Exercises for TA sessions 2016/11/29

I We are given the following bivariate data.

x	y
x_1	y_1
$\vdots$	$\vdots$
x_n	y_n

We define a variable

$$z = ax + by + c$$

i.e. $z_j = a x_j + b y_j + c$

Then show that

$$V(z) = a^2 V(x) + 2ab\, C_{xy} + b^2 V(y)$$

by defining the vector

$$\vec{z} = \frac{1}{\sqrt{n}} \begin{pmatrix} z_1 - \bar{z} \\ \vdots \\ z_n - \bar{z} \end{pmatrix}$$

II Show that the system of equations

$$\begin{cases} x - 2y + z = -1 \\ x + 3y + 2z = 1 \\ 2x + 11y + 5z = 5 \end{cases}$$

has no solution.

III Solve the system of equations

(1)
$$\begin{cases} x + y + z + w = 1 \\ 5x + 6y + 3z + 7w = 1 \\ 2x + y + 4z = 6 \end{cases}$$

(2)
$$\begin{cases} x - y + 2z + w = 2 \\ 3x + 2y + w = 1 \\ 4x + y + 2z + 2w = 3 \end{cases}$$

(3)
$$\begin{cases} 2x + 6y = 11 \\ 6x + 20y - 6z = 3 \\ 6y - 18z = 1 \end{cases}$$

IV Find the condition with respect to c for the system of equations

$$\begin{cases} 2x + y - z = 1 \\ x - 3y + 2z = 4 \\ 3x - 2y + z = c \end{cases}$$

V Find the inverse matrix for A if any.

(1) $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 3 & 4 & 1 \\ 3 & 3 & 3 & 1 \\ 1 & 2 & 3 & 1 \end{pmatrix}$ (2) $A = \begin{pmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{pmatrix}$

(3) $A = \begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix}$ (4) $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{pmatrix}$

(5) $A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 6 \\ -1 & 5 & 1 \end{pmatrix}$