2016/11/29

I

$$(A \mid I_3) = \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \qquad \begin{array}{l} R_2 \to (-2)R_1 + R_2 \\ R_3 \to (-1)R_1 + R_3 \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -2 \\ 0 & 1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right) \qquad \begin{array}{l} R_1 \to R_1 + (-2)R_3 \\ R_2 \to R_2 + R_3 \end{array}$$

It follows from the above sequence of row elementary operation that

$$A^{-1} = \begin{pmatrix} 3 & 0 & -2 \\ -3 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

II

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 3 & -1 & 2 & 4 \\ 5 & 3 & -4 & 2 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 7 \\ 0 & -7 & 11 & 7 \end{array}\right) \qquad \begin{array}{l} R_2 \to (-3)R_1 + R_2 \\ R_3 \to (-5)R_1 + R_3 \end{array}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -7 & 11 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad R_3 \to -R_2 + R_3$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -\frac{11}{7} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad R_2 \to -\frac{1}{7} R_2.$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{7} & \overset{1}{\cancel{-3}} \\ 0 & 1 & -\frac{11}{7} & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

From the above sequence of row elementary operations it follows that the system of equations is equivalent to

$$\begin{cases} x \qquad - \frac{1}{7} z = \cancel{-3}\ 1 \\ \qquad y - \frac{11}{7} z = -1 . \end{cases}$$

Put $z = t$. Then

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{7} t \cancel{-3}\ +1 \\ \frac{11}{7} t - 1 \\ t \end{pmatrix}$$