Jan. 17, 2017

(1) As is shown in Exercise I of Dec. 27,

(2) $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ is diagonalized by the rotation

$P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ and the quadratic form defined

by A is expressed by

$$\left(A \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right) = -\xi^2 + 3\eta^2$$

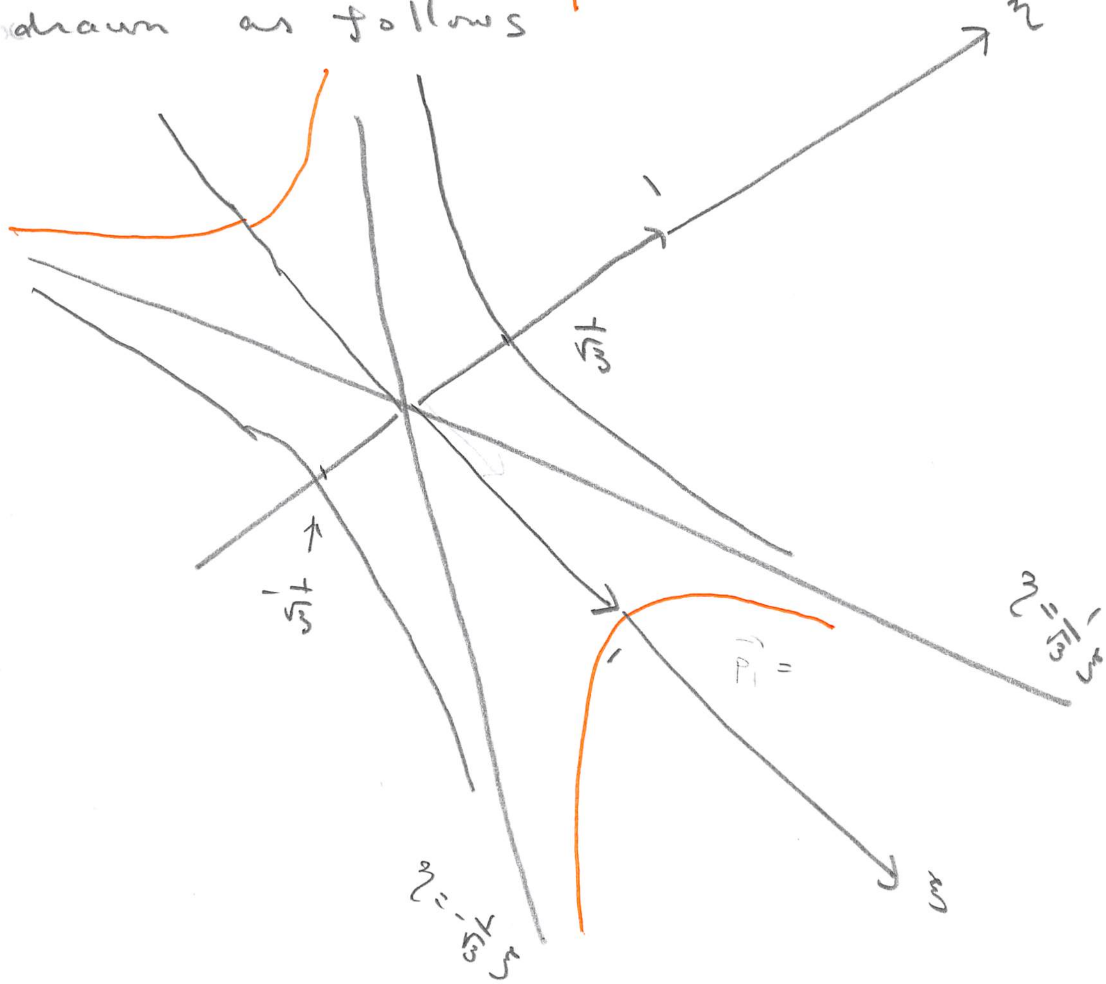
interms of the coordinate system

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix}.$$

The quadratic curve

$$\left(A \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right) = 1$$

is drawn as follows -1



(3) As shown in Exercise I of Dec. 27, $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$

(4)

is diagonalized by a rotation

$$P = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

and the quadratic form by A is expressed

$$\left(A \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right) = -\xi^2 + 9\eta^2$$

in terms of the coordinate system

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix}.$$

The quadratic curve

$$\left(A \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right) = 1$$

-1

is drawn as follows