Exercises for TA session Nov. 22, 2016

I Given a square matrix A of size n and assume that A is regular. Show that ${}^tA$ is also regular. Moreover show that

$$({}^tA)^{-1} = {}^t(A^{-1})$$

II Given a square matrix A of size n. Assume that A is symmetric in the sense that ${}^tA = A$. Show that if A is regular, A^{-1} is also symmetric.

III Find a row reduced echelon matrix for A:

(1) $A = \begin{pmatrix} 0 & 1 & 3 & 2 \\ 1 & 2 & 2 & 0 \\ 2 & 4 & 4 & 3 \\ -1 & -1 & 1 & 0 \end{pmatrix}$ (2) $A = \begin{pmatrix} 1 & 2 & -1 & 4 \\ 3 & 2 & 0 & 2 \\ 0 & 1 & 3 & 2 \\ 3 & 3 & 3 & 4 \end{pmatrix}$

IV Find all the solutions to the system of equations.

(1) $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 3 \\ 3 & 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \vec{0}$ (2) $\begin{pmatrix} 1 & -3 & 0 & 2 \\ 2 & -6 & 2 & 2 \\ 4 & -12 & 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \vec{0}$