plin 2016/11/08

Solution to Quiz.

$$ {}^t A A = \begin{pmatrix} \|\vec{a}\|^2 & (\vec{a},\vec{e}) \\ (\vec{e},\vec{a}) & \|\vec{e}\|^2 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 1 & 7 \end{pmatrix} $$

$$ {}^t A \vec{c} = \begin{pmatrix} (\vec{a},\vec{c}) \\ (\vec{e},\vec{c}) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} $$

$$ {}^t A A \begin{pmatrix} s \\ t \end{pmatrix} = {}^t A \vec{c} \iff \begin{pmatrix} 4 & 1 \\ 1 & 7 \end{pmatrix} \begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} $$

Since we have

$$ \begin{pmatrix} 4 & 1 \\ 1 & 7 \end{pmatrix}^{-1} = \frac{1}{27} \begin{pmatrix} 7 & -1 \\ -1 & 4 \end{pmatrix} $$

$$ \begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 1 & 7 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{27} \begin{pmatrix} 7 & -1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} $$

$$ = \frac{1}{27} \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 2 \\ 1 \end{pmatrix}. $$

Accordingly $\|\vec{c} - s\vec{a} - t\vec{e}\|^2$ takes minimum value at $(s,t) = \left(\frac{2}{9}, \frac{1}{9}\right)$.