

I Find t for which the following matrices are not regular.

$$(1) \begin{pmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{pmatrix}$$

$$(2) \begin{pmatrix} t-1 & 3 & -3 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{pmatrix}$$

$$(3) \begin{pmatrix} t+3 & -1 & 1 \\ 7 & t-5 & 1 \\ 6 & -6 & t+2 \end{pmatrix}$$

II Find $\tilde{A}$ and A^{-1} for the following A .

$$(1) A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$(2) A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$(3) A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(4) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix}$$

III Compute the determinant of each matrix

$$(1) \begin{pmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 2 & 5 & -1 \end{pmatrix}$$

$$(2) \begin{pmatrix} 2 & 0 & 1 \\ 4 & 2 & -3 \\ 5 & 3 & 1 \end{pmatrix}$$

$$(3) \begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & -3 \\ -1 & -3 & 5 \end{pmatrix}$$

$$(4) \begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & -4 \\ 4 & 1 & 3 \end{pmatrix}$$

I (i)

$$\begin{vmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{vmatrix} = (t-4) \begin{vmatrix} t-2 & 4 \\ 1 & t+1 \end{vmatrix}$$

$$= (t-4)(t^2 - t - 6) = (t-2)(t-3)(t-4)$$

The matrix is not regular iff $t = 2, 3, 4$.

$$(ii) \quad \begin{vmatrix} t-1 & 3 & -3 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{vmatrix} \xrightarrow{R_1 \rightarrow R_1 + (-1)R_2} \begin{vmatrix} t+2 & -t-2 & 0 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{vmatrix}$$

$$= (t+2) \begin{vmatrix} 1 & -1 & 0 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{vmatrix}$$

$$= (t+2) \begin{vmatrix} 1 & -1 & 0 \\ 0 & t+2 & -3 \\ 0 & 0 & t-4 \end{vmatrix} = (t+2)^2 (t-4)$$

The matrix is not regular iff $t \neq -2, 4$

$$(iii) \quad \begin{vmatrix} t+3 & -1 & 1 \\ 7 & t-5 & 1 \\ 6 & -6 & t+2 \end{vmatrix} \xrightarrow{R_1 \rightarrow R_1 + (-1)R_2} \begin{vmatrix} t-4 & 4-t & 0 \\ 7 & t-5 & 1 \\ 6 & -6 & t+2 \end{vmatrix}$$

$$= (t-4) \begin{vmatrix} 1 & -1 & 0 \\ 7 & t-5 & 1 \\ 6 & -6 & t+2 \end{vmatrix} = (t-4) \begin{vmatrix} 1 & -1 & 0 \\ 0 & t+2 & 1 \\ 0 & 0 & t+2 \end{vmatrix}$$

$$= (t-4)(t+2)^2$$

The matrix is not regular iff $t \neq -2, 4$.

$$\text{II. }^{(1)} |A| = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2$$

$$\tilde{A} = \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$$

$$\tilde{A} = -\frac{1}{2} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$$

$$^{(2)} |A| = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 0 & -2 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix}$$

$$= -1$$

$$\tilde{A} = \begin{pmatrix} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} \\ \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -2 \\ -3 & -1 & 6 \\ 2 & 1 & -5 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & -6 \\ -2 & -1 & 5 \end{pmatrix}$$

(3)

$$|A| = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$$

$$A_2 = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = A_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(4)

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & -2 \\ 0 & 3 \end{vmatrix} = 2$$

$$A_2 = \begin{pmatrix} 1 & 3 & 4 & 5 \\ 2 & 4 & 5 & 7 \\ 3 & 4 & 5 & 7 \\ 2 & 3 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 4 & 5 \\ -1 & 1 & 1 & 3 \\ -1 & 1 & 1 & 3 \\ -1 & 1 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 4 & 5 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 & -1 & -1 \\ -10 & 4 & -2 & -2 \\ 4 & -3 & -1 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -1 & -1 & -1 \\ -10 & 4 & -2 & -2 \\ 4 & -3 & -1 & -1 \end{pmatrix}$$

III

$$(1) \begin{vmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 2 & 5 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -10 & -9 \\ 0 & 1 & -7 \end{vmatrix} = 1 \cdot \begin{vmatrix} -10 & -9 \\ 1 & -7 \end{vmatrix} \\ = 63$$

$$(2) \begin{vmatrix} 2 & 0 & 1 \\ 4 & 2 & -3 \\ 5 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 1 \\ -2 & 2 & 0 \\ 3 & 3 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} -2 & 2 \\ 3 & 3 \end{vmatrix} \\ = -12$$

$$(3) \begin{vmatrix} 2 & 0 & 1 \\ 3 & 2 & -3 \\ -1 & -3 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 1 \\ 9 & 2 & 0 \\ -11 & -3 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 9 & 2 \\ -11 & -3 \end{vmatrix} \\ = -5$$

$$(4) \begin{vmatrix} 1 & 0 & 0 \\ 3 & 2 & -4 \\ 4 & 1 & 3 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix} = 10$$