2016/08/07
3rd Rec

$f'(a) = ?$ $f'(a)$ を求めよ.

(i) $f(x) = \dfrac{1}{x-1}$, $a \neq 1$

$$\frac{\dfrac{1}{x-1} - \dfrac{1}{a-1}}{x-a} = \frac{(a-1)-(x-1)}{(a-1)(x-1)} \cdot \frac{1}{(x-a)}$$

$$= \frac{a-x}{(a-1)(x-1)(x-a)} = -\frac{1}{a-1} \cdot \frac{1}{x-1}$$

$x \to a$ のとき $x-1 \to a-1 \neq 0$

$$\longrightarrow -\frac{1}{a-1} \cdot \frac{1}{a-1}$$

$$= -\frac{1}{(a-1)^2}$$

$$\left(\frac{1}{x-1}\right)' = -\frac{(x-1)'}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

(ii) $f(x) = Ax^2 + Bx + C$ 　　A, B, C 定数.

$$\frac{f(x) - f(a)}{x-a} = \frac{(Ax^2 + Bx + C) - (Aa^2 + Ba + C)}{x-a}$$

$$= A\frac{x^2 - a^2}{x-a} + B\frac{x-a}{x-a}$$

$$\boxed{\begin{aligned} & x^2 - a^2 \\ & = (x+a)(x-a) \end{aligned}}$$

$$= A(x+a) + B$$

$$\longrightarrow A \cdot (a+a) + B = 2Aa + B$$

$$(Ax^2 + Bx + C)' = A \cdot 2x + B \cdot 1 + 0$$

$$= 2Ax + B.$$

$$\boxed{\begin{aligned} & C \text{ 定数} \quad (C)' = 0 \\ & (c f(x))' = c f'(x) \end{aligned}}$$

(iii) $f(x) = \dfrac{1}{x^2 + x + 1}$

$$\frac{\dfrac{1}{x^2+x+1} - \dfrac{1}{a^2+a+1}}{x-a} = \frac{(a^2+a+1)-(x^2+x+1)}{(x^2+x+1)(a^2+a+1)} \cdot \frac{1}{x-a}$$

$$= \frac{(a^2-x^2)+(a-x)}{(x^2+x+1)(a^2+a+1)} \cdot \frac{1}{x-a}$$

$$= \frac{1}{a^2+a+1} \cdot \frac{1}{x^2+x+1} \cdot \left\{-(a+x)-1\right\}$$

$$\longrightarrow \frac{1}{a^2+a+1} \cdot \frac{1}{a^2+a+1} \left\{-(a+a)-1\right\} = -\frac{2a+1}{(a^2+a+1)^2}$$

$$\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$$

$$\left(\frac{1}{x^2+x+1}\right)' = -\frac{(x^2+x+1)'}{(x^2+x+1)^2} = -\frac{2x+1}{(x^2+x+1)^2}$$

(iv) $f(x) = \dfrac{1}{(x^2+1)^2}$

$A^2 - B^2 = (A-B)(A+B)$

$$\frac{\dfrac{1}{(x^2+1)^2} - \dfrac{1}{(a^2+1)^2}}{x-a} = \frac{(a^2+1)^2 - (x^2+1)^2}{(x^2+1)^2(a^2+1)^2} \cdot \frac{1}{x-a}$$

$$= \frac{(a^2+1+x^2+1)\boxed{\{(a^2+1)-(x^2+1)\}}}{(x^2+1)^2(a^2+1)^2} \cdot \frac{1}{x-a}$$

$= a^2 - x^2$

$= (a-x)(a+x)$

$$= \frac{(a^2+1+x^2+1)}{(x^2+1)^2(a^2+1)^2}(-a-x)$$

$2(a^2+1)$

$$\longrightarrow \frac{a^2+1+a^2+1}{(a^2+1)^2(a^2+1)^2} \cdot (-a-a)$$

$$= \frac{2}{(a^2+1)^3}(-2a)$$

$$= -\frac{4a}{(a^2+1)^3}$$

86p 演習 3.3

$$(\sqrt{x})' = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$$

$$\boxed{(fg)' = f'g + fg'}$$

(1) $$(x^2\sqrt{x})' = (x^2)'\sqrt{x} + x^2(\sqrt{x})'$$

$$= 2x\sqrt{x} + x^2\cdot\frac{1}{2}\frac{1}{\sqrt{x}}$$

$$= 2x\sqrt{x} + \frac{1}{2}x\sqrt{x} = \frac{5}{2}x\sqrt{x}$$

$$\left(x^{\frac{5}{2}}\right)' = \frac{5}{2}x^{\frac{3}{2}} = \frac{5}{2}x^{\frac{5}{2}-1}$$

$$\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$$

$$\boxed{(x\sqrt{x})' = \frac{3}{2}\sqrt{x}}$$

(2) $$\left(\frac{1}{x\sqrt{x}}\right)' = -\frac{(x\sqrt{x})'}{(x\sqrt{x})^2} = -\frac{\frac{3}{2}\sqrt{x}}{x^3}$$

$$= -\frac{3}{2}\cdot\frac{1}{x^2\sqrt{x}}$$

$$\left(x^{-\frac{3}{2}}\right)' = -\frac{3}{2}x^{-\frac{5}{2}} = -\frac{3}{2}x^{-\frac{3}{2}-1}$$

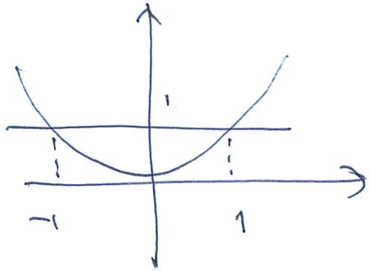
$$\boxed{\begin{array}{c}(x^\alpha)' = \alpha x^{\alpha-1}\\ x>0\\ \alpha\in\mathbb{R}\end{array}}$$

(3) $y = \dfrac{1}{1+x^2}$ $\quad y' = -\dfrac{(1+x^2)'}{(1+x^2)^2} = -\dfrac{2x}{(1+x^2)^2}$

(4) $y = \dfrac{x}{1+x^2}$ $\quad y' = \dfrac{(x)'(1+x^2) - x(1+x^2)'}{(1+x^2)^2}$ $\quad \left(\dfrac{g}{f}\right)' = \dfrac{g'f - gf'}{f^2}$

$$= \frac{1 \cdot (1+x^2) - x \cdot 2x}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2}$$

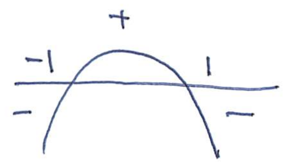


y' の符号.

$(1+x^2)^2 > 0.$

$$y' \gtreqless 0 \iff 1-x^2 \gtreqless 0 \iff x^2 \lesseqgtr 1$$

$$\iff \begin{cases} -1 < x < 1 \\ x = \pm 1 \\ x < -1 \text{ OR } x > 1 \end{cases}$$



(6) $y = \dfrac{x-1}{x+1}$

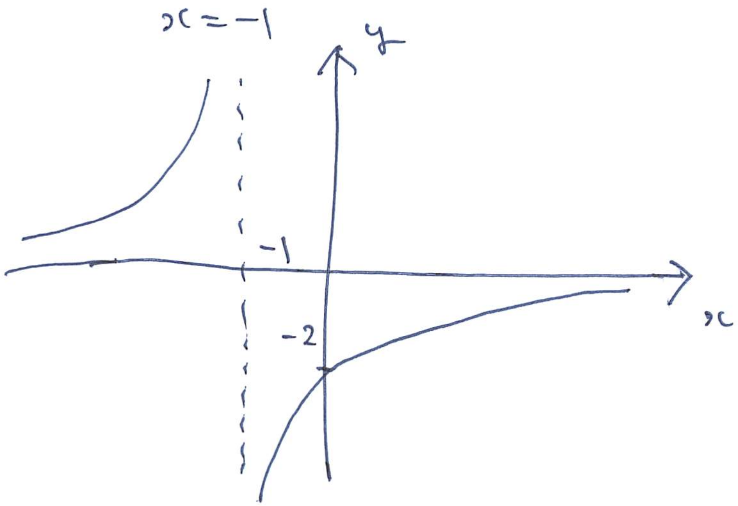
$$y' = \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2}$$

$$\boxed{\left(\frac{g}{f}\right)' = \frac{g'f - gf'}{f^2}}$$

$$= \frac{1\cdot(x+1) - (x-1)\cdot 1}{(x+1)^2}$$

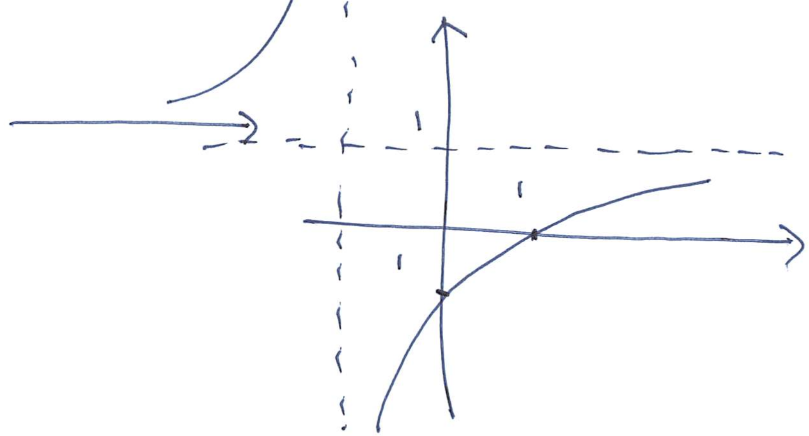
$$= \frac{2}{(x+1)^2} > 0$$

$$y = \frac{(x+1) - 2}{x+1} = 1 - \frac{2}{x+1}$$



$y = -\dfrac{2}{x+1}$

$y = 1 - \dfrac{2}{x+1}$



(1)

$$\begin{cases} a_{n+2} - \overset{1}{a_{n+1}} - 2a_n = 0 \\ a_0 = c_0,\ a_1 = c_1 \end{cases}$$

特性方程式 $\lambda^2 - \lambda - 2\lambda = 0 \iff (\lambda-2)(\lambda+1) = 0$

$\iff \lambda = -1, 2.$

$1 = 2 + (-1), \quad -2 = (-1)\cdot 2$

$$\rightarrow \begin{cases} a_{n+2} - 2a_{n+1} = (-1)(a_{n+1} - 2a_n) \\ a_{n+2} + a_{n+1} = 2(a_{n+1} + a_n) \end{cases}$$

$\{a_{n+1} - 2a_n\}$ 公比 (-1) の等比数列.

$\{a_{n+1} + a_n\}$ は公比 2 の等比数列.

$$\rightarrow \begin{cases} a_{n+1} - 2a_n = (-1)^n (a_1 - 2a_0) \\ a_{n+1} + a_n = 2^n (a_1 + a_0) \end{cases}$$

$-)$

$$-3a_n = (-1)^n (a_1 - 2a_0) - 2^n (a_1 + a_0)$$

$$a_n = \frac{1}{3} 2^n (a_1 + a_0) - \frac{1}{3} (-1)^n (a_1 - 2a_0)$$

$a_{n+2} + p a_{n+1} + q a_n = 0 \quad \longleftrightarrow \quad \lambda^2 + p\lambda + q = 0$ 特性方程式.

$\downarrow$

$$\| \\ (\lambda-\alpha)(\lambda-\beta) = \lambda^2 - (\alpha+\beta)\lambda + \alpha\beta.$$

$a_{n+2} - (\alpha+\beta) a_{n+1} + \alpha\beta a_n = 0 \qquad \alpha+\beta = -p, \ \alpha\beta = q.$

$$\begin{cases} a_{n+2} - \alpha a_{n+1} = \beta (a_{n+1} - \alpha a_n) \\ a_{n+2} - \beta a_{n+1} = \alpha (a_{n+1} - \beta a_n) \end{cases}$$

14p. に解が有る.

$\alpha \neq \beta$ とする. $a_n = C\alpha^n + C'\beta^n$ と書ける

15p の上.

C, C' は n に依らない.

· 연습 문제 1.12 31p.

(1)

$$\sum_{n=0}^{+\infty}\left(\frac{1}{3}\right)^n \qquad S_n = \sum_{k=0}^{n}\left(\frac{1}{3}\right)^k = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}$$

$$-\Big)\ \frac{1}{3}S_n = \qquad = \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n} + \frac{1}{3^{n+1}}$$

$$\frac{2}{3}S_n = \qquad = 1 - \left(\frac{1}{3}\right)^{n+1}$$

$$S_n = \frac{3}{2}\cdot\left(1 - \left(\frac{1}{3}\right)^{n+1}\right) \longrightarrow \frac{3}{2}\left(1 - \frac{1}{3}\cdot 0\right) = \frac{3}{2}$$

$$\boxed{|r| < 1 \text{ 이면 } \quad r^n \to 0 \quad (n \to +\infty)}$$

$$\sum_{n=0}^{+\infty} r^n = \frac{1}{1-r}$$

(3) $\displaystyle\sum_{n=0}^{+\infty} n\left(\frac{2}{3}\right)^{n}$

$\displaystyle = \frac{2}{3}\cdot\sum_{n=1}^{+\infty} n\left(\frac{2}{3}\right)^{n-1}$

$\displaystyle\sum_{n=1}^{+\infty} n r^{n-1} = \frac{1}{(1-r)^2} = \frac{2}{3}\cdot 9 = 6$

第n部分和.

$$S_n = \sum_{k=0}^{n} k\left(\frac{2}{3}\right)^{k} = 1\cdot\frac{2}{3} + 2\cdot\left(\frac{2}{3}\right)^2 + \cdots + n\left(\frac{2}{3}\right)^n$$

$$\frac{2}{3}S_n = 1\cdot\left(\frac{2}{3}\right)^2 + \cdots + (n-1)\left(\frac{2}{3}\right)^n + n\left(\frac{2}{3}\right)^{n+1}$$

$$\frac{1}{3}S_n = \frac{2}{3} + \cdots + \left(\frac{2}{3}\right)^n - n\left(\frac{2}{3}\right)^{n+1}$$

$|r|<1$

$nr^n \to 0$
$(n\to+\infty)$

$$S_n = 3\left(\frac{2}{3} + \cdots + \left(\frac{2}{3}\right)^n\right) - 3n\left(\frac{2}{3}\right)^{n+1}$$

$$= 3\cdot\frac{\frac{2}{3}-\left(\frac{2}{3}\right)^{n+1}}{1-\frac{2}{3}} - 3n\left(\frac{2}{3}\right)^{n+1}$$

$$= 9\cdot\left(\frac{2}{3}-\left(\frac{2}{3}\right)^{n+1}\right) - 3n\left(\frac{2}{3}\right)^{n+1}$$

$$T = \frac{2}{3} + \cdots + \left(\frac{2}{3}\right)^n$$

$$\frac{2}{3}T = \left(\frac{2}{3}\right)^2 + \cdots + \left(\frac{2}{3}\right)^n + \left(\frac{2}{3}\right)^{n+1}$$

$$\to 9\cdot\left(\frac{2}{3}-0\right) - 3\cdot 0$$

$$= 6.$$

III $x \neq -1$, $a_n = \dfrac{x^n}{1+x^n}$

$x = 1$ のとき $a_n = \dfrac{1}{1+1} = \dfrac{1}{2} \longrightarrow \dfrac{1}{2} \quad (n \to +\infty)$

$|x| < 1$ のとき $x^n \longrightarrow 0 \quad (n \to +\infty)$

$$a_n \longrightarrow \frac{0}{1+0} = 0$$

$|x| > 1$ のとき $a_n = \dfrac{1}{\left(\frac{1}{x}\right)^n + 1}$ $\left|\frac{1}{x}\right| < 1$ だから $\left(\frac{1}{x}\right)^n \to 0 \ (n \to +\infty)$

$$a_n \longrightarrow \frac{1}{0+1} = 1$$

IV

(1) $$\left(\frac{x}{1-x}\right)' = \frac{(x)'(1-x) - x(1-x)'}{(1-x)^2} = \frac{1\cdot(1-x) - x(-1)}{(1-x)^2}$$

$$= \frac{1}{(1-x)^2}$$

$y = \dfrac{x}{1-x}$ のグラフは?

(2)

$$(x^3\sqrt{x})' = (x^3)'\sqrt{x} + x^3(\sqrt{x})'$$

$$=$$

$$= 3x^2\cdot\sqrt{x} + x^3\cdot\frac{1}{2}\frac{1}{\sqrt{x}}$$

$$= 3x^2\sqrt{x} + \frac{1}{2}x^2\sqrt{x} = \frac{7}{2}x^2\sqrt{x}$$

$$\left(x^{\frac{7}{2}}\right)' = \frac{7}{2}x^{\frac{5}{2}} = \frac{7}{2}x^{\frac{7}{2}-1}$$

合成関数の微分.

微分の小手調.

$y=\sqrt{x+1}$ の $x=a$ における微分.

$x_n > -1$

$x_n \neq a$

$x_n \to a.\ (n\to+\infty)$

$y_n = x_n+1$ とおく.

$b = a+1$

$$\frac{\sqrt{x_n+1}-\sqrt{a+1}}{x_n-a} = \frac{\sqrt{y_n}-\sqrt{b}}{y_n-b}\cdot\frac{(x_n+1)-(a+1)}{x_n-a}$$

x_n+1
$\shortparallel$
$y_n \longrightarrow a+1=b.$

$$\longrightarrow g'(b)\cdot 1 = \frac{1}{2}\frac{1}{\sqrt{a+1}}$$

$g(t)=\sqrt{t}$

$g'(t)=\frac{1}{2}\frac{1}{\sqrt{t}}$

$$\boxed{(\sqrt{x+1})' = \frac{1}{2}\frac{1}{\sqrt{x+1}}}$$

$$\frac{\sqrt{x+1}-\sqrt{a+1}}{x-a} = \frac{(x+1)-(a+1)}{\left(\sqrt{x+1}+\sqrt{a+1}\right)(x-a)}$$

$$= \frac{1}{\sqrt{x+1}+\sqrt{a+1}} \longrightarrow \frac{1}{\sqrt{a+1}+\sqrt{a+1}} = \frac{1}{2\sqrt{a+1}}$$

$x \to a$ のとき $x+1 \to a+1$ のとき $\sqrt{x+1} \to \sqrt{a+1}$

$$\boxed{\sqrt{x} \to \sqrt{b} \quad (x \to b)}$$

88p

$z = \dfrac{1}{(2x-1)^3}$

$\left.\begin{array}{l} x_n \to a. \\ x_n \neq \frac{1}{2} \end{array}\right\}$ を満たすとする.

$$\frac{\dfrac{1}{(2x_n-1)^3} - \dfrac{1}{(2a-1)^3}}{x_n - a.}$$

$y_n = 2x_n - 1.$

$\longrightarrow 2a-1 = A$ とおく.

$$= \boxed{\frac{\dfrac{1}{y_n^3} - \dfrac{1}{A^3}}{y_n - A}} \cdot \overbrace{\frac{(2x_n-1)-(2a-1)}{x_n - a_n}}^{=\,2}$$

$g(y) = \dfrac{1}{y^3}$ $\longrightarrow g'(A)\cdot 2 = -\dfrac{3}{A^4}\cdot 2 = -\dfrac{6}{(2a-1)^4}$

$$= \frac{g(y_n) - g(A)}{y_n - A.}$$

$\left(\dfrac{1}{y^3}\right)' = -\dfrac{3}{y^4}$

$$\left(\frac{1}{(2x-1)^3}\right)' = -\frac{3}{(2x-1)^4}\cdot 2$$

$$z = \frac{1}{(2x-1)^3} = \frac{1}{y^3} \qquad y = 2x-1$$

$$\frac{dz}{dx} = \frac{d\left(\frac{1}{y^3}\right)}{dy} \cdot \frac{dy}{dx}.$$

$$= \frac{dz}{dy} \cdot \frac{dy}{dx}.$$

$$y = \frac{1}{(1+x^2)^2} = \frac{1}{u^2}$$

$u = 1 + x^2$.

$x_n \longrightarrow a$.

$u_n = 1 + x_n^2 \longrightarrow 1 + a^2 = A$ とおく.

$$\frac{\dfrac{1}{(1+x_n^2)^2} - \dfrac{1}{(1+a^2)^2}}{x_n - a}$$

$$= \frac{\dfrac{1}{u_n^2} - \dfrac{1}{A^2}}{u_n - A} \cdot \frac{(1+x_n^2) - (1+a^2)}{x_n - a}$$

$h(x) = 1 + x^2$

$g(u) = \frac{1}{u^2}$ $\longrightarrow$ $g'(A) = -\frac{2}{A^3}$

$g'(u) = -\frac{2}{u^3}$

$h'(a) = 2a$.

$$\longrightarrow \quad g'(A) \cdot h'(a) = -\frac{2}{(1+a^2)^3} \cdot 2a$$

$$= -\frac{4a}{(1+a^2)^3}$$

$$y = \frac{1}{(1+x^2)^2} = \frac{1}{u^2} \qquad u = 1+x^2$$

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$ 合成関数の微分.

$$y = f(u) = \frac{1}{u^2}, \qquad u = g(x) = 1+x^2$$

$$= f(g(x)) = \frac{1}{(1+x^2)^2}$$

$$y = \frac{1}{t^2+1} = \frac{1}{u} \qquad u = t^2+1 \qquad \boxed{\left(\frac{1}{x^n}\right)' = -\frac{n}{x^{n+1}}}$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt} = -\frac{1}{u^2}\cdot 2t = -\frac{2t}{(1+t^2)^2}$$

$$y = \frac{1}{(1+t+t^2)^{10}} = \frac{1}{u^{10}} \qquad (u = 1+t+t^2)$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt} = -\frac{10}{u^{11}}\cdot(1+2t) = -\frac{10(1+2t)}{(1+t+t^2)^{11}}$$

$$y = \sqrt{1+t^2} = \sqrt{u} \qquad u = 1+t^2$$

$$\frac{dy}{dt} = \frac{dy}{du}\cdot\frac{du}{dt} = \frac{1}{2}\cdot\frac{1}{\sqrt{u}}\cdot 2t = \frac{t}{\sqrt{1+t^2}}$$

$y = f(u)$, $u = g(x)$ $\quad x = a$. $\quad A = g(a)$ とおく.

$y = f(g(x))$

$x_n \to a$; $\quad u_n = g(x_n) \longrightarrow g(a) = A$.

微分できたら連続

$$\frac{f(g(x_n)) - f(g(a))}{x_n - a} = \frac{f(u_n) - f(A)}{u_n - A} \cdot \frac{g(x_n) - g(a)}{x_n - a}$$

$g'(a)$ が存在するならば

$g(x) \to g(a) \quad (x \to a)$

$$\longrightarrow f'(A) \cdot g'(a)$$

$$= f'(g(a)) \cdot g'(a)$$

$$\boxed{\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$y = \frac{1}{\sqrt{1+t^2}} = \frac{1}{\sqrt{u}}, \quad u = 1+t^2$$

$$\left(\frac{1}{\sqrt{u}}\right)' = -\frac{(\sqrt{u})'}{(\sqrt{u})^2} = -\frac{\frac{1}{2}\frac{1}{\sqrt{u}}}{u} = -\frac{1}{2 \cdot u\sqrt{u}}$$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = -\frac{1}{2u\sqrt{u}} \cdot 2t$$

$$= -\frac{t}{u\sqrt{u}} = -\frac{t}{(1+t^2)\sqrt{1+t^2}}$$

演習 $f'(x)$ を求める

(1) $\dfrac{x}{x^2+x+1}$ (2) $\dfrac{1}{(3x+1)^3}$ (3) $(1-2x)^5$

(4) $\dfrac{1}{(3x-2)^5}$ (5) $\left(\dfrac{x-1}{x}\right)^5$ (6) $\dfrac{1}{\sqrt{1+x+x^2}}$

ネピアの数

年利 r 単利 元利合計. $1+r$.

$\frac{1}{2}$年複利 $\left(1+\frac{r}{2}\right)^2$

$\frac{1}{3}$年複利 $\left(1+\frac{r}{3}\right)^3$

$\frac{1}{4}$年 〃 $\left(1+\frac{r}{4}\right)^4$

$\frac{1}{n}$年 〃 $\left(1+\frac{r}{n}\right)^n$

⋮

瞬間複利 $\downarrow$ e^r

$r = 1$ $\qquad a_n = \left(1 + \frac{1}{n}\right)^n < 3.$

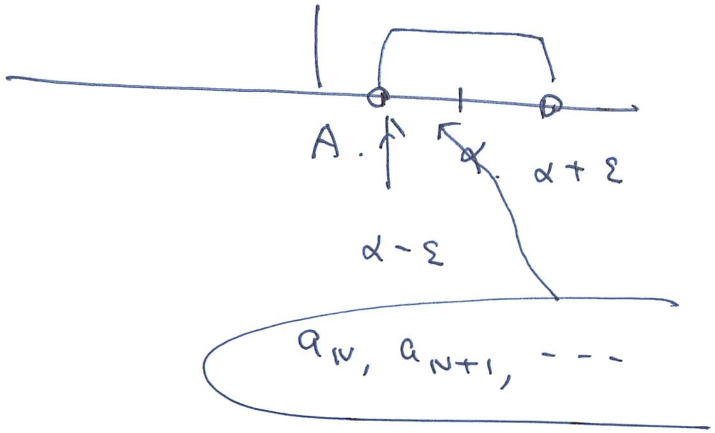
$a_n \leqq A. \quad (n = 0, 1, 2, \cdots)$

$a_n \to \alpha \quad (n \to +\infty)$ $\qquad \Longrightarrow \qquad \alpha \leqq A$

$\alpha > A$ とする.

$\varepsilon = \frac{1}{2}(\alpha - A)$

$\exists N$ が存在して ・・・



有り得ない

$\alpha \leqq A$ とならなければいけない

$a_n = \left(1 + \frac{1}{n}\right)^n$ は単調増加

$a_1 < a_2 < \cdots a_n < a_{n+1} < a_{n+2}$

定理（実数の完備性）

1) あるMに対して

$$a_n \le M \quad (n = 0, 1, 2, \cdots)$$

2) $\cdots < a_n < a_{n+1} < \cdots$

$$\Longrightarrow \{a_n\} \text{ は収束.}$$

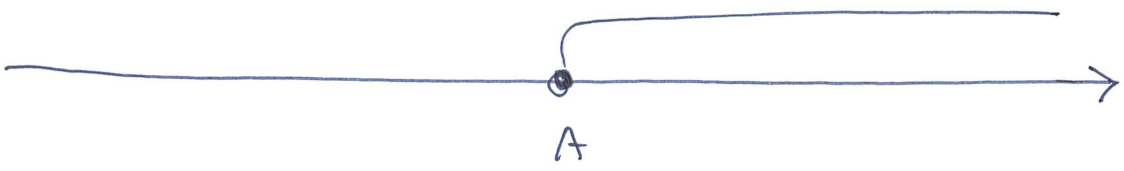
$a_n = \left(1 + \frac{1}{n}\right)^n$ は収束して $a_n \to e \quad (n \to +\infty)$ とすると

$$e \le 3.$$

$$\begin{array}{c} e \\ \parallel \\ 2.71\cdots \end{array}$$

$$\left(1+\frac{1}{t}\right)^t \longrightarrow e \quad (t \to +\infty)$$

93p.



$f : (A, +\infty) \longrightarrow \mathbb{R}$

$\quad\;\; \|$

$\{x ;\; x > A\}$

$$f(t) \longrightarrow \alpha \quad (t \to +\infty)$$

$\Longleftrightarrow$ $\begin{cases} \boxed{t_n \to +\infty} \ ? \\ t_n > A. \end{cases}$ ← $f(t_n)$ が定義できる.

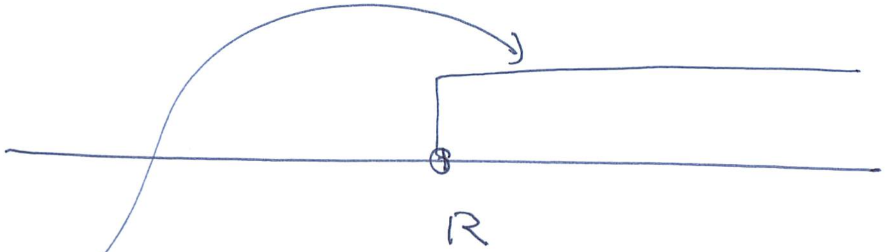
ならば $f(t_n) \to \alpha.$ $(n \to +\infty)$

27p.(発背2)

$t_n \to +\infty$ とは $\forall R > 0$ に対して $\exists N$

$a_N, a_{N+1}, a_{N+2}, \ldots$

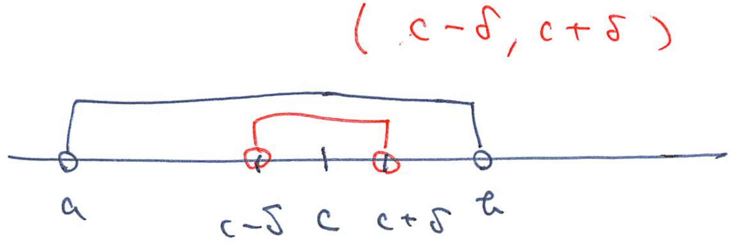
$a_n > R \quad (n \geq N)$



(続きは明日) (3.10) から.

極大・極小の条件.

極大・極小の定義.



$f : (a, b) \longrightarrow \mathbb{R}$

f が $t = c$ で（広義の）極小．（$\Longleftrightarrow$ $t = c$ の近くで最小．）

狭義

ある $\delta > 0$ に対して

$$f(t) \geqq f(c) \quad (c-\delta < t < c+\delta)$$

$$f(t) > f(c) \quad (c-\delta < t < c+\delta,\ t \neq c)$$

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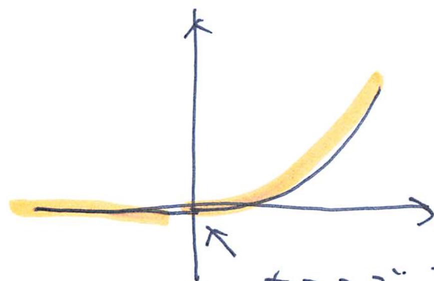
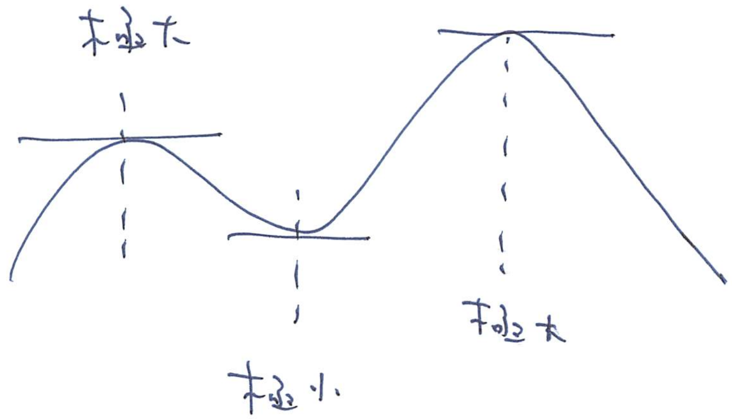
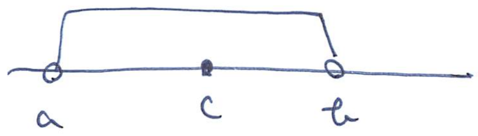


$t=0$ で広義の極小





定理4.1

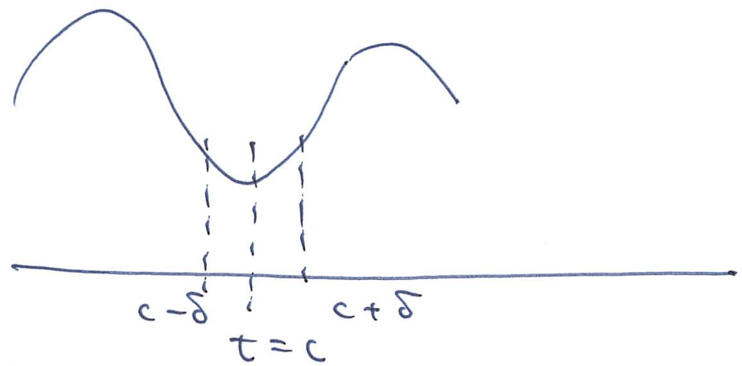
$f : (a, b) \longrightarrow \mathbb{R}$ 各点で微分可能とする.

$t = c \in (a, b)$ で広義の極小（大） $\Longrightarrow f'(c) = 0$.

極小の場合 $\exists \delta > 0$

$$f(t) \geqq f(c) \quad (c-\delta < t < c+\delta)$$

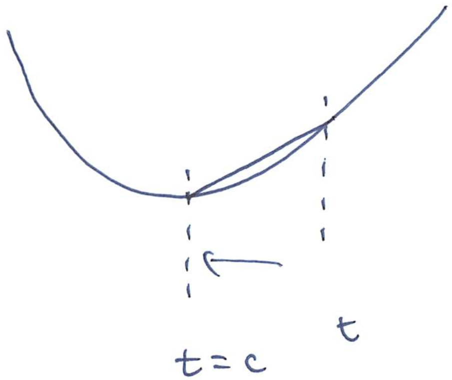
が成立する



（右極限を使う）

$c < t < t+\delta$ とする.

$$\frac{\overset{\geq 0}{f(t)-f(c)}}{\underset{> 0}{t-c}} \geq 0.$$



$t \to c+0$ とする

（$t > c$ として $t \to c$）

$t_n \to c$ を

$t_n > c$ で $t_n \to c$

と置きかえる.

$$\frac{f(t_n)-f(c)}{t_n - c} \geq 0$$

$$\downarrow \qquad \downarrow$$

$$f'(c) \geq 0$$

$\rightsquigarrow$ $f'(c) \geq 0.$

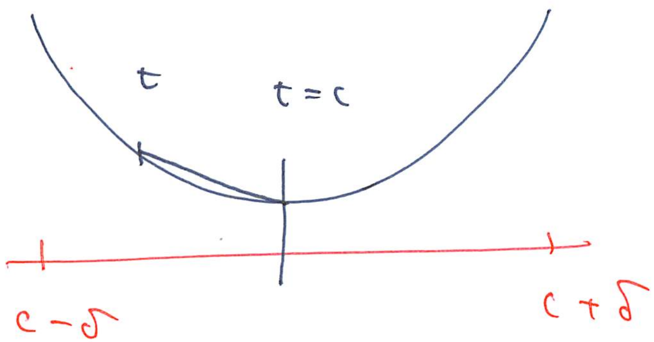
(左極限)

$t-\delta < t < c$ とする.

$\geqq 0$ ← 極小.

$$\frac{f(t)-f(c)}{t-c} \leqq 0.$$

$t - c < 0$



$t-\delta < t_n < c$, $t_n \to c$

$$\frac{f(t_n)-f(c)}{t_n-c} \leqq 0$$

$$\downarrow \qquad \downarrow$$

$$f'(c) \leqq 0 \quad \rightsquigarrow \quad f'(c) \leqq 0$$

$$\boxed{f'(c) = 0}$$

Ⅰ 微分，公式を使って計算

f'(x)を

(1) $f(x) = \dfrac{x^2}{1+x^2}$ (2) $f(x) = \dfrac{x}{2x-1}$

(3) $f(x) = \dfrac{x}{1+x+x^2}$ (4) $f(x) = (x-1)\sqrt{x}$

Ⅱ

(1) $f(x) = \left(\dfrac{x-1}{x}\right)^4$ (2) $f(x) = (1+x^2)^6$

(3) $f(x) = \sqrt{3x+1}$

Ⅲ.

$$\begin{cases} a_{n+2} - 4a_{n+1} + 3a_n = 0 \\ a_0 = c_0,\ a_1 = c_1 \end{cases}$$

を解く。