2016/10/11

Exercises for TA Session

I For $\vec{a} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, find the minimum value for

$$f(t) = \|\vec{b} - t\vec{a}\|^2 .$$

II The same question as I for

$$\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

III Prove the identity

$$\|\vec{a} + \vec{b} + \vec{c}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 + \|\vec{c}\|^2 + 2(\vec{a}, \vec{b}) + 2(\vec{b}, \vec{c}) + 2(\vec{c}, \vec{a})$$

IV Assume that $\vec{f}_1, \vec{f}_2, \vec{f}_3 \in \mathbb{R}^4$ satisfy the condition

$$(\vec{f}_i, \vec{f}_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j . \end{cases}$$

(1) Show the identities

$$\|x\vec{f}_1 + y\vec{f}_2\|^2 = x^2 + y^2$$

$$\|x\vec{f}_1 + y\vec{f}_2 + z\vec{f}_3\|^2 = x^2 + y^2 + z^2$$

(2) $\| \vec{g} - x\vec{f}_1 - y\vec{f}_2 \|^2$

$$= \|\vec{g}\|^2 + x^2 + y^2 - 2x(\vec{g}, \vec{f}_1) - 2y(\vec{g}, \vec{f}_2)$$

V Assume that $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}$ satisfy

$$\vec{a} \perp \vec{b}, \quad \vec{a} \perp \vec{c}.$$

Then show that

$$\vec{a} \perp (\lambda\vec{b} + \mu\vec{c})$$

VI Assume that $\vec{f}_1, \vec{f}_2, \vec{f}_3 \in \mathbb{R}^n$ satisfy the condition

$$(\vec{f}_i, \vec{f}_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j. \end{cases}$$

Then show that $\vec{f}_1, \vec{f}_2, \vec{f}_3$ are linearly independent.

VII Find the orthogonal projection $\vec{w}$ of $\vec{b}$ in the direction of $\vec{a}$.

(1) $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

(2) $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

(3) $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}$

VIII Calculate the following products of matrices

(i) $\begin{pmatrix} 1 & \lambda \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & \mu \\ 0 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ \lambda & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \mu & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 0 \\ 0 & \lambda \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix}$ (iv) $\begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mu & 0 \\ 0 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} \alpha & \gamma \\ 0 & \beta \end{pmatrix}\begin{pmatrix} \alpha' & \gamma' \\ 0 & \beta' \end{pmatrix}$ (vi) $\begin{pmatrix} \alpha & 0 \\ \gamma & \beta \end{pmatrix}\begin{pmatrix} \alpha' & 0 \\ \gamma' & \beta' \end{pmatrix}$

IX For a 2×2 matrix A, A^2 denote $A \cdot A$ and A^3 denotes $A^2 \cdot A = AAA$. Calculate A^2 and A^3 for

(i) $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$.

X Calculate A^{-1} if A is regular.

(i) $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ (ii) $A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$

(iii) $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ (iv) $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$