2016/12/27 P

I Diagonalize the following symmetric matrix A by a rotation matrix and simplify the quadratic form defined by A.

(1) $A = \begin{pmatrix} -1 & 3 \\ 3 & 7 \end{pmatrix}$ (2) $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ (3) $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$

II Let A be a 2×2 matrix. Show that

$$\Phi_A(\lambda) = \Phi_{P^{-1}AP}(\lambda)$$

for a 2×2 regular matrix P.

III A vector $\vec{p} \in \mathbb{R}^n$ satisfies the condition

$$c\vec{p} = \vec{0} \quad \text{and} \quad \vec{p} \neq \vec{0}.$$

Show that $c = 0$.

IV Find a symmetric matrix A which defines the quadratic form

(1) $x^2 + xy + y^2$

(2) $3x^2 + 4xy + 2y^2$

(3) $x^2 - 6xy + 4y^2$

V. Recall the definition of the rotation matrix with angle θ by

$$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

Show that

$$R_\theta R_{\theta'} = R_{\theta+\theta'}$$

and that

$$R_\theta^{-1} = R_{-\theta}.$$

VI Let $Q = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$. Show that

${}^tQ\,Q = Q\,{}^tQ = I_2$ and that

$$(Q\vec{v}, Q\vec{w}) = (\vec{v}, \vec{w})$$

for any $\vec{v}, \vec{w} \in \mathbb{R}^2$