Exercise for TA session 2016/11/01

I. Given 3×3 matrices A, B, C and Assume that they are regular.

(i) show that AB is regular.

(ii) show That ABC is regular.

(iii) Show that A^{-1} is regular.

II. Find a 4×4 matrix P satisfying the given identity

(i)
$$P\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{d}\end{pmatrix} = \begin{pmatrix}\mathbf{c}\\ \mathbf{b}\\ \mathbf{a}\\ \mathbf{d}\end{pmatrix}$$

(ii)
$$P\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{d}\end{pmatrix} = \begin{pmatrix}\mathbf{d}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{a}\end{pmatrix}$$

(iii)
$$P\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{d}\end{pmatrix} = \begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \lambda\mathbf{a}+\mathbf{c}\\ \mathbf{d}\end{pmatrix}$$

(iv)
$$P\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{d}\end{pmatrix} = \begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \lambda\mathbf{b}+\mathbf{d}\end{pmatrix}$$

(v)
$$P\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\\ \mathbf{d}\end{pmatrix} = \begin{pmatrix}\mathbf{a}\\ \mathbf{b}\\ \lambda\mathbf{c}\\ \mathbf{d}\end{pmatrix}$$

III Let $A = \begin{pmatrix} a & p & q \\ 0 & b & r \\ 0 & 0 & c \end{pmatrix}$ and assume that $abc \neq 0$.

Show that A is regular and find the inverse of A.

IV $\vec{a}_1, \ldots, \vec{a}_\ell \in \mathbb{R}^n$ and a regular $n \times n$ matrix P is given. We define the vectors b_j by

$$\vec{b}_1 = P\vec{a}_1, \ldots, \vec{b}_j = P\vec{a}_j, \ldots, \vec{b}_\ell = P\vec{a}_\ell.$$

Show that $\vec{a}_1, \ldots, \vec{a}_\ell$ are linearly independent iff $\vec{b}_1, \ldots, \vec{b}_\ell$ are linearly independent.