解答　　2016/12/14

I (1) $(e^{-2t})' = -2e^{-2t}$ から $\left(-\frac{1}{2}e^{-2t}\right)' = e^{-2t}$.

$$\int_0^1 e^{-2t}\,dt = \left[-\frac{1}{2}e^{-2t}\right]_0^1 = -\frac{1}{2}(e^{-2}-1)$$

$$= \frac{e^2-1}{2e^2}$$

(2) $$\int_0^{\frac{\pi}{2}} \sin t\,dt = \left[-\cos t\right]_0^{\frac{\pi}{2}} = -(0-1) = 1$$

(2) $(\cos 2t)' = -2\sin 2t$ から $\left(-\frac{1}{2}\cos 2t\right)' = \sin 2t$

$$\int_0^{\frac{\pi}{2}} \sin 2t\,dt = \left[-\frac{1}{2}\cos 2t\right]_0^{\frac{\pi}{2}} = -\frac{1}{2}(\cos\pi - \cos 0)$$

$$= -\frac{1}{2}(-1-1) = 1.$$

(3) $\left(\frac{1}{x^2}\right)' = -\frac{2}{x^3}$ から $\left(-\frac{1}{2}\cdot\frac{1}{x^2}\right)' = \frac{1}{x^3}$

$$\int_1^2 \frac{1}{x^3}\,dx = \left[-\frac{1}{2}\cdot\frac{1}{x^2}\right]_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right)$$

$$= \frac{3}{8}$$

(4) $\left(x^{\frac{4}{3}}\right)' = \frac{4}{3}x^{\frac{1}{3}}$ から $\left(\frac{3}{4}x^{\frac{4}{3}}\right)' = x^{\frac{1}{3}}$

$$\int_1^8 x^{\frac{1}{3}}\,dx = \left[\frac{3}{4}x\sqrt[3]{x}\right]_1^8 = \frac{3}{4}(8\cdot 2-1)$$

$$= \frac{45}{4}$$

(5) $\{(x-1)^5\}' = 5(x-1)^4$

より

$$\left\{\frac{1}{5}(x-1)^5\right\}' = (x-1)^4$$

$$\int_1^2 (x-1)^4\,dx = \left[\frac{1}{5}(x-1)^5\right]_1^2 = \frac{1}{5}$$

(6) $\{\log(2x+1)\}' = 2\cdot\dfrac{1}{2x+1}$

より

$$\left\{\frac{1}{2}\log(2x+1)\right\}' = \frac{1}{2x+1}$$

$$\int_1^2 \frac{1}{2x+1}\,dx = \left[\frac{1}{2}\log(2x+1)\right]_1^2$$

$$= \frac{1}{2}(\log 5 - \log 3) = \frac{1}{2}\log\frac{5}{3}.$$