x}\,dx$

$\varphi(t) = 3t+2$

$\varphi'(t) = 3.$

$f(x) = \dfrac{1}{x}.$

| $t$ | $0 \nearrow 1$ |
| --- | --- |
| $x$ | $2 \nearrow 5$ |

$x = \varphi(t)$

$$= \frac{1}{3}\Big[\log x\Big]_2^5$$

$$= \frac{1}{3}(\log 5 - \log 2)$$

$$= \frac{1}{3}\log\frac{5}{2}$$

$$(\log(3t+2))' = \frac{1}{3t+2}\cdot \underbrace{3}_{=\,(3t+2)'}$$

$$\left(\frac{1}{3}\log(3t+2)\right)' = \frac{1}{3t+2}$$

$$= \left[\frac{1}{3}\log(3t+2)\right]_0^1 = \cdots$$

(7)

$$\int_0^1 \frac{dt}{(3t+2)^2} = \frac{1}{3}\int_2^5 \frac{1}{x^2}\,dx$$

$$= \frac{1}{3}\left[-\frac{1}{x}\right]_2^5 = \frac{1}{3}\left(-\frac{1}{5}+\frac{1}{2}\right) = \frac{1}{10}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\leadsto \left(-\frac{1}{x}\right)' = \frac{1}{x^2}$$

$$= \frac{1}{2}\varphi'(t)$$

(8)

$$\int_0^1 \sqrt{1+t^2}\,t\,dt = \frac{1}{2}\int_0^1 \sqrt{1+t^2}\cdot(1+t^2)'\,dt.$$

$\varphi(t) = 1+t^2$, $\varphi'(t) = 2t$

$f(x) = \sqrt{x}$.

$$= \frac{1}{2}\int_1^2 \sqrt{x}\,dx = \frac{1}{2}\left[\frac{2}{3}x\sqrt{x}\right]_1^2$$

$$= \frac{1}{3}\cdot(2\sqrt{2}-1)$$

| $t$ | $0 \nearrow 1$ |
|---|---|
| $x$ | $1 \nearrow 2$ |

$$(x\sqrt{x})' = \frac{3}{2}\sqrt{x} \leadsto \left(\frac{2}{3}x\sqrt{x}\right)' = \sqrt{x}.$$

$$\| $$

$$(x^{\frac{3}{2}})'$$

$$\int_{-2}^{1} \frac{x}{\sqrt{x+3}}\,dx = \int_{-2}^{1} \frac{x+3-3}{\sqrt{x+3}}\,dx = \int_{-2}^{1} \sqrt{x+3}\,dx - 3\int_{-2}^{1} \frac{dx}{\sqrt{x+3}}$$

$$= \left[\frac{2}{3}(x+3)\sqrt{x+3}\right]_{-2}^{1} - \cdots$$

We find $\varphi(t)$ so that

$x = \varphi(t)$

$x + 3 = t$

$\rightsquigarrow \varphi(t) + 3 = t$

$\rightsquigarrow \varphi(t) = t - 3$

| $t$ | $1 \nearrow 4$ |
|---|---|
| $x$ | $-2 \nearrow 1$ |

$(\sqrt{t})' = \frac{1}{2}\frac{1}{\sqrt{t}}$.

$(2\sqrt{t})' = \frac{1}{\sqrt{t}}$

$$= \int_{1}^{4} \frac{t-3}{\sqrt{t}}\,dt = \int_{1}^{4} \sqrt{t}\,dt - 3\int_{1}^{4} \frac{1}{\sqrt{t}}\,dt$$

$$= \left[\frac{2}{3}t\sqrt{t}\right]_{1}^{4} - 3\left[2\sqrt{t}\right]_{1}^{4}$$

$$= \frac{2}{3}(8-1) - 6(2-1) = \cdots = \frac{4}{3}.$$

## Example

$$\int_{-2}^{1} \frac{x}{\sqrt{x+3}}\,dx = \int_{1}^{2} \frac{t^2-3}{t} \cdot 2t\,dt$$

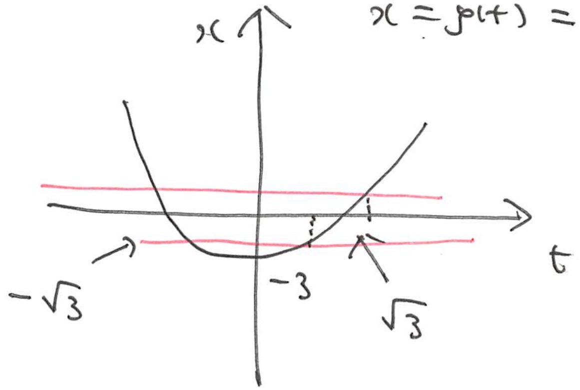
Try to find

$x = \varphi(t)$ gives you $t = \sqrt{x+3} \rightsquigarrow t^2 = x+3$.

$x = \varphi(t) = t^2 - 3;\quad dx = \varphi'(t)\,dt = 2t\,dt$

$t = \sqrt{x+3} \rightsquigarrow t \geq 0.$

$x = \varphi(t) = t^2 - 3$




| $t$ | $1 \nearrow 2$ |
|---|---|
| $x$ | $-2 \nearrow 1$ |

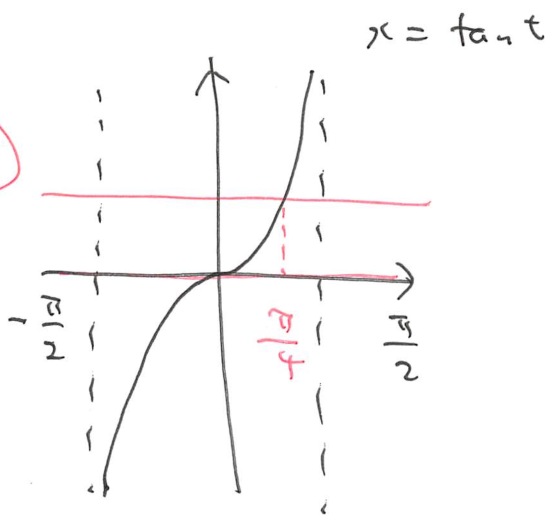
$x = -2;\ t^2 = -2+3 \to t = 1$

$x = 1;\ t^2 = 1+3 \to t = 2$

$$= 2\int_{1}^{2} (t^2-3)\,dt$$

$$= 2\left[\frac{t^3}{3} - 3t\right]_1^2 = \cdots$$

$$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \cos^2 t \cdot \frac{1}{\cos^2 t}\, dt$$




$x = \tan t$.

$$1 + x^2 = 1 + \frac{\sin^2 t}{\cos^2 t} = \frac{\cos^2 t + \sin^2 t}{\cos^2 t}$$

$$= \frac{1}{\cos^2 t}.$$

$$(\tan t)' = \left(\frac{\sin t}{\cos t}\right)' = \frac{(\sin t)' \cos t - \sin t (\cos t)'}{\cos^2 t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$= \int_0^{\frac{\pi}{4}} dt = \frac{\pi}{4}.$$

$$(1+t^2)' = 2t$$

(1) $\displaystyle\int_0^1 \log(1+t^2)\, t\, dt$

(2) $\displaystyle\int_{-1}^1 (x+1)(x-1)^4\, dx =$ $\int$ by integration by parts

(3) $\displaystyle\int_0^1 \frac{1}{\sqrt{3x+2}}\, dx$

Find $x = \varphi(t)$ s.t. $t = 3x+2$

(4) $\displaystyle\int_0^1 (2x+1)\, e^x\, dx$

$$e^x = (e^x)'$$

(5) $\displaystyle\int_1^2 \log(2x+1)\, dx$

① Find $x = \varphi(t)$ s.t. $t = 2x+1$

②

$$= \int_1^2 (x)' \log(2x+1)\, dx$$

(6) $\int_1^2 \frac{1}{(2x+1)^4} dx$ Find $x = \varphi(t)$ s.t. $t = 2x+1$

(7) $\int_0^1 \sqrt{2x+1}\, dx$

(8) $\int_0^1 \frac{x-3}{(2-x)^2} dx.$ $t = 2-x$ OR $t = x-2$

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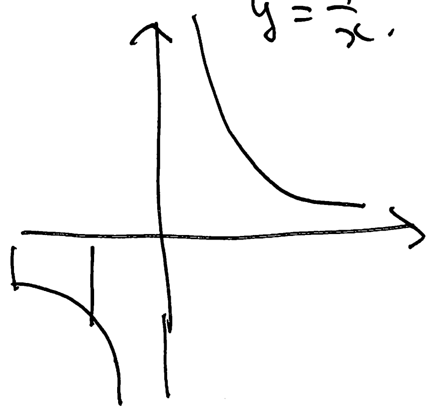
$(\log x)' = \frac{1}{x}.$ $(x>0)$





when $x<0$

$(\log(-x))' = \frac{1}{-x} \cdot (-1) = \frac{1}{x}.$

$(\log|x|)' = \frac{1}{x}$

$$\int_{-2}^{-1} \frac{1}{x} dx = \Big[\log|x|\Big]_{-2}^{-1} = \log 1 - \log 2$$

$$= -\log 2.$$