$y = px^2 + qx + r$

Σ (a, A), (b, B), (c, C) が通る $\Leftrightarrow$ とおく

$$\begin{cases} pa^2 + qa + r = A \\ pb^2 + qb + r = B \\ pc^2 + qc + r = C \end{cases}$$

とおく. $\begin{pmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}$

とおく

$$D = \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} = \begin{vmatrix} a^2 & a & 1 \\ b^2-a^2 & b-a & 0 \\ c^2-a^2 & c-a & 0 \end{vmatrix} = \begin{vmatrix} b^2-a^2 & b-a \\ c^2-a^2 & c-a \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} b+a & 1 \\ c+a & 1 \end{vmatrix}$$

$$= (b-a)(c-a)(b-c)$$

$$= -(a-b)(b-c)(c-a) \neq 0$$

より, クラメルの公式より

$$p = \frac{1}{D}\begin{vmatrix} A & a & 1 \\ B & b & 1 \\ C & c & 1 \end{vmatrix}, \quad q = \frac{1}{D}\begin{vmatrix} a^2 & A & 1 \\ b^2 & B & 1 \\ c^2 & C & 1 \end{vmatrix}, \quad r = \frac{1}{D}\begin{vmatrix} a^2 & a & A \\ b^2 & b & B \\ c^2 & c & C \end{vmatrix}$$

とおく

$$y = \frac{1}{D}\begin{vmatrix} A & a & 1 \\ B & b & 1 \\ C & c & 1 \end{vmatrix} x^2 + \frac{1}{D}\begin{vmatrix} a^2 & A & 1 \\ b^2 & B & 1 \\ c^2 & C & 1 \end{vmatrix} x + \frac{1}{D}\begin{vmatrix} a^2 & a & A \\ b^2 & b & B \\ c^2 & c & C \end{vmatrix}$$

$$= \frac{A}{D}\begin{vmatrix} b & 1 \\ c & 1 \end{vmatrix} x^2 - \frac{A}{D}\begin{vmatrix} b^2 & 1 \\ c^2 & 1 \end{vmatrix} x + \frac{A}{D}\begin{vmatrix} b^2 & b \\ c^2 & c \end{vmatrix}$$

$$- \frac{B}{D}\begin{vmatrix} a & 1 \\ c & 1 \end{vmatrix} x^2 + \frac{B}{D}\begin{vmatrix} a^2 & 1 \\ c^2 & 1 \end{vmatrix} x - \frac{B}{D}\begin{vmatrix} a^2 & a \\ c^2 & c \end{vmatrix}$$

$$+ \frac{C}{D}\begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} x^2 - \frac{C}{D}\begin{vmatrix} a^2 & 1 \\ b^2 & 1 \end{vmatrix} x + \frac{C}{D}\begin{vmatrix} a^2 & a \\ b^2 & b \end{vmatrix}$$

$$= \frac{A}{D}\left((b-c)x^2 - (b^2-c^2)x + \frac{A}{D} bc(b-c) \right)$$

$$+ \frac{B}{D}\left((c-a)x^2 - (c^2-a^2)x + ac(c-a) \right)$$

$$+ \frac{C}{D}\left((a-b)x^2 - (a^2-b^2)x + ab(a-b) \right)$$

$$= A \frac{x^2 - (b+c)x + bc}{(a-b)(a-c)}$$

$$+ B \frac{x^2 - (a+c)x + ac}{(b-a)(b-c)}$$

$$+ C \frac{x^2 - (a+b)x + ab}{(c-a)(c-b)}$$

$$= A \frac{(x-b)(x-c)}{(a-b)(a-c)} + B \frac{(x-a)(x-c)}{(b-a)(b-c)}$$

$$+ C \frac{(x-a)(x-b)}{(c-a)(c-b)}$$