2016/12/13

I Find $\tilde{A}$ for $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 3 \end{pmatrix}$

$$\tilde{A} = \begin{pmatrix} |A_{11}| & -|A_{21}| & |A_{31}| \\ -|A_{12}| & |A_{22}| & -|A_{32}| \\ |A_{13}| & -|A_{23}| & |A_{33}| \end{pmatrix}$$

$$= \begin{pmatrix} \begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 & 0 \\ -3 & 3 & 0 \\ -1 & -1 & 2 \end{pmatrix}$$

II (i) $\left| \lambda I_2 - A \right| = \begin{vmatrix} \lambda-1 & -2 \\ -4 & \lambda-3 \end{vmatrix}$

$= (\lambda-1)(\lambda-3) - 8$

$= \lambda^2 - 4\lambda - 5$

$= (\lambda-5)(\lambda+1)$

(ii) $\Phi(\lambda) = 0 \iff \lambda = -1, 5.$

<u>In case $\lambda = -1$</u>

$A\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix} \iff (-I_2 - A)\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$

$\iff \begin{pmatrix} -2 & -2 \\ -4 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$

$\iff x + y = 0$

The solution is expressed by y

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = y\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

<u>In case $\lambda = 5$</u>

$A\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix} \iff (5I_2 - A)\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$

$\iff \begin{pmatrix} 4 & -2 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$

$\iff 2x - y = 0$

The solution is expressed by x

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix}.$