Exercises October 18, 2016

I We assume that 2×2 matrices A and B are regular. Show that

AB and A^{-1} are also regular.

II We are given $\vec{a} \in \mathbb{R}^n$ and it satisfies the condition

$$(\vec{a}, \vec{x}) = 0 \quad (\text{for any } \vec{x} \in \mathbb{R}^n).$$

Show that $\vec{a} = \vec{0}$

III We are given a 2×2 matrix A. We assume that

$$A\vec{x} = \vec{0} \quad (\text{for any } \vec{x} \in \mathbb{R}^2).$$

Then show that $A = O_2$.

IV Calculate A^{-1} if A is regular

(i) $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ (iv) $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

V. Calculate the following

(i) $\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$

VI Find A^n without using C-H.

(i) $A = \begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 3 & -9 \\ 1 & -3 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (iv) $A = \begin{pmatrix} 1 & 6 \\ 0 & -1 \end{pmatrix}$

VII Find a 2×2 matrix X satisfying the following.

(i) $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} X = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$

(ii) $X \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

VIII Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} -4 & 4 \\ 1 & 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}.$$

Find a 2×2 matrix X satisfying

$$AX + B = C.$$

IX calculate A^n by using C-H.

(i) $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ (ii) $A = \begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$

X calculat A^n by dividing λ^n by $\Phi_A(\lambda)$ and using C-H.

(i) $A = \begin{pmatrix} 7 & 4 \\ -1 & 3 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 5 & -1 \\ 9 & -1 \end{pmatrix}$.

XI calculate $\Phi_A(\lambda)$ and solve the equation $(\lambda I_2 - A)\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0}$ for λ satisfying $\Phi_A(\lambda) = 0$.

(i) $A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$ (ii) $A = \begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$

XII Calculate $A^3 - A^2 - 8A + I_2$ for $A = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$. You are <u>kindly</u> asked to use C-H.