2016/08/06
2nd Lec

8p. 演習 1.2.

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(1)

$$a_0 = 1, \quad a_{n+1} = 3a_n - 2$$

$$\lambda = 3\lambda - 2 \iff \lambda = 1$$

だから

$$\begin{array}{rcl} & a_{n+1} = 3a_n - 2 \\ -) & 1 = 3 \times 1 - 2 \\ \hline & a_{n+1} - 1 = 3(a_n - 1) \end{array}$$

n step

$$a_0 - 1 \xrightarrow{\times 3} a_1 - 1 \longrightarrow \cdots \xrightarrow{\times 3} a_n - 1$$

$\{a_n - 1\}$ は公比 3 の等比数列

$$a_n - 1 = 3^n (a_0 - 1)$$

だから

$$a_n = 3^n (1 - 1) + 1 = 1.$$

(2) $a_0 = 1, \quad a_{n+1} = -2a_n + 1$

$$\lambda = -2\lambda + 1 \iff \lambda = \frac{1}{3}$$

だから

$$a_{n+1} = -2a_n + 1$$

$$-)\quad \frac{1}{3} = -2 \times \frac{1}{3} + 1$$

$$a_{n+1} - \frac{1}{3} = -2\left(a_n - \frac{1}{3}\right)$$

$\left\{a_n - \frac{1}{3}\right\}$ は公比 (-2) の等比数列.

$$a_n - \frac{1}{3} = (-2)^n \left(a_0 - \frac{1}{3}\right)$$

だから

$$a_n = \frac{1}{3} + (-2)^n\left(1 - \frac{1}{3}\right) = \frac{1}{3} - \frac{2}{3}(-2)^n$$

$\sum_{k=1}^{n} k^3$ 省略. → 1回目のノートに書いてある

${}_{n}C_{k} = {}_{n-1}C_{k-1} + {}_{n-1}C_{k}$ を示す.　　${}_{n}C_{k} = \dfrac{n!}{k!\,(n-k)!}$

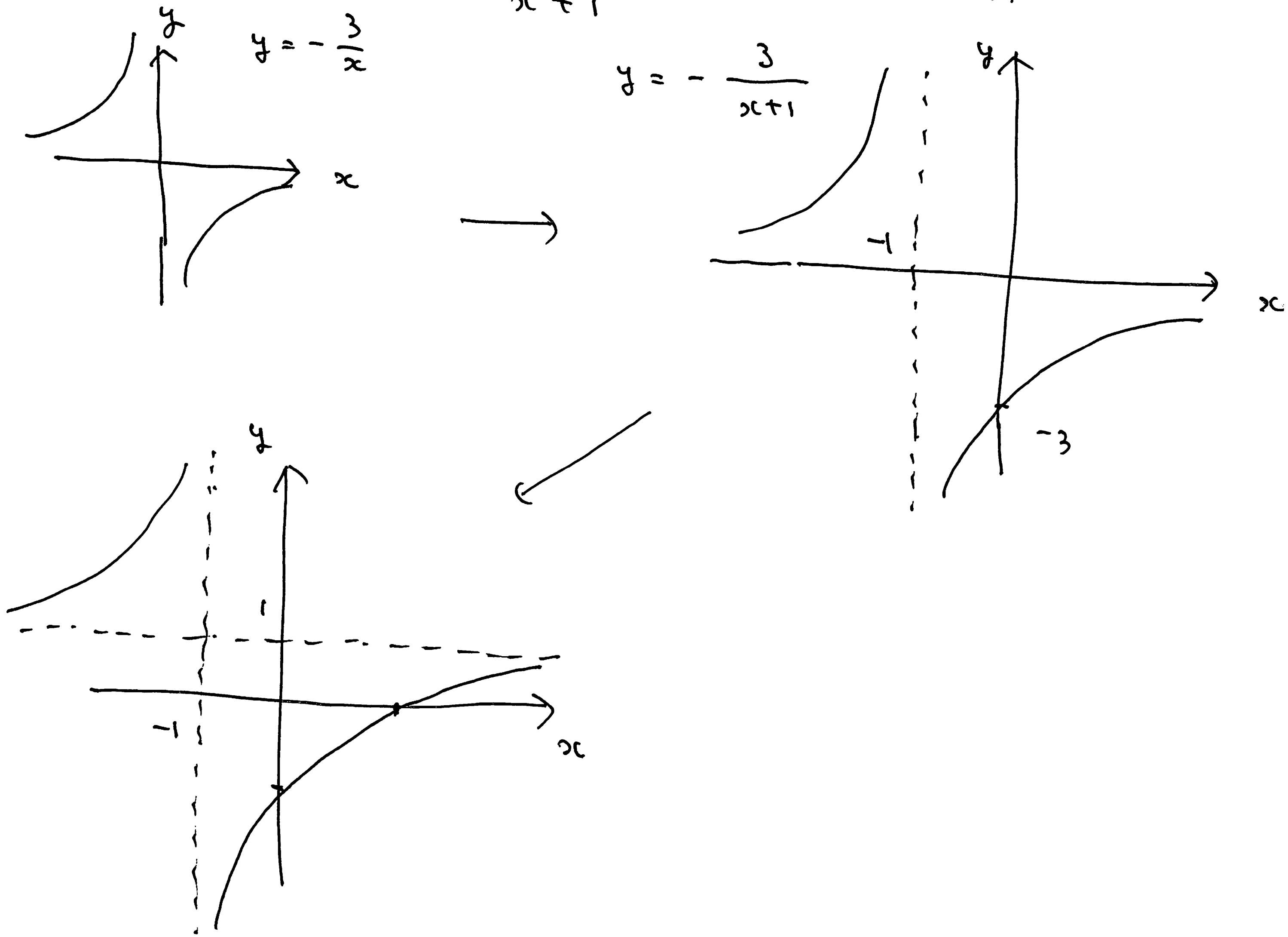
$$(右) = \frac{(n-1)!}{(k-1)!\,\underbrace{\boxed{\{n-1-(k-1)\}}}_{= n-k}!} + \frac{(n-1)!}{k!\,(n-1-k)!}$$

$$= (n-1)!\,\frac{k + n - k}{k!\,(n-k)!} = \frac{(n-1)!\,n}{k!\,(n-k)!}$$

$$= \frac{n!}{k!\,(n-k)!} = {}_{n}C_{k} = (左)$$

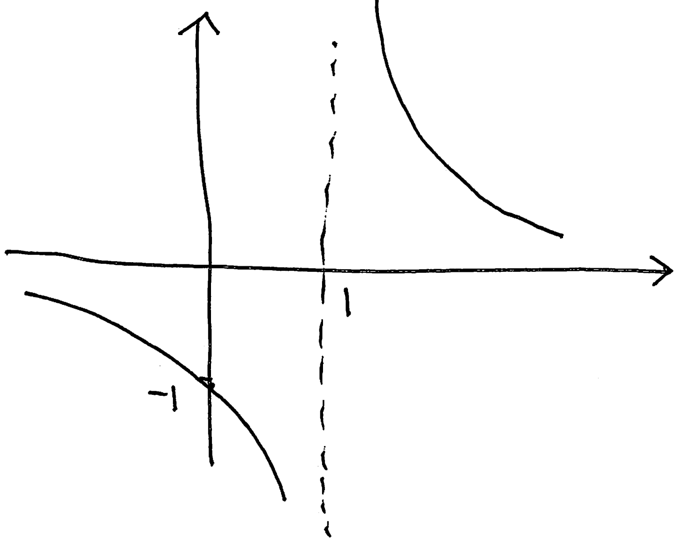
$$\boxed{n! = n \cdot (n-1)!}$$

(1) $y = \dfrac{x-2}{x+1} = \dfrac{x+1-3}{x+1} = 1 - \dfrac{3}{x+1}$

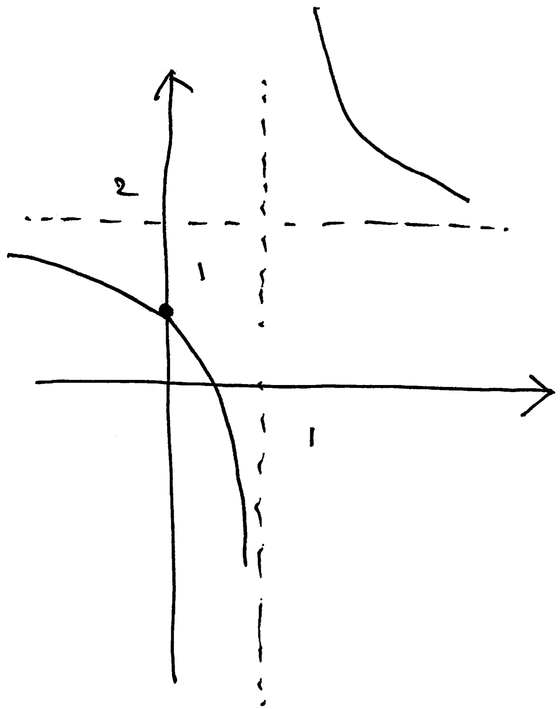


(2) $y = \dfrac{2x-1}{x-1} = \dfrac{2(x-1)+1}{x-1}$

$= 2 + \dfrac{1}{x-1}$



$y = \dfrac{1}{x-1}$



$y = \dfrac{cx+d}{ax+b}$

$|r|<1$ のとき $\quad n r^n \to 0 \quad (n \to +\infty)$

$0<r<1$ のとき. $\quad s = \dfrac{1}{r} > 1 \quad s = 1+\theta$ とおくと $\theta > 0$

$$\underset{\substack{\uparrow \\ \text{2項定理}}}{s^n =} 1 + {}_nC_1\,\theta + \boxed{{}_nC_2\,\theta^2} + \cdots + {}_nC_{n-1}\,\theta^{n-1} + \theta^n$$

${}_nC_k\,\theta^k > 0$ より

$$s^n > {}_nC_2\,\theta^2 = \frac{n(n-1)}{2}\theta^2$$

$${}_nC_3\,\theta^3 = \frac{n(n-1)(n-2)}{6}\theta^3$$

$$0 < r^n = \frac{1}{s^n} < \frac{2}{n(n-1)}\cdot\frac{1}{\theta^2}$$

$$0 < n r^n < \frac{1}{n-1}\cdot\frac{2}{\theta^2}$$

において $\dfrac{1}{n-1} = \dfrac{1}{n}\cdot\dfrac{1}{1-\frac{1}{n}} \longrightarrow 0\cdot\dfrac{1}{1-0} = 0$

$\downarrow \quad \downarrow \quad \downarrow$

$0 \quad 0 \quad 0\cdot\dfrac{2}{\theta^2}$

by はさみうち.

$k = 1, 2, 3, \cdots$

$|r| < 1$

$$n^k r^n \longrightarrow 0 \quad (n \to +\infty)$$

$r = 0$ の場合　$n r^n = 0 \longrightarrow 0$

$-1 < r < 0$ のとき　$s = -r$ とする　$0 < s < 1$.

$$-n s^n \le n r^n \le n s^n$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 \end{array}$$

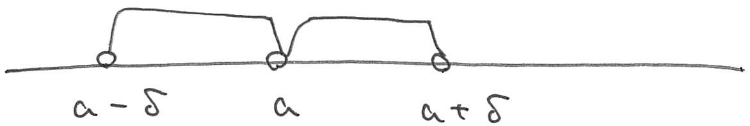
by はさみうち.

関数の極限と微分係数

44p.

$\delta > 0.$

$$f : (a-\delta, a) \cup (a, a+\delta) \longrightarrow \mathbb{R}$$



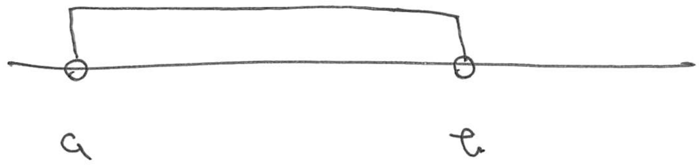
例

$$f(x) = \frac{x^2 - a^2}{x - a}$$

$a < b$

(a, b) 　　 $]a, b[$

$= \{x \,;\, a < x < b\}$



$x \longrightarrow a$ のとき $f(x) \longrightarrow A.$

$\Longleftrightarrow$ 数列 $\{x_n\}$ が

$f(x_n)$ が定義されるため。 $\longrightarrow$ ① $x_n \in (a-\delta, a) \cup (a, a+\delta)$

$(a-\delta < x_n < a+\delta, \; x_n \neq a)$

② $x_n \longrightarrow a \quad (n \longrightarrow +\infty)$

を満たすならば

$f(x_n) \longrightarrow A.$

$$\frac{x_n^2 - a^2}{x_n - a}$$

x_n は ① $a-\delta < x_n < a+\delta$, $x_n \neq a$.

② $x_n \to a$.

$$= x_n + a. \longrightarrow a + a = 2a$$

$x \to a$ のとき $\dfrac{x^2 - a^2}{x - a} \longrightarrow 2a.$

x_n に依らない

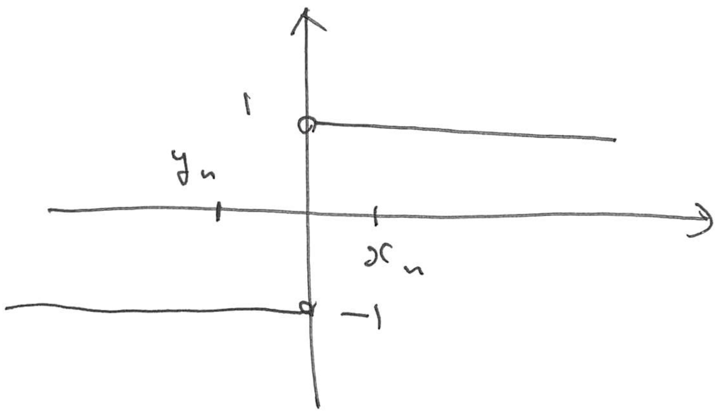
$x_n \to \alpha,\ y_n \to \beta$

① $x_n \pm y_n \longrightarrow \alpha \pm \beta$

② $x_n y_n \longrightarrow \alpha\beta$

③ $y_n \neq 0,\ \beta \neq 0$ とすると

$$\frac{x_n}{y_n} \longrightarrow \frac{\alpha}{\beta}$$



$x \to 0$ のとき $f(x) \longrightarrow$?

$x_n = \dfrac{1}{n}$ のとき $x_n \to 0$ $(n \to +\infty)$

$f(x_n) = 1 \longrightarrow 1$ $(n \to +\infty)$

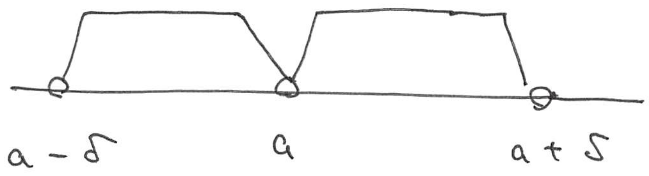
$y_n = -\dfrac{1}{n} \longrightarrow 0$ $(n \to +\infty)$

$f(y_n) = -1 \longrightarrow -1$

$\lim_{x \to 1} f(x)$ は存在しない

$$f : (a-\delta, a) \cup (a, a+\delta) \longrightarrow \mathbb{R}$$

$$g : \qquad 〃 \qquad \longrightarrow \mathbb{R}.$$



定理 2.1

$x \to a$ のとき $f(x) \longrightarrow A$

$g(x) \longrightarrow B.$

$\Rightarrow$ (i) $f(x) \pm g(x) \longrightarrow A \pm B$

(ii) $f(x) \cdot g(x) \longrightarrow A \cdot B$

(iii) $g(x) \neq 0 \quad (x \in (\quad) \cup (\quad))$, $B \neq 0$

$$\frac{f(x)}{g(x)} \longrightarrow \frac{B}{A}$$

(ii) を示す. $x_n \to a \quad (n \to +\infty)$, $a-\delta < x_n < a+\delta$, $x_n \neq a$.

とすると $f(x_n) \longrightarrow A$, $g(x_n) \longrightarrow B$.

このとき

$$f(x_n)\, g(x_n) \longrightarrow A \cdot B.$$

81 P.

$P(x, x^2) \neq A$

APの傾き $= \dfrac{x^2 - a^2}{x - a}$

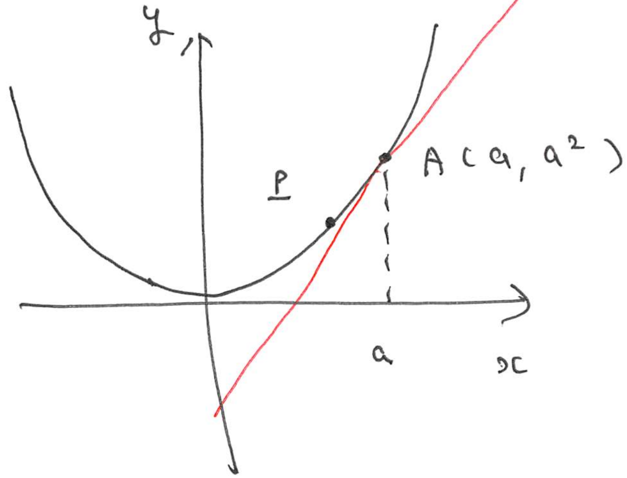
||

$= x + a$

xとaの間の平均変化率 $\longrightarrow 2a.$

$x \to a$

$y = f(x) = x^2$



$$\boxed{f'(a) = 2a}$$

$(x^2)' = 2x.$

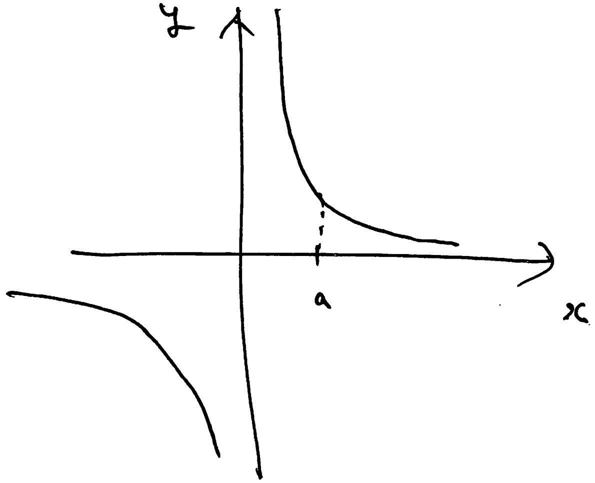
fの$x = a$における微分係数

$y = f(x)$の$x = a$における接線の傾き.

49 p. はさみうちは後で.

50 p. $a > 0$ のとき $\sqrt{x} \to \sqrt{a}$ $(x \to a)$ は後回し.

13113.2 $\quad g(x) = \frac{1}{x} \quad (x \neq 0)$



$x = a \neq 0$

$$\frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{a - x}{ax(x-a)}$$

$$= -\frac{1}{a}\cdot\frac{1}{x} \longrightarrow -\frac{1}{a}\cdot\frac{1}{a} = -\frac{1}{a^2}$$

$x \longrightarrow a$ aとす. $\frac{1}{x} \rightarrow \frac{1}{a}$

$$\boxed{g'(a) = -\frac{1}{a^2}}$$

$f(x) = x^3$.

$$\frac{x^3 - a^3}{x - a} = \frac{(x-a)(x^2 + ax + a^2)}{x - a}$$

$$= x^2 + ax + a^2 \longrightarrow a^2 + a\cdot a + a^2 = 3a^2$$

$x_n \neq a$

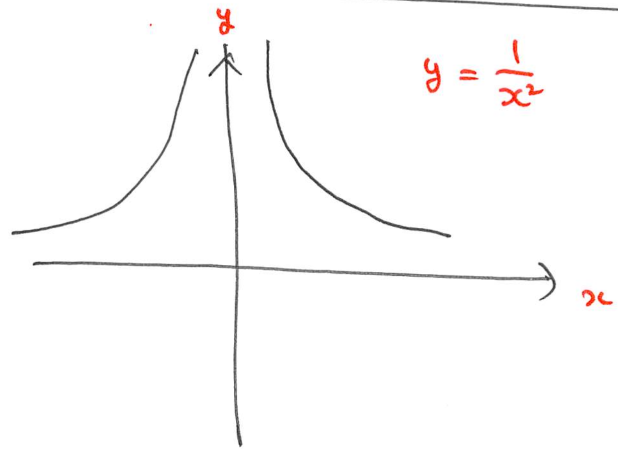
$x_n \to a$

$$\frac{x_n^3 - a^3}{x_n - a} = x_n^2 + a x_n + a^2 \longrightarrow a^2 + a\cdot a + a^2$$

$$= 3a^2.$$

$$(x^3)' = 3x^2.$$

$$f(x) = \frac{1}{x^2} \qquad a \neq 0$$



$$\frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a} = \frac{a^2 - x^2}{x^2 a^2 (x-a)}$$

$$= \frac{(a-x)(a+x)}{x^2 a^2 (x-a)} = -\frac{a+x}{x^2 a^2}$$

$$= -\frac{1}{a^2} \cdot (a+x) \cdot \frac{1}{x^2} \longrightarrow -\frac{1}{a^2} \cdot (a+a) \cdot \frac{1}{a^2}$$

$$= -\frac{2a}{a^4} = -\frac{2}{a^3}$$

$$\boxed{\left(\frac{1}{x^2}\right)' = -\frac{2}{x^3}}$$

$$f(x) = \frac{1}{x^2+1}$$

$$\frac{\dfrac{1}{x^2+1} - \dfrac{1}{a^2+1}}{x-a} = \frac{(a^2+1)-(x^2+1)}{(x^2+1)(a^2+1)(x-a)}$$

$$= (a-x)(a+x)$$

$$= \frac{a^2-x^2}{(x^2+1)(a^2+1)(x-a)} = -\frac{x+a}{(x^2+1)(a^2+1)}$$

$$= -\frac{1}{a^2+1} \cdot (x+a) \cdot \frac{1}{x^2+1}$$

$$\longrightarrow -\frac{1}{a^2+1} \cdot (a+a) \cdot \frac{1}{a^2+1} = -\frac{2a}{(a^2+1)^2}.$$

演習 微分係数の定義を用いて $f'(a)$ を求めよ.

(i) $f(x) = \dfrac{1}{x-1}$ $(a \neq 1)$

(ii) $f(x) = Ax^2 + Bx + C$ A, B, C は定数

(iii) $f(x) = \dfrac{1}{x^2+x+1}$

$x^2 + x + 1$
$= (x+\frac{1}{2})^2 + \frac{3}{4} > 0.$

(iv) $f(x) = \dfrac{1}{(x^2+1)^2}$

はさみうちの定理.

$\delta > 0$

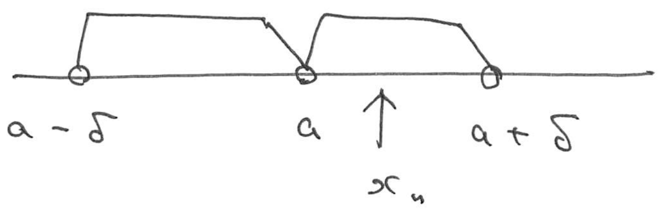
$f : (a-\delta, a) \cup (a, a+\delta) \longrightarrow \mathbb{R}$

$g :$ 〃

$h :$ 〃

$f(x) \leq g(x) \leq h(x)$

$(x \in (\quad) \cup (\quad))$



$x \to a$ のとき $f(x) \longrightarrow A$, $h(x) \to A$

$\Longrightarrow$ $g(x) \longrightarrow A$.

$\{x_n\}$ s.t. $x_n \in (a-\delta, a) \cup (a, a+\delta)$, $x_n \to a$

$$\begin{array}{ccccc} f(x_n) & \leq & g(x_n) & \leq & h(x_n) \\ \downarrow & & \downarrow & & \downarrow \\ A & & A & & A \end{array}$$

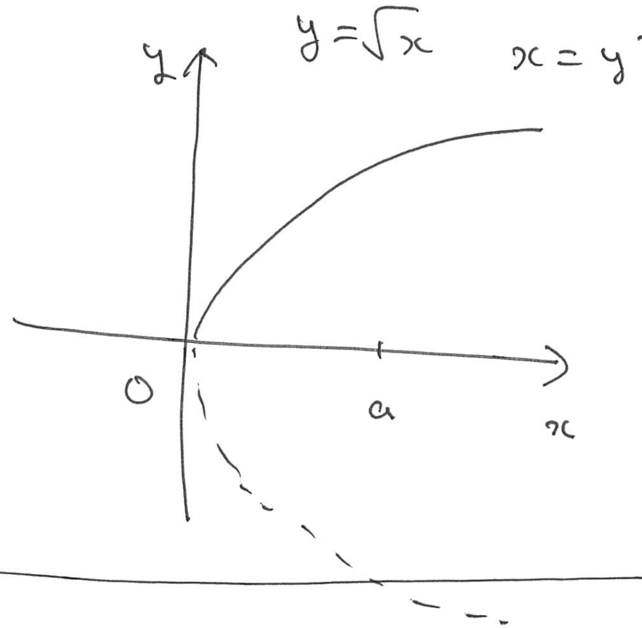
よって $g(x) \longrightarrow A$ $(x \to a)$

by はさみうち.

50p.

$a > 0$

$\sqrt{x} \to \sqrt{a} \quad (x \to a)$



$\sqrt{x} - \sqrt{a}$

$$= \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{\sqrt{x} + \sqrt{a}}$$

$$= \frac{x - a}{\sqrt{x} + \sqrt{a}}.$$

$$|\sqrt{x} - \sqrt{a}| = \frac{|x - a|}{\sqrt{x} + \sqrt{a}} < \frac{|x - a|}{\sqrt{a}}$$

$B \geq 0$ のとき

$|A| \leq B$

$<$

$\Updownarrow$

$-B \leq A \leq B$

$<$ $<$

$$-\frac{|x-a|}{\sqrt{a}} < \sqrt{x} - \sqrt{a} < \frac{|x-a|}{\sqrt{a}}$$

$$\sqrt{a} - \frac{|x-a|}{\sqrt{a}} < \sqrt{x} < \sqrt{a} + \frac{|x-a|}{\sqrt{a}}$$

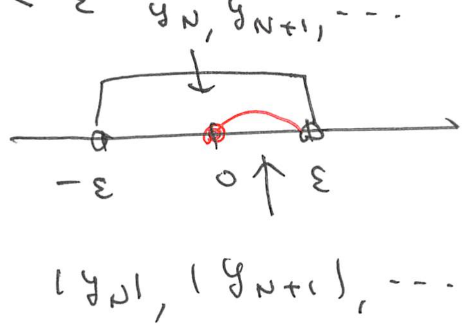
$\downarrow$ $\quad$ はさみうち $\downarrow$ $\quad$ $\downarrow$

$\sqrt{a} - \frac{0}{\sqrt{a}} = \sqrt{a} \quad \sqrt{a} \quad \sqrt{a} + \frac{0}{\sqrt{a}}$

$y \to 0$ ならば $|y| \to 0$ $\rightsquigarrow$ $x \to a$ ならば $|x-a| \to 0$

$y_n \to 0 \iff \forall \varepsilon > 0\ \exists N$

$n \geq N \Rightarrow -\varepsilon < y_n < \varepsilon$



$-\varepsilon < 0 \leq |y_n| < \varepsilon \quad (n \geq N)$

$\rightsquigarrow (|y_n| \to 0)$

演習 2.2*

$a > 0,\ x > 0$

$$\sqrt[3]{x} - \sqrt[3]{a} = \frac{x-a}{(\sqrt[3]{x})^2 + \sqrt[3]{x}\cdot\sqrt[3]{a} + (\sqrt[3]{a})^2}$$

$$|\sqrt[3]{x} - \sqrt[3]{a}| = \frac{|x-a|}{(\sqrt[3]{x})^2 + \sqrt[3]{x}\cdot\sqrt[3]{a} + (\sqrt[3]{a})^2} < \frac{|x-a|}{(\sqrt[3]{a})^2}$$

から

$$-\frac{|x-a|}{(\sqrt[3]{a})^2} + \sqrt[3]{a} < \sqrt[3]{x} < \frac{|x-a|}{(\sqrt[3]{a})^2} + \sqrt[3]{a}$$

$x \to a$ のとき $|x-a| \to 0$ となるので はさみうちの定理から

$$\sqrt[3]{x} \to \sqrt[3]{a}$$

が言う.

83p.

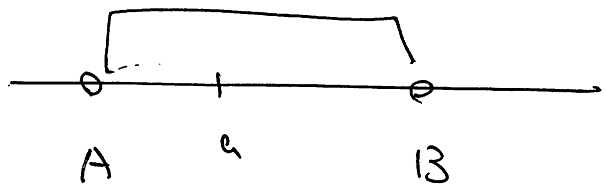
$f(x) = \sqrt{x}$. $a > 0$

$$\frac{\sqrt{x} - \sqrt{a}}{x - a} = \frac{x - a}{(\sqrt{x} + \sqrt{a})(x - a)} = \frac{1}{\sqrt{x} + \sqrt{a}}$$

$x \to a$ a とき. $\sqrt{x} \to \sqrt{a}$ $\longrightarrow \frac{1}{2\sqrt{a}}$.

$$\boxed{(\sqrt{x})' = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}}$$

定理3.1　(微分可能ならば連続)



$f(x)$ が $x=a$ で微分可能

$$\left(\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \text{ が存在} \right)$$

$\Longrightarrow$ $f(x)$ は $x=a$ で連続

$x \to a$ のとき $f(x) \to f(a)$

$$\frac{f(x)-f(a)}{x-a} \times (x-a) + f(a) \longrightarrow f'(a) \times 0 + f(a)$$

$$= f(a)$$

$$= f(x) - f(a) + f(a)$$

$$= f(x)$$

定理 3.2.

1) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

2) Leibnitz Rule ライプニッツの公式

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

3) $$\left(\frac{g(x)}{f(x)}\right)' = \frac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$$

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2) $$\frac{f(x)g(x) - f(a)g(a)}{x-a} = \frac{f(x)g(x) - f(x)g(a) + f(x)g(a) - f(a)g(a)}{x-a}$$

$$= \frac{g(x)-g(a)}{x-a} \times \underbrace{f(x)}_{} + \frac{f(x)-f(a)}{x-a} \times g(a)$$

$\downarrow$ $g'(a)$ $\downarrow$ 定理 3.1. $f(a)$ $\searrow$ $f'(a)$ $\searrow$ $g(a)$

$\longrightarrow g'(a)f(a) + f'(a)g(a)$

$y = x\sqrt{x}.$

$$y' = (x)'\sqrt{x} + x\cdot(\sqrt{x})'$$
$$= 1\cdot\sqrt{x} + x\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x}}$$
$$= \sqrt{x} + \frac{1}{2}\sqrt{x} = \frac{3}{2}\sqrt{x}.$$

$$\boxed{(\sqrt{x})' = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}}$$

$$\downarrow$$

$$(x^{\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}}$$
$$= \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\boxed{(x^{\frac{3}{2}})' = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}x^{\frac{3}{2}-1}}$$

$y = x^n$ $\quad (x^n)' = nx^{n-1}$ とする.

$(x)' = 1.$

$(x^2)' = 2x$

$(x^3) = 3x^2$

$$(x^{n+1})' = (x^n\cdot x)'$$
$$= (x^n)'x + x^n\cdot(x)'$$
$$= nx^{n-1}\cdot x + x^n\cdot 1 = (n+1)x^n$$
$$= (n+1)x^{n+1-1}$$

$$\boxed{(x^n)' = nx^{n-1}}$$

$$\left(\frac{x-1}{3x+2}\right)' = \frac{(x-1)'(3x+2) - (x-1)(3x+2)'}{(3x+2)^2}$$

$$= \frac{1\cdot(3x+2) - (x-1)\cdot 3}{(3x+2)^2}$$

$$= \frac{5}{(3x+2)^2}$$

$$y = \frac{1}{x^n} \qquad \left(\frac{1}{x^n}\right)' = -\frac{n x^{n-1}}{x^{2n}} = -n\frac{1}{x^{n+1}}$$

$$\boxed{\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}} \qquad \boxed{(x^{-n})' = -n x^{-n-1}}$$

演習 3.3

+

(5) $y = x^2\sqrt{x}$. (6) $y = \dfrac{x-1}{x+1}$ (7) $y = \dfrac{1}{1+x+x^2}$

$y = \sqrt{x+1}$ $a > -1$. $\dfrac{\sqrt{x+1} - \sqrt{a+1}}{x-a} \to ?$ $(x \to a)$

$= f(x)$

($f'(a) = ?$ ということ)

13 p.

$$\begin{cases} a_{n+2} - 3a_{n+1} + 2a_n = 0 \quad (n = 0, 1, 2, \cdots) \\ a_0 = c_0, \quad a_1 = c_1 \end{cases}$$

← 初期条件.

$n=0$ $\quad a_2 = 3a_1 - 2a_0$

$n=1$ $\quad a_3 = 3a_2 - 2a_1$

$n=2$ $\quad a_4 = 3a_3 - 2a_2$

$\vdots$

$\lambda^2 - 3\lambda + 2 = 0$ 特性方程式.

$a_n = C\lambda^n$ とおくと

$$C\lambda^{n+2} - 3 \cdot C\lambda^{n+1} + 2C\lambda^n = 0 \quad (\text{すべての}n)$$

$$\parallel$$

$$C\lambda^n(\lambda^2 - 3\lambda + 2)$$

$\lambda^2 - 3\lambda + 2 = 0 \iff \lambda = 1, 2.$

$$\boxed{a_{n+2} - 3a_{n+1} + 2a_n = 0}$$

$$1+2=3, \quad 1\cdot 2 = 3$$

$$a_{n+2} - (1+2)\,a_{n+1} + 1\cdot 2\,a_n = 0.$$

(1.16) $\quad a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) \quad \nearrow$ $a_{n+1} - a_n$ は公比2の等比数列

(1.17) $\quad a_{n+2} - 2a_{n+1} = a_{n+1} - 2a_n \quad \searrow$ $a_{n+1} - 2a_n$ 定数列

$$-\,\begin{cases} a_{n+1} - a_n = 2^n(a_1 - a_0) \\ a_{n+1} - 2a_n = a_1 - 2a_0 \end{cases}$$

$$a_n = 2^n(a_1 - a_0) - (a_1 - 2a_0)$$

$\lambda^2 - 3\lambda + 2 = 0 \quad \lambda = 1, 2 \quad \leftarrow$ 特性根.

$$a_{n+2} + p\,a_{n+1} + q\,a_n = 0 \qquad p, q : 定数$$

$$\lambda^2 + p\lambda + q = 0.$$

λが虚数の場合、周期的な
動きがでてくる。

演習

(1) $a_{n+2} - a_{n+1} - 2a_n = 0$

$a_0 = c_0,\ a_1 = c_0$

(2) $a_{n+2} - a_{n+1} - 6a_n = 0$

$a_0 = c_0,\ a_1 = c_0.$

$|r| < 1$ のとき $\displaystyle\sum_{n=0}^{+\infty} r^n = \frac{1}{1-r}$

$r \neq 1$

$(n-1)\,r^{n-2}$

$$T_n = \sum_{k=1}^{n} k\,r^{k-1} = 1 + 2r + 3r^2 + \cdots + n\,r^{n-1}$$

$$-)\ r\,T_n = r + 2r^2 + \cdots + (n-1)\,r^{n-1} + n\,r^n$$

$$(1-r)\,T_n = 1 + r + r^2 + \cdots + r^{n-1} - n\,r^n$$

$$= \frac{1-r^n}{1-r} - n\,r^n$$

$$T_n = \frac{1-r^n}{(1-r)^2} - \frac{n\,r^n}{1-r}$$

$|r| < 1$ のとき

$$= \frac{1}{(1-r)^2} - \frac{1}{(1-r)^2}\,\underset{\displaystyle\downarrow\atop\displaystyle 0}{r^n} - \frac{1}{1-r}\,\underset{\displaystyle\downarrow\atop\displaystyle 0}{n\,r^n}$$

$$\longrightarrow \frac{1}{(1-r)^2} - \frac{1}{(1-r)^2}\cdot 0 - \frac{1}{1-r}\cdot 0$$

$$= \frac{1}{(1-r)^2}$$

$|r|<1$ のとき

$$\sum_{n=1}^{+\infty} n r^{n-1} = \frac{1}{(1-r)^2}$$

使える.

$$\boxed{\sum_{n=0}^{+\infty} r^n = \frac{1}{1-r}}$$

演習 1.12. (31p)

I　$a_0 = 1,\quad a_{n+1} = 3a_n - 1$　を解く

II.　$y = \dfrac{2x-1}{x+1}$　のグラフを描く.

III　$x \neq -1.$　$a_n = \dfrac{x^n}{x^n+1}$　の極限値を求める.

$x = 1,\quad |x| < 1,\quad |x| > 1$

このとき $a_n = \dfrac{1}{1+\dfrac{1}{x^n}}$

IV.

(1)　$\left(\dfrac{x}{1-x}\right)' = ?$　　(2)　$(x^3\sqrt{x})' = ?$

$(\sqrt{x})' = \dfrac{1}{2}\cdot\dfrac{1}{\sqrt{x}}$