plin 2016/10/18

I

(1)
$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$$

(2)
$$\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}$$

(3)
$$\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}^{-1} = -\frac{1}{3}\begin{pmatrix} 1 & -1 \\ -4 & 1 \end{pmatrix}$$

(4)
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} = \frac{1}{\cos^2\theta + \sin^2\theta}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

<u>Remark</u>
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^{-1} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$

II By C-H we get
$$A^2 - 7A + 6I_2 = O_2.$$
we consider this as
$$A^2 - (1+6)A + 1\cdot 6 I_2 = O_2.$$
to get
$$\begin{cases} A(A - I_2) = 6(A - I_2) & \cdots ① \\ A(A - 6I_2) = A - 6I_2 & \cdots ② \end{cases}$$

By using ① and ② repeatedly we get

$$\begin{cases} A^n (A - I_2) = 6^n (A - I_2) \\ A^n (A - 6I_2) = A - 6I_2 \end{cases}$$

This is

$$\begin{cases} A^{n+1} - A^n = 6^n (A - I_2) & \cdots ③ \\ A^{n+1} - 6A^n = A - 6I_2 & \cdots ④ \end{cases}$$

③ - ④ leads us to

$$5A^n = 6^n (A - I_2) - (A - 6I_2)$$

Accordingly $A^n = \frac{1}{5} 6^n (A - I_2) - \frac{1}{5} (A - 6I_2)$