Oct. 25, 2016

I We have by C-H

$$A^2 - 8A + 7I_2 = O_2$$

which is viewed as

$$\begin{cases} A(A - I_2) = (A - I_2) & \cdots (1) \\ A(A - 7I_2) = (A - 7I_2) & \cdots (2) \end{cases}$$

We apply (1) and (2) repeatedly to get

$$\begin{cases} A^n(A - I_2) = 7^n(A - I_2) & \cdots (1)' \\ A^n(A - 7I_2) = (A - 7I_2) & \cdots (2)' \end{cases}$$

which is rewritten by

$$\begin{cases} A^{n+1} - A^n = 7^n(A - I_2) & \cdots (1)'' \\ A^{n+1} - 7A^n = A - 7I_2 & \cdots (2)'' \end{cases}$$

Then $(1)'' - (2)''$ leads us to

$$6A^n = 7^n(A - I_2) - (A - 7I_2)$$

Accordingly $A^n = \dfrac{7^n}{6}(A - I_2) - \dfrac{1}{6}(A - 7I_2)$

II

(i) $$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \lambda & 0 & 1 \end{pmatrix}\begin{pmatrix} x & p \\ y & q \\ z & r \end{pmatrix} = \begin{pmatrix} x & p \\ y & q \\ \lambda x + z & \lambda p + r \end{pmatrix}$$

(ii) $$\begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} x & p \\ y & q \\ z & r \end{pmatrix} = \begin{pmatrix} \alpha x & \alpha p \\ \beta y & \beta q \\ \gamma z & \gamma r \end{pmatrix}$$

(iii) $$\begin{pmatrix} x & p & \alpha \\ y & q & \beta \\ z & r & \gamma \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \alpha & p & x \\ \beta & q & y \\ \gamma & r & z \end{pmatrix}$$