2016/10/04

I Determine whether or not the vectors $\vec{a}, \vec{b}, \vec{c}$ are linear independent.

$$\vec{a} = \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$(\#) \quad x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$$

$$\Longleftrightarrow \begin{cases} 2x = 0 & (1) \\ 4x + y + z = 0 & (2) \\ -3x + y - z = 0 & (3) \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = 0 & (1)' = (1) \times \frac{1}{2} \\ y + z = 0 & (2)' = (2) + (1) \times \left(-\frac{1}{2}\right) \\ y - z = 0 & (3)' = (3) + (1) \times 3 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x = 0 & (1)' \\ y + z = 0 & (2)' \\ -2z = 0 & (3)'' = (3)' + (2)' \times (-1) \end{cases}$$

Then it follows from $(3)''$ and $(1)'$ that $x = z = 0$. Moreover in this situation $y = -z = 0$. Accordingly $(\#)$ implies $x = y = z = 0$. $\vec{a}, \vec{b}, \vec{c}$ are linearly independent.

II The same question for

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad \vec{c} = \begin{pmatrix} 2 \\ 3 \\ -11 \end{pmatrix}$$

$$(\#) \quad x\vec{a} + y\vec{b} + z\vec{c} = \vec{0}$$

$$\Leftrightarrow \begin{cases} x + y + 2z = 0 & (1) \\ 3x + 4y + 3z = 0 & (2) \\ -4x - 3y - 11z = 0 & (3) \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + 2z = 0 & (1) \\ y - 3z = 0 & (2)' = (2) + (1) \times (-3) \\ y - 3z = 0 & (3)' = (3) + (1) \times 4 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + 2z = 0 & (1) \\ y - 3z = 0 & (2)' \end{cases}$$

$(y, z) = (3, 1)$ satisfies $(2)'$. Moreover

$$x = -y - 2z = -3 - 2 \cdot 1 = -5$$

in this case. Thus $(x, y, z) = (-5, 3, 1)$ is $\underline{\underline{a}}$ solution to (1), (2), (3). We have

$$-5\vec{a} + 3\vec{b} + \vec{c} = \vec{0},$$

which shows that $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent.