Exercises in TA session on Oct. 13

I We consider the system of linear equation

$$\begin{cases} ax + by = \alpha & \cdots (1) \\ cx + dy = \beta & \cdots (2) \end{cases}$$

Try to deduce the equation

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} y = \begin{vmatrix} a & \alpha \\ c & \beta \end{vmatrix}$$

II Evaluate the following determinants.

(i) $\begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 5 \\ 4 & 1 \end{vmatrix}$ (iii) $\begin{vmatrix} 6 & 1 \\ 3 & -2 \end{vmatrix}$

III Compute the following determinants.

(i) $\begin{vmatrix} t-2 & -3 \\ -4 & t-1 \end{vmatrix}$ (ii) $\begin{vmatrix} t-5 & 7 \\ -1 & t+3 \end{vmatrix}$

IV Solve by determinants

(i) $\begin{cases} 3x + 5y = 8 \\ 4x - 2y = 1 \end{cases}$ (ii) $\begin{cases} 2x - 3y = -1 \\ 4x + 7y = -1 \end{cases}$

V. (x, y, z) satisfies the system of linear equation

$$\begin{cases} x + 2y + z = -2 \\ 2x - y - z = 1 \end{cases}$$

Try to express x and y by z.

VI Try to show the following identities.

(iii) $|\vec{a}\ \vec{b}+\vec{c}| = |\vec{a}\ \vec{b}| + |\vec{a}\ \vec{c}|$

(iv) $|\vec{a}\ \lambda\vec{b}| = \lambda\,|\vec{a}\ \vec{b}|$

for $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$.

VII Calculate the following products of matrices

(i) $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$

(ii) $\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$

(iii) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(iv) $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(v) $\begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix}\begin{pmatrix} \cos\vartheta' & -\sin\vartheta' \\ \sin\vartheta' & \cos\vartheta' \end{pmatrix}$

(vi) $\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ \sin\vartheta & -\cos\vartheta \end{pmatrix}\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ \sin\vartheta & -\cos\vartheta \end{pmatrix}$