2016/08/09

(2) e^{-x} $\quad y = e^{-x} = e^{u} \quad u = -x$

$(e^{t})' = e^{t}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^{u} \cdot (-1) = -e^{-x}.$$

$$\boxed{(e^{ax})' = a e^{ax} \quad a: 定数}$$

(3) $y = f(x) = e^{-\frac{1}{2}x^2} = e^{u} \quad \left(u = -\frac{1}{2}x^2\right)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^{u} \cdot (-x)$$

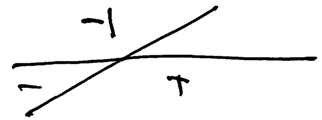
$$= -x e^{-\frac{1}{2}x^2}$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

標準正規分布の密度関数.

$$P(0 \leqq X \leqq x) = F(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt.$$

(4) $y = f(x) = xe^x$

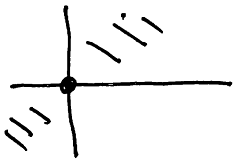


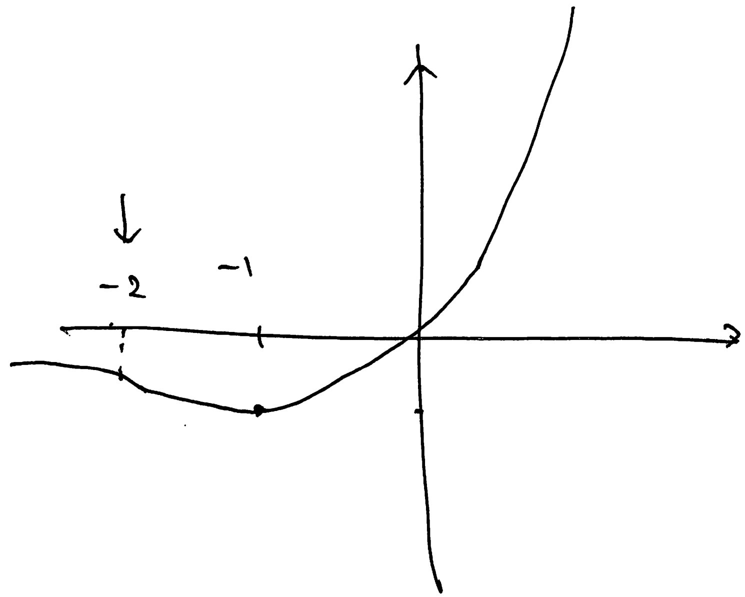
$$y' = (x)'e^x + x(e^x)'$$
$$= 1 \cdot e^x + x \cdot e^x$$
$$= (x+1)e^x$$

x		-1	
y'	$-$	0	$+$
y	$\searrow$	$-\frac{1}{e}$	$\nearrow$

$$y' \gtreqless 0 \iff x \gtreqless -1$$
$$\uparrow$$
$$e^x > 0.$$

$$xe^x \gtreqless 0 \iff x \gtreqless 0$$
$$\uparrow$$
$$e^x > 0$$





(5) $y = \dfrac{1}{1+e^x}$ $\qquad$ $y' = -\dfrac{(1+e^x)'}{(1+e^x)^2}$

$$\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$$

$$= -\frac{e^x}{(1+e^x)^2}$$

逆から $y = \dfrac{e^x}{1+e^x}$ の増減表.

$$\boxed{(\log x)' = \frac{1}{x}}$$

(7) $y = x^2 \log x.$ $\qquad$ $y' = (x^2)' \log x + x^2 (\log x)'$

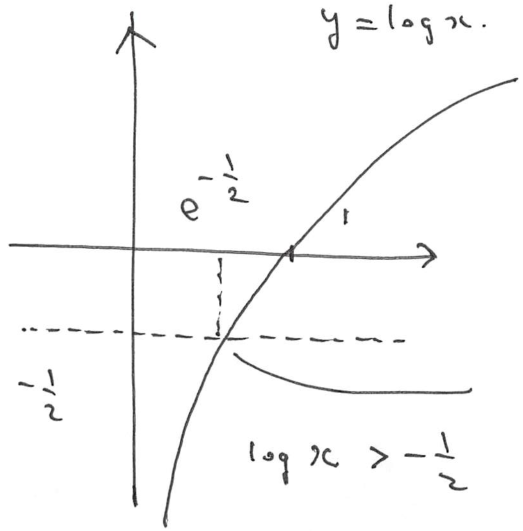
$(x > 0)$

$$= 2x \log x + x^2 \cdot \frac{1}{x} \quad = x$$

$$= \underbrace{2x}_{>0} \left(\log x + \frac{1}{2}\right)$$

$y' \gtreqless 0 \iff \log x \gtreqless -\dfrac{1}{2}$

$\iff x \gtreqless e^{-\frac{1}{2}}$



$x\log x \to 0 \quad (x \to +0)$

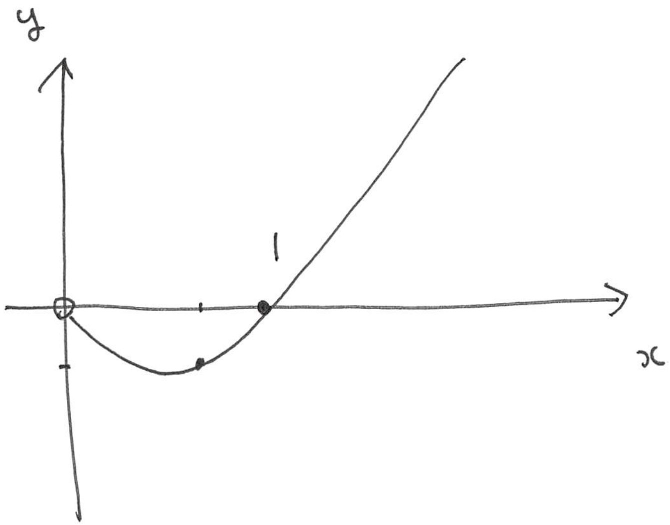
$x^2 \log x \to 0 \cdot 0 = 0$

$\quad \shortparallel$

$x \cdot x\log x$

$x^2 \log x \gtreqless 0 \iff \log x \gtreqless 0$

$\iff x \gtreqless 1$



x	(0)		$e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$	
y'		$-$	0	$+$
y	(0)	$\searrow$	$-\frac{1}{2e}$	$\nearrow$

$$\left(e^{-\frac{1}{2}}\right)^2 \cdot \log e^{-\frac{1}{2}}$$

$$= \frac{1}{e} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2e}$$

$$y' = 2x\left(\log x + \frac{1}{2}\right)$$

$$y'' = 2\left\{1 \cdot \left(\log x + \frac{1}{2}\right) + x \cdot \frac{1}{x}\right\}$$

$$= 2\left(\log x + \frac{3}{2}\right)$$

(8) $y = \dfrac{\log x}{x}$　　$y' = \dfrac{(\log x)' x - \log x \cdot (x)'}{x^2}$　　$\left(\dfrac{g}{f}\right)' = \dfrac{g'f - gf'}{f^2}$

$$= \frac{\frac{1}{x} \cdot x - \log x \cdot 1}{x^2}$$

$$= -\frac{\log x - 1}{x^2}$$

$$y' \gtreqless 0 \iff \log x \lesseqgtr 1 \iff x \lesseqgtr e$$

x	(0)		e	
y'		$+$	0	$-$
y		$\nearrow$	$\frac{1}{e}$	$\searrow$

グラフは凹凸を入れて書く。

(9) $y = \log(1+x^2) = \log u \quad u = 1+x^2.$ 　　$x \in \mathbb{R}$ で定義できる.

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}\cdot 2x = \frac{2x}{1+x^2}.$$

$\mathbb{R} := \{$実数$\}$

$y = \dfrac{x}{1+x^2}$ → あとで凹凸を入れて計算.

(1) $y = f(x) = (x^2-1)^5 = u^5 \quad (u = x^2-1)$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 5u^4\cdot 2x = 10(x^2-1)^4 x.$$

(2) $f(x) = \dfrac{x}{(1+x^2)^4}$

$$f'(x) = \frac{(x)'(1+x^2)^4 - x\{(1+x^2)^4\}'}{(1+x^2)^8}$$

$$= \frac{(1+x^2)^4 - 8x^2(1+x^2)^3}{(1+x^2)^8}$$

$$= \frac{1+x^2-8x^2}{(1+x^2)^5} = \frac{1-7x^2}{(1+x^2)^5}$$

$y = (1+x^2)^4 = u^4$

$(u = 1+x^2)$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 4u^3\cdot 2x = 4(1+x^2)^3\cdot 2x. = 8x(1+x^2)^3$$

(3) $y = \left(\dfrac{x+2}{x-1}\right)^3 = u^3 \qquad u = \dfrac{x+2}{x-1}$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 3u^2 \cdot \frac{(x+2)'(x-1) - (x+2)(x-1)'}{(x-1)^2}$$

$$= 3\left(\frac{x+2}{x-1}\right)^2 \frac{1\cdot(x-1) - (x+2)\cdot 1}{(x-1)^2}$$

$$= 3\left(\frac{x+2}{x-1}\right)^2 \cdot \frac{-3}{(x-1)^2} = -\frac{9}{(x-1)^2}\left(\frac{x+2}{x-1}\right)^2$$

(4) $f(x) = x^2 e^{-x}.$

$$f'(x) = (x^2)' e^{-x} + x^2 (e^{-x})'$$

$$\boxed{(e^{-x})' = -e^{-x}.}$$

$$= 2x e^{-x} + x^2(-e^{-x})$$

$$= e^{-x}(2x - x^2)$$

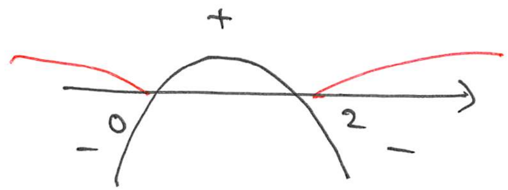
$(e^{ax})' = a e^{ax}$

$$= x(2-x)e^{-x}.$$

a : 定数.

$\vee$
0

$$f'(x) \gtreqless 0 \iff x(2-x) \gtreqless 0$$

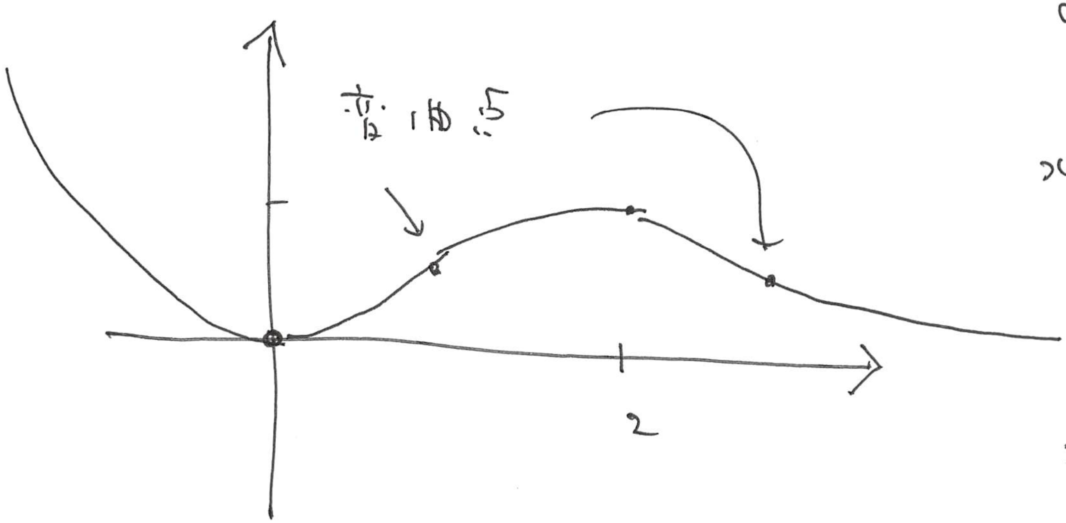


$$\iff x(x-2) \lesseqgtr 0$$

$$f(x) = x^2 e^{-x}$$

$$\iff \begin{cases} 0 < x < 2 \\ x = 0, 2 \\ \boxed{x < 0,\ x > 2} \end{cases}$$

x		0		2	
y'	$-$	0	$+$	0	$-$
y	$\searrow$	0	$\nearrow$	$\frac{4}{e^2}$	$\searrow$



$x \to +\infty$

$$x^2 e^{-x} = \frac{x^2}{e^x} \longrightarrow 0$$

$$(x \to +\infty)$$

$x \to -\infty$, $-x \to +\infty$

$x^2 \to +\infty$

$e^{-x} \to +\infty$

$\leadsto x^2 e^{-x} \to +\infty.$

(5) $y = \dfrac{\log x}{x^2}$ $\qquad f'(x) = \dfrac{(\log x)' x^2 - \log x \, (x^2)'}{x^4}$

$$= \frac{\frac{1}{x} \cdot x^2 - \log x \cdot 2x}{x^4}$$

$$= \frac{2x\left(\frac{1}{2} - \log x\right)}{x^4}.$$

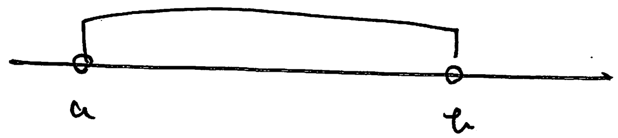
増減表を求める.

(6) $y = f(x) = \log(2x+1) = \log u. \qquad u = 2x+1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 2 = \frac{2}{2x+1}$$

凹凸

$f : (a, b) \longrightarrow \mathbb{R}$



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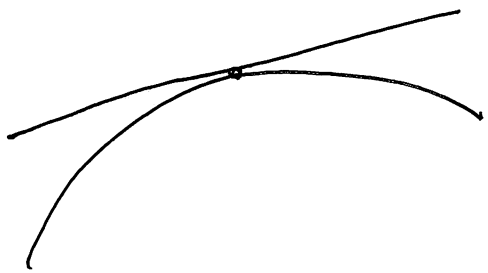

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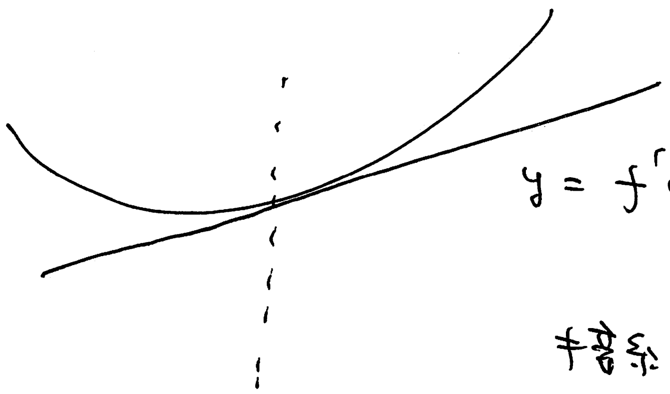


$x = c$

$$y = f'(c)(x - c) + f(c)$$

接線の方程式.

$f''(x) > 0$ とする. $(x \in (a, b))$

$F(x) = f(x) - f'(c)(x-c) - f(c)$ とおく

$F'(x) = f'(x) - f'(c)$

$F''(x) = f''(x) > 0.$ $\longrightarrow$ $F'(x)$ (a, b) 上で狭義の単調増加.

$a < x_1 < c < x_2 < b \Longrightarrow F'(x_1) < \underset{\substack{\parallel \\ 0}}{F'(c)} < F'(x_2)$

x		c	
F'	$-$	0	$+$
F	$\searrow$	0	$\nearrow$

$F(c) = 0$

$F(x) > 0 \quad (x \neq c)$

$\longrightarrow$ $f(x) > f'(c)(x-c) + f(c)$

$(x \neq c)$

定理 $f : (a, b) \to \mathbb{R}$ 2階微分可能.

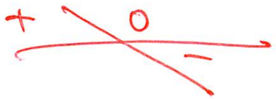
(i)
(ii)
$$f''(x) \underset{<}{>} 0 \quad (x \in (a,b)), \quad c \in (a,b)$$

$$\Longrightarrow \quad f(x) \underset{<}{>} f'(c)(x-c) + f(c) \quad (x \neq c)$$

y'	y''	
+	+	
+	−	
−	+	
−	−	

$y = e^{-\frac{1}{2}x^2}$

$y' = -x\, e^{-\frac{1}{2}x^2}$ $\quad e^{-\frac{1}{2}x^2} > 0$

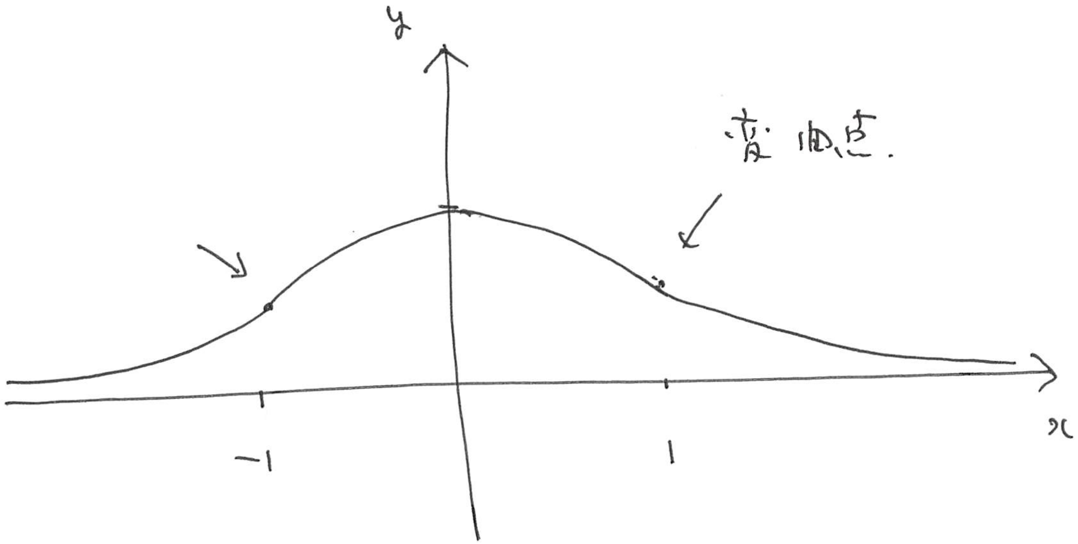


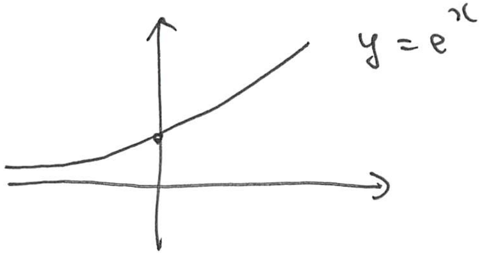
$y'' = (-x)'\, e^{-\frac{1}{2}x^2} + (-x)\left(e^{-\frac{1}{2}x^2}\right)'$

$= -e^{-\frac{1}{2}x^2} + (-x)(-x)\, e^{-\frac{1}{2}x^2}$

$= (x^2 - 1)\, e^{-\frac{1}{2}x^2}$ $\quad e^{-\frac{1}{2}x^2} > 0$

x		-1		0		1	
y'	+	+	+	0	−	−	−
y''	+	0	−	−	−	0	+
y	↗	$\frac{1}{\sqrt{e}}$	↗	1	↘	$\frac{1}{\sqrt{e}}$	↘





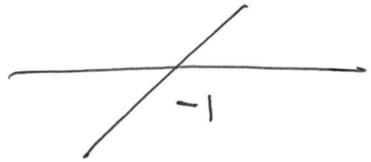
$x \to +\infty$ のとき
$-\infty$

$-\frac{1}{2}x^2 \to -\infty$

$e^{-\frac{1}{2}x^2} \to 0$

$y = x e^x$

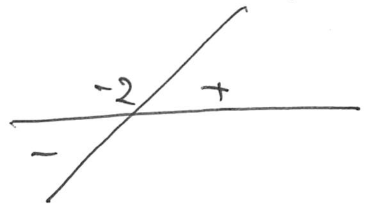
$y' = (x+1) e^x$



$$
\begin{aligned}
y'' &= (x+1)' e^x + (x+1)(e^x)' \\
&= 1 \cdot e^x + (x+1) e^x \\
&= (x+2)\, e^x
\end{aligned}
$$

$e^x > 0$

$y = x + 2$



x		-2		-1	
y'	$-$	$-$	$-$	0	$+$
y''	$-$	0	$+$	$+$	$+$
y	$\searrow$	$-\frac{2}{e^2}$	$\searrow$	$-\frac{1}{e}$	$\nearrow$

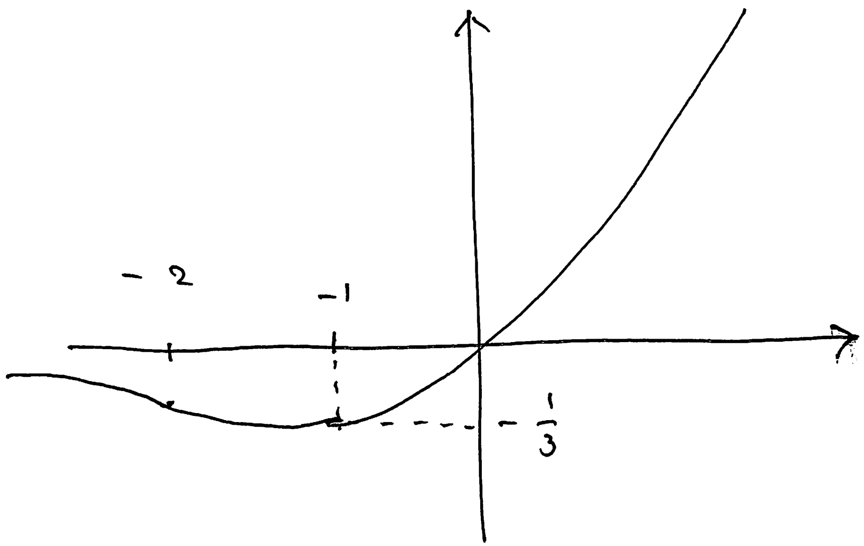
$x \to +\infty$ alors $e^x \to +\infty$ $\quad x e^x \to +\infty$

$x \to -\infty$ alors $t = -x \to +\infty$

$$x e^x = -t e^{-t} = -\frac{t}{e^t}$$

$$\to -0 = 0$$

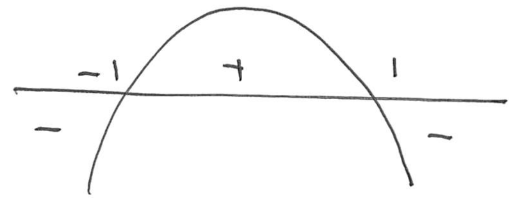
$$\boxed{\frac{t}{e^t} \to 0 \quad (t \to +\infty)}$$



$$y = \frac{x}{1+x^2} \qquad y' = \frac{(x)'(1+x^2) - x(1+x^2)'}{(1+x^2)^2}$$

$$= \frac{1\cdot(1+x^2) - x\cdot 2x}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$



$$y'' = \frac{(1-x^2)'(1+x^2)^2 - (1-x^2)\{(1+x^2)^2\}'}{(1+x^2)^4}$$

$$u = 1+x^2.$$

$$\frac{d(u^2)}{dx} = \frac{d(u^2)}{du}\cdot\frac{du}{dx}$$

$$= 2u\cdot 2x$$

$$= 4x(1+x^2)$$

$$= \frac{-2x(1+x^2)^2 - (1-x^2)\cdot 4x(1+x^2)}{(1+x^2)^4}$$

$$= \frac{-2x(1+x^2) - 4x(1-x^2)}{(1+x^2)^3}$$

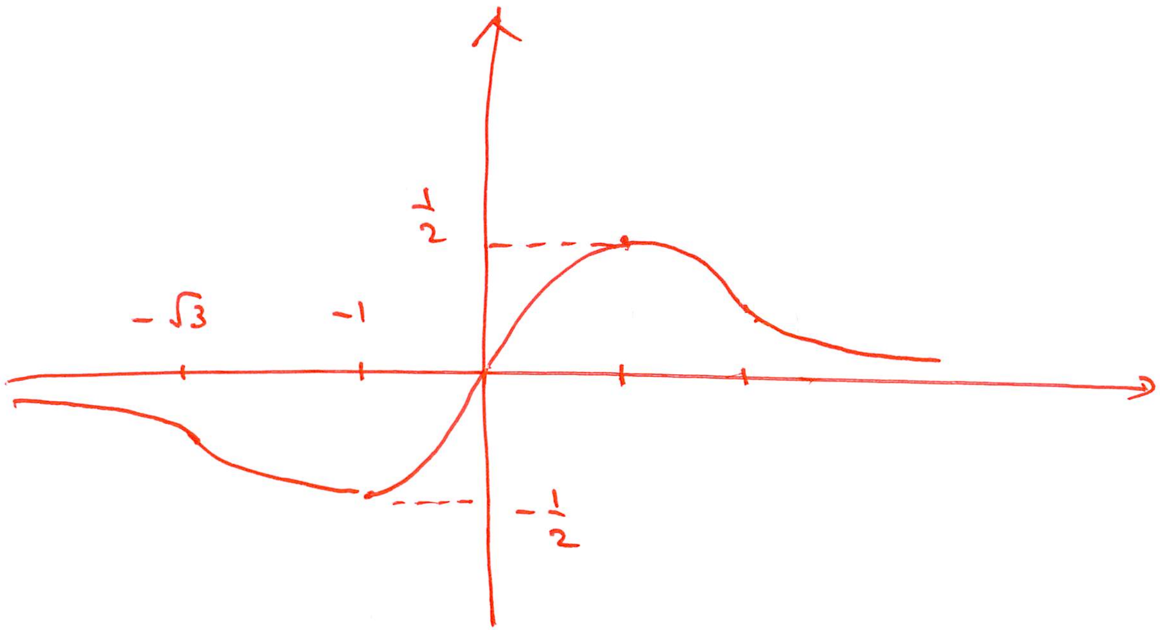
$$= \frac{-2x\{(1+x^2) + 2(1-x^2)\}}{(1+x^2)^3} = \frac{-2x(3-x^2)}{(1+x^2)^3}$$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
$3-x^2$	$-$	0	$+$	$+$	$+$	0	$-$
$-x$	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$-$	0	$+$	0	$-$	0	$+$

演習 続いて 増減表を求めよ.

x		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
y'	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
y	↘	$-\frac{\sqrt{3}}{4}$	↘	$-\frac{1}{2}$	↗	0	↗	$\frac{1}{2}$	↘	$\frac{\sqrt{3}}{4}$	↘

$x \to \pm\infty$ のとき $\quad \dfrac{x}{1+x^2} = \dfrac{\frac{1}{x}}{\left(\frac{1}{x}\right)^2+1} \longrightarrow \dfrac{0}{0+1} = 0$



1/2
-√3
-1
-1/2

演習

次の $y = f(x)$ の増減表を求めよ.

(1) $y = x^2 \log x.$ (2) $y = x \log x.$ (3) $y = \dfrac{\log x}{x}.$

(4) $y = x^2 e^x$ (5) $y = x e^{-x}.$

コーシーの平均値の定理.

$$f : [a, b] \longrightarrow \mathbb{R}$$
$$g : [a, b] \longrightarrow \mathbb{R}$$

$[a, b]$ の各点で連続
(a, b) ―― で微分可能

$f'(x)$, $g'(x)$ は同時に 0 でない.
$g(b) - g(a) \neq 0$.

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

をみたす $c \in (a, b)$ は存在.

$A = f(b) - f(a)$, $B = g(b) - g(a)$

$\varphi(x) = B(f(x) - f(a)) - A(g(x) - g(a))$ とおく

$$\varphi(b) = B(\underbrace{f(b) - f(a)}_{\shortparallel \atop A}) - A(\underbrace{g(b) - g(a)}_{B}) = AB - AB = 0$$

$\shortparallel$

$\varphi(a) = 0$

φに対して Rolle の定理が使える.

$$\varphi'(c) = 0$$

を満たす $c \in (a, b)$ が存在.

$$\varphi'(x) = B f'(x) - A g'(x)$$

$$= B f'(c) - A g'(c) = 0$$

$$\overset{\neq 0}{(g(b) - g(a))} f'(c) = (f(b) - f(a)) \overset{\neq 0}{g'(c)}$$

$g'(c) = 0 \longrightarrow f'(c) = 0 \longrightarrow$ 仮定に反す. $g'(c) \neq 0$

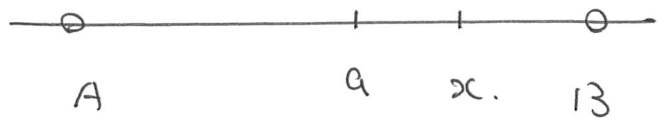
$\uparrow$

$g(b) - g(c) \neq 0$

$$\frac{f(b) - f(a)}{g(b) - g(c)} = \frac{f'(c)}{g'(c)}$$

Taylorの定理
(2階)

$f : (A, B) \longrightarrow \mathbb{R}$



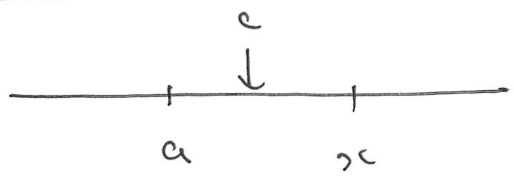
2階微分可能.

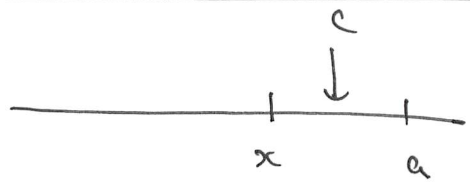
$x \neq a$.

$$f(x) = f(a) + f'(a)(x-a) + \boxed{\frac{1}{2} f''(c)(x-a)^2}$$

剰余項.

を満たす c が a と x の間に存在する.



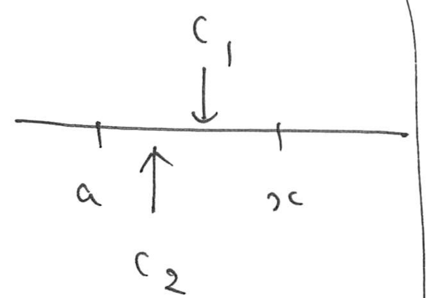


$$F(x) = f(x) - f'(a)(x-a) - f(a) \qquad F(a) = 0$$

$$G(x) = (x-a)^2 \qquad G(a) = 0$$

$$\frac{F(x)}{G(x)} = \frac{F(x) - \overbrace{F(a)}^{0}}{G(x) - \underbrace{G(a)}_{0}} = \frac{F'(c_1)}{G'(c_1)}$$

を満たす c_1 が a と x の間に存在

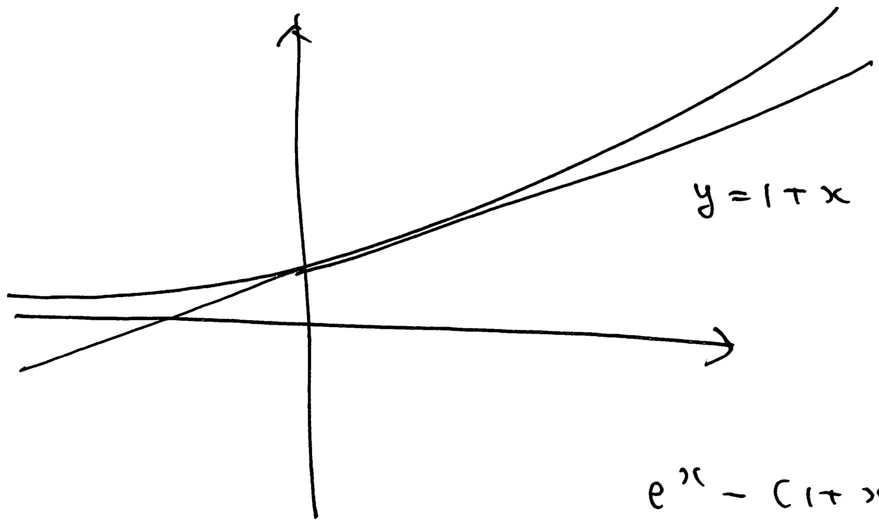


$$F'(x) = f'(x) - f'(a) \qquad F'(a) = 0$$

$$G'(x) = 2(x-a) \qquad G'(a) = 0$$

$$= \frac{F'(c_1) - F'(a)}{G'(c_1) - G'(a)} = \frac{F''(c_2)}{G''(c_2)} = \frac{f''(c_2)}{2}$$

を満たす c_2 が a と c_1 の間に存在



$f''(x) = e^x > 0$

$y = f(x)$ は $\mathbb{R}$ 上

下に凸

$$e^x - (1+x) = \frac{e^c}{2}x^2 > 0 \quad (x \neq 0)$$

② $f(x) = \log(1+x)$ $\qquad f'(x) = \dfrac{1}{1+x}$

$a = 0$

$u = 1 + x$

$$\frac{d(\log u)}{dx} = \frac{d(\log u)}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 1 = \frac{1}{1+x}$$

$$f''(x) = -\frac{1}{(1+x)^2}$$

$$f^{(3)}(x) = \frac{2}{(1+x)^3}$$

$$\frac{d\left(\frac{1}{u}\right)}{dx} = \frac{d\left(\frac{1}{u}\right)}{du} \cdot \frac{du}{dx} = -\frac{1}{u^2}$$

演習 $f^{(n)}(x) = ?$

$u = 1+x$.

---

③ $f(x) = \sqrt{1+x}$ $\quad f'(x) = \dfrac{1}{2\sqrt{1+x}}$

$a = 0$

$$\frac{d\sqrt{u}}{dx} = \frac{d\sqrt{u}}{du}\cdot\frac{du}{dx}.$$

$$= \frac{1}{2}\frac{1}{\sqrt{u}}\cdot 1$$

$$= \frac{1}{2\sqrt{1+x}}.$$

$$f''(x) = \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{(1+x)^{\frac{3}{2}}}$$

$f(0) = 1$

$f'(0) = \dfrac{1}{2}$

$$f^{(3)}(x) = \left(\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{(1+x)^{\frac{5}{2}}}$$

演習 $f^{(4)}, f^{(5)}$ を計算.

$$\left((1+x)^{\alpha}\right)' = \alpha(1+x)^{\alpha-1}$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(c)(x-a)^2$$

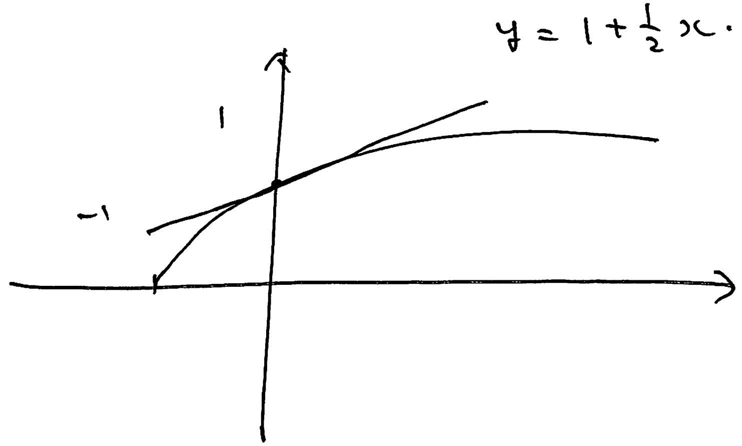
を満たす c が a と x の間に存在

$$\sqrt{1+x} = 1 + \frac{1}{2}x + \frac{1}{2}\left(-\frac{1}{4}\right)\frac{1}{(1+c)^{\frac{3}{2}}}\,x^2$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}\cdot\frac{1}{(1+c)^{\frac{3}{2}}}\,x^2$$

$$\sqrt{1+x} \fallingdotseq 1 + \frac{1}{2}x.$$

(1次近似)



$$F''(x) = f''(x)$$

$$G''(x) = 2$$

$$\frac{f(x) - f'(a)(x-a) - f(a)}{(x-a)^2} = \frac{f''(c_2)}{2}$$

$$\rightarrow \quad f(x) = f(a) + f'(a)(x-a) + \frac{f''(c_2)}{2}(x-a)^2$$

ここで c_2 は a と x の間にある。

問5.3.4 ① e^x $a=0$

$$f(x) = e^x, \quad f'(x) = e^x, \quad f''(x) = e^x$$

$$\boxed{f^{(3)}(x) = e^x, \cdots, f^{(n)}(x) = e^x.}$$

$$f(0) = 1, \quad f'(0) = 1$$

$$e^x = 1 + x + \frac{e^c}{2}x^2$$ をみたす c が

0 と x の間に存在する。

定理

$$f : (a, b) \longrightarrow \mathbb{R}$$

$$f''(x) > 0 \qquad (x \in (a, b))$$

$$\Longrightarrow \quad f(x) > f(a) + f'(a)(x-a) \qquad (x \neq a)$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!} \underset{\substack{\vee \\ 0}}{f''(c)} \underset{\substack{\vee \\ 0}}{(x-a)^2}$$

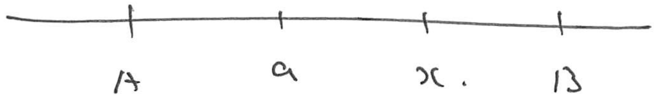
$$> f(a) + f'(a)(x-a)$$

3階のTaylorの定理

$f : (A, B) \longrightarrow \mathbb{R}$

3階微分可能

$x \neq a$, $x, a \in (A, B)$



$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!} f''(a)(x-a)^2$$

$$+ \frac{1}{3!} f^{(3)}(c)(x-a)^3$$

を満たす c が a と x の間に存在.

$$F(x) = f(x) - f(a) - f'(a)(x-a) - \frac{1}{2!} f''(a)(x-a)^2 \qquad F(a) = 0$$

$$G(x) = (x-a)^3 \qquad G(a) = 0$$

$$\frac{F(x)}{G(x)} = \frac{F(x) - \overbrace{F(a)}^{0}}{G(x) - \underbrace{G(a)}_{0}} \overset{\text{コーシー}}{=} \frac{F'(c_1)}{G'(c_1)} = \cdots$$

自習 続きを示す.

3回コーシーの定理を使う.

$\frac{(3)}{2}$ 1手 くるり

① e^x　$a = 0$.

$$f(x) = e^x,\quad f'(x) = f''(x) = f^{(3)}(x) = e^x$$

$$f(0) = f'(0) = f''(0) = 1$$

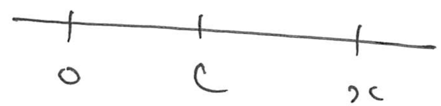
$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}e^c x^3$$

を満たす c が 0 と x の間に存在.

$$\boxed{x > 0 \Rightarrow e^x > 1 + x + \frac{1}{2}x^2}$$

$x > 0$ とする.

$$e^x = \underbrace{\boxed{1 + x}}_{\substack{\cup \\ 0}} + \frac{1}{2}x^2 + \underbrace{\boxed{\frac{1}{3!}e^c x^3}}_{\substack{\cup \\ 0}}$$



$$> \frac{1}{2}x^2$$

$$\underset{\downarrow \atop 0}{0} < \underset{\searrow \atop 0}{\frac{x}{e^x}} < \frac{x}{\frac{1}{2}x^2} = 2 \cdot \underset{\downarrow \atop 0}{\frac{1}{x}}$$

by はさみうち.

$$\boxed{\frac{x}{e^x} \to 0 \quad (x \to +\infty)}$$

(2) $f(x) = \log(1+x)$

$f'(x) = \dfrac{1}{1+x}$

$f''(x) = -\dfrac{1}{(1+x)^2}$

$f^{(3)}(x) = \dfrac{2}{(1+x)^3}$

$f(0) = \log 1 = 0$

$f'(0) = 1$

$f''(0) = -1$

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2} f''(a)(x-a)^2 + \frac{1}{3!} f^{(3)}(c)(x-a)^3$$

$$\log(1+x) = 0 + 1 \cdot x + \frac{1}{2}(-1)x^2 + \frac{1}{3!} \cdot \frac{2}{(1+c)^3} x^3$$

$$= x - \frac{1}{2}x^2 + \frac{1}{3!} \cdot \frac{2}{(1+c)^3} x^3$$

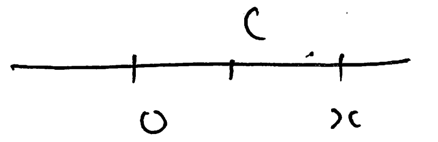
演習

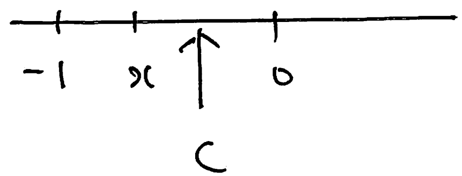
$$\log(1+x) \lessgtr x - \frac{1}{2}x^2$$

$\uparrow$

$x > 0$ の場合

$x < 0$. の場合.

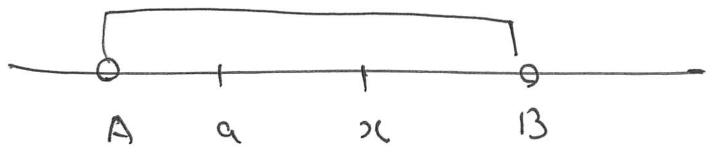




演習 $f(x) = \sqrt{1+x}$, $a = 0$ の場合の3階のTaylorの定理.

n階のTaylor　　$f: (A, B) \longrightarrow \mathbb{R}$

n階微分可能.



$x \neq a$,

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \frac{1}{3!}f^{(3)}(a)(x-a)^3$$

$$+ \cdots + \frac{1}{k!}f^{(k)}(a)(x-a)^k + \cdots + \frac{1}{(n-1)!}f^{(n-1)}(a)(x-a)^{n-1}$$

$$+ \frac{1}{n!}f^{(n)}(c)(x-a)^n$$

を満たす c が a と x の間に存在.

ここだけ

$f(x) = e^x$, $a = 0$ $\qquad f^{(k)}(0) = 1$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \cdots + \frac{1}{k!}x^k + \cdots + \frac{1}{(n-1)!}x^{n-1}$$

$$+ \frac{1}{n!}e^c x^n$$

を満たす c が 0 と x の間に存在.

$n \to +\infty$.

演習

$n = 4$, $f(x) = \log(1+x)$, $a = 0$

$n = 5$ $f(x) = \log(1+x)$, $a = 0$

I　$y = f(x) = x e^{2x}$　の増減表　y''を含めて

$(e^{ax})' = a e^{ax}$.