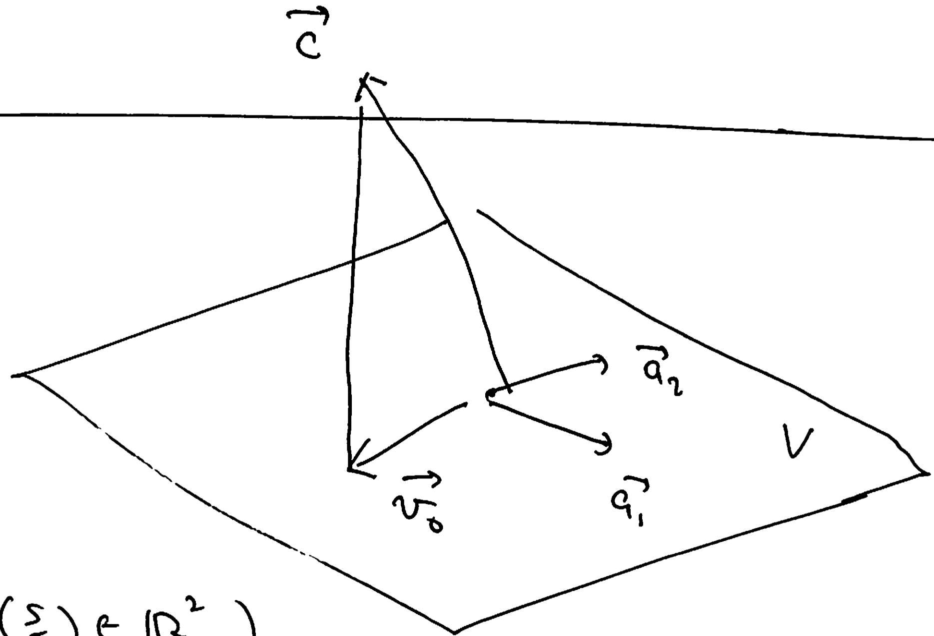
emath 2015 10/20

$\vec{a_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$, $\vec{a_2} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad A = (\vec{a_1}\ \vec{a_2})$

$\vec{c}$ の $\underset{\substack{\| \\ V}}{\mathrm{Im}(A)} = \{ x\vec{a_1} + y\vec{a_2} \,;\, x, y \in \mathbb{R} \}$ への直交射影。

$\{ x\vec{a_1} + y\vec{a_2} \} \subset \mathbb{R}^4$



① $\vec{v_0} \in V$

② $\vec{c} - \vec{v_0} \perp V$

<u>方法1.</u> $\vec{v_0} = A\begin{pmatrix} x \\ y \end{pmatrix}$

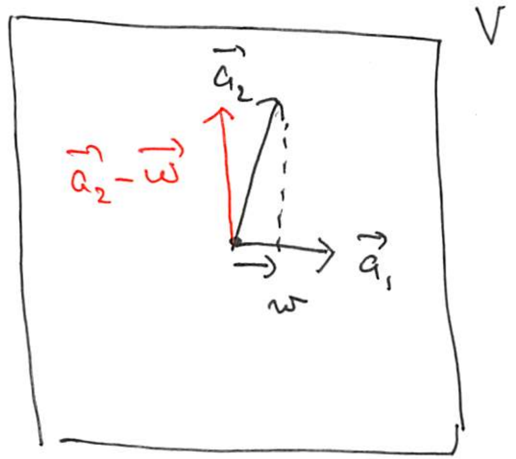
$$\left(\vec{c} - A\begin{pmatrix} x \\ y \end{pmatrix},\ A\begin{pmatrix} s \\ t \end{pmatrix}\right) = 0 \quad \left(\begin{pmatrix} s \\ t \end{pmatrix} \in \mathbb{R}^2\right)$$

$$\Updownarrow$$

$${}^tAA\begin{pmatrix} x \\ y \end{pmatrix} = {}^tA\vec{c} \quad \cdots$$

$V = \mathrm{Im}(A)$ の正規直交基底を求める.

$\|\vec{a}_1\| = 2$, $\|\vec{a}_2\| = \sqrt{6}$; $(\vec{a}_1, \vec{a}_2) = 2$

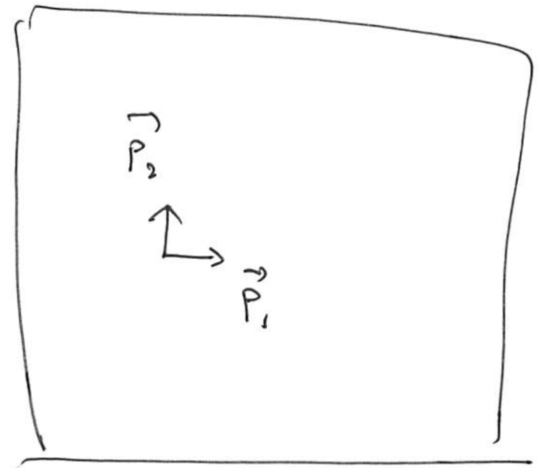


$\vec{a}_2$ の $\vec{a}_1$ 方向への直交射影 $\vec{w}$

$$\boxed{\vec{w} = \frac{(\vec{a}_1, \vec{a}_2)}{\|\vec{a}_1\|^2}\vec{a}_1} = \frac{2}{4}\vec{a}_1 = \frac{1}{2}\vec{a}_1$$

$$\vec{a}_2 - \vec{w} \perp \vec{a}_1$$

$$\vec{a}_2 - \vec{w} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 3 \\ 3 \end{pmatrix}$$



$$\vec{p}_1 = \frac{1}{\|\vec{a}_1\|}\vec{a}_1 = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \quad \vec{p}_2 = \frac{1}{\sqrt{20}}\begin{pmatrix} 1 \\ -1 \\ 3 \\ 3 \end{pmatrix}$$

$$\longrightarrow \|\vec{p}_1\| = \|\vec{p}_2\| = 1, \quad (\vec{p}_1, \vec{p}_2) = 0$$

V の正規直交基底

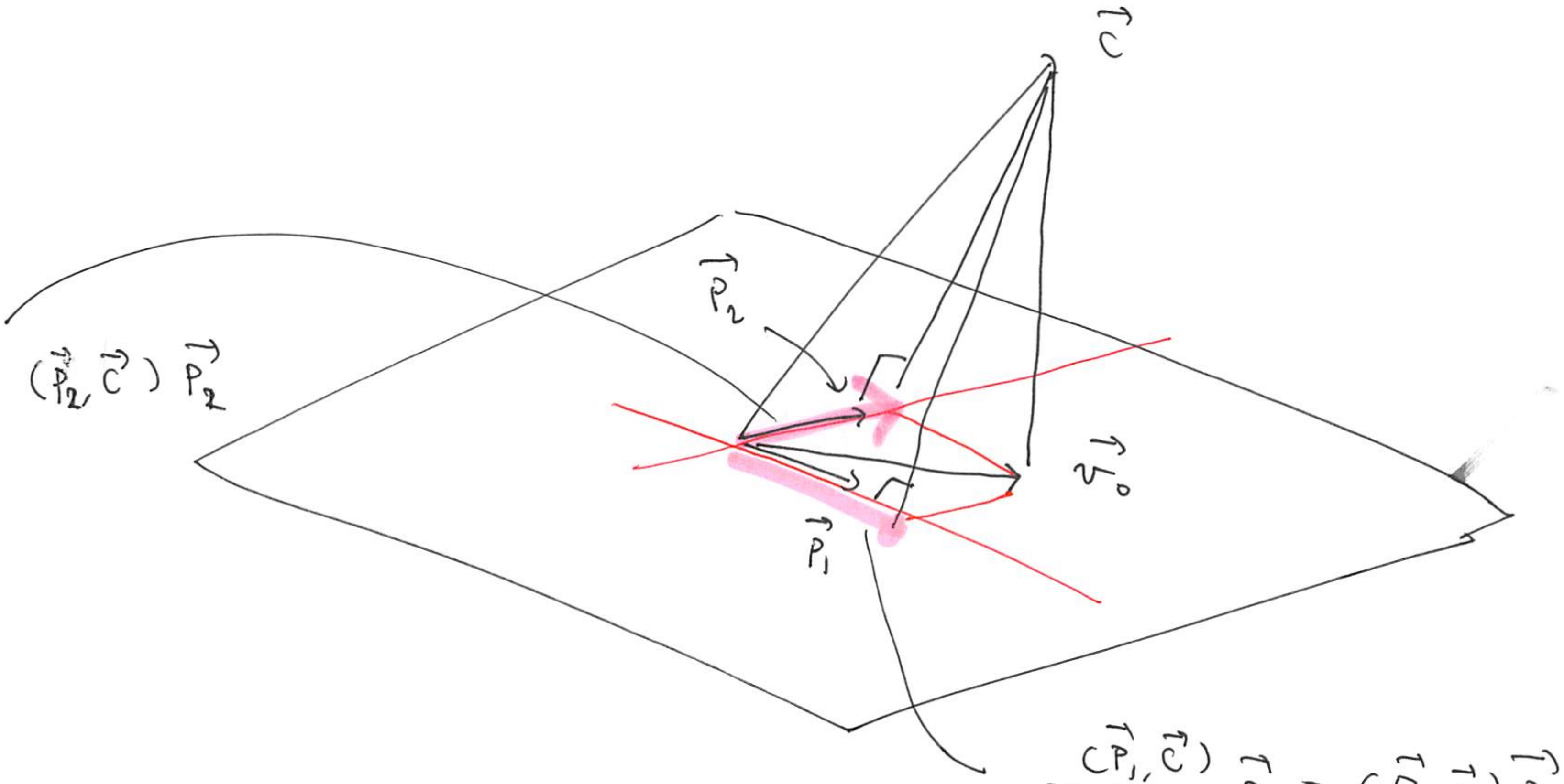
$\vec{c} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ の V への直交射影

$$\boxed{\vec{v_0} = (\vec{p_1}, \vec{c})\,\vec{p_1} + (\vec{p_2}, \vec{c})\,\vec{p_2}}$$

$$= \left(\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right) \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} + \left(\frac{1}{\sqrt{20}}\begin{pmatrix} 1 \\ -1 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}\right) \cdot \frac{1}{\sqrt{20}}\begin{pmatrix} 1 \\ -1 \\ 3 \\ 3 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} + \frac{1}{20}\begin{pmatrix} 1 \\ -1 \\ 3 \\ 3 \end{pmatrix} = \frac{1}{20}\begin{pmatrix} 5+1 \\ 5-1 \\ 5+3 \\ -5+3 \end{pmatrix} = \frac{1}{20}\begin{pmatrix} 6 \\ 4 \\ 8 \\ -2 \end{pmatrix}$$

$$= \frac{1}{10}\begin{pmatrix} 3 \\ 2 \\ 4 \\ -1 \end{pmatrix}$$



$$\frac{(\vec{p}_1, \vec{c})}{\|\vec{p}_1\|^2}\,\vec{p}_1 = (\vec{p}_1, \vec{c})\,\vec{p}_1$$

$\|\vec{p}_1\| = 1$

$$W = \{ x\vec{a}_1 + y\vec{a}_2 + z\vec{c} \;;\; x, y, z \in \mathbb{R} \}$$

Wの基底で $\vec{p}_1, \vec{p}_2, \vec{p}_3$　$\|\vec{p}_i\|^2 = 1$　Wの正規直交基底

$(\vec{p}_i, \vec{p}_j) = 0 \quad (i \neq j)$

$\vec{c} - \vec{v_0} \perp V$

$\vec{v_0} = * \vec{p_1} + * \vec{p_2} \in V$

$\left.\begin{array}{l} \vec{p_1} = * \vec{a_1} \\ \vec{p_2} = * \vec{a_1} + * \vec{a_2} \end{array}\right) \rightsquigarrow \vec{p_1}, \vec{p_2} \in V$

$= \frac{1}{\Delta} (\vec{a_2} - * \vec{a_1})$

エライ

17鈴木

$\vec{c} - \vec{v_0} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{1}{10} \begin{pmatrix} 3 \\ 2 \\ 4 \\ -1 \end{pmatrix}$

$= \frac{1}{10} \begin{pmatrix} 7 \\ -2 \\ -4 \\ 1 \end{pmatrix}$

$\vec{p_3} = \frac{1}{\sqrt{70}} \begin{pmatrix} 7 \\ -2 \\ -4 \\ 1 \end{pmatrix}$

$$\begin{array}{r} 49 \\ 4 \\ 16 \\ 1 \\ \hline \end{array}$$

$\| \vec{p_1} \| = \| \vec{p_2} \| = \| \vec{p_3} \| = 1$

$(\vec{p_1}, \vec{p_2}) = 0$

$(\vec{p_1}, \vec{p_3}) = (\vec{p_2}, \vec{p_3}) = 0 \leftarrow \vec{p_3} \perp V \leftarrow \vec{c} - \vec{v_0} \perp V$

Lagrange 未定乗数法

3変数. 1制約条件.　　CT 329p〜.

$g(x,y,z)=0$ の下で $w=f(x,y,z)$

陰関数定理

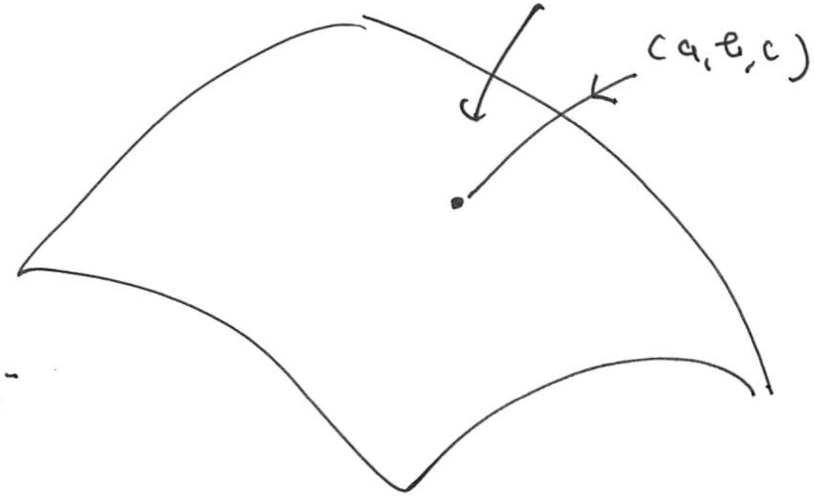
$\longrightarrow$ $\boxed{g_z(a,b,c)\neq 0}$

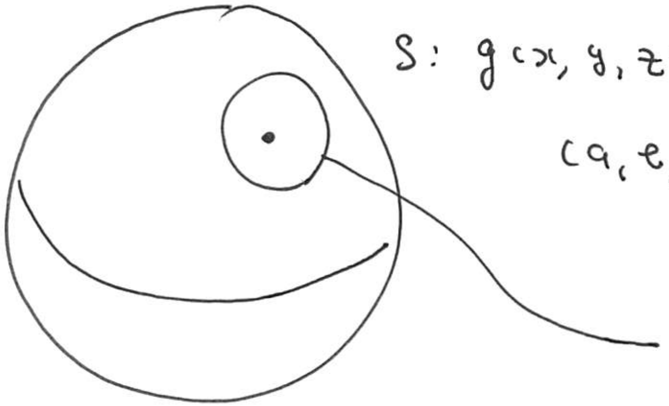
$g(a,b,c)=0$

$\Rightarrow$ (a,b,c) の近くで

曲面 $g(x,y,z)=0$ は

$z=\varphi(x,y)$ と書ける.





$S: g(x,y,z)=x^2+y^2+z^2-1=0$

$(a,b,c)\in S$　　$g_z=2z\neq 0$

$\rightsquigarrow c\neq 0$

ここでは　$\varphi(x,y)=\sqrt{1-x^2-y^2}$.

$$F(x, y) = f(x, y, \varphi(x, y))$$

(a, b, c) で極値をとる $\longrightarrow$ $F_x(a, b) = F_y(a, b) = 0$

Chain Rule

$$\frac{d}{dt} f(\underbrace{x(t), y(t), z(t)}_{= P_t})$$

$$= f_x(P_t)\, x'(t) + f_y(P_t)\, y'(t) + f_z(P_t)\, z'(t).$$

$$F(s, t) = f(s, t, \varphi(s, t))$$

$$F_s = f_x(\ )\cdot 1 + f_y(\ )\cdot 0 + f_z(\ )\cdot \varphi_s(s, t).$$

$\uparrow$

t は止まっている

← s が止まっている

$$F_t = f_x(\ )\cdot 0 + f_y(\ )\cdot 1 + f_z(\ )\cdot \varphi_t(s, t)$$

$$\begin{cases} F_s(a, b) = f_x(a, b, c) + f_z(a, b, c)\cdot \varphi_s(a, b) = 0 \\ F_t(a, b) = f_y(a, b, c) + f_z(a, b, c)\cdot \varphi_t(a, b) = 0 \end{cases}$$

$$g(s,t,\varphi(s,t)) \equiv 0 \qquad ((a,b,c) \text{ の近くで常に})$$

sとtで微分

$$\begin{cases} g_x(\quad)\cdot 1 + g_y(\quad)\cdot 0 + g_z(\quad)\cdot \varphi_s(s,t) \equiv 0 \\ g_x(\quad)\cdot 0 + g_y(\quad)\cdot 1 + g_z(\quad)\cdot \varphi_t(s,t) \equiv 0 \end{cases}$$

$s=a$, $t=b$ を代入. $\quad P_0(a,b,c)$

$$\begin{cases} g_x(P_0) + g_z(P_0)\,\varphi_s(a,b) = 0 \\ g_y(P_0) + g_z(P_0)\,\varphi_t(a,b) = 0 \end{cases}$$

$$\rightarrow \quad \varphi_s(a,b) = -\frac{g_x(P_0)}{g_z(P_0)}, \quad \varphi_t(a,b) = -\frac{g_y(P_0)}{g_z(P_0)}$$

代入

$$\begin{cases} f_x(P_0) + \boxed{\left(-\frac{f_z(P_0)}{g_z(P_0)}\right)} g_x(P_0) = 0 \\ f_y(P_0) + \boxed{\left(-\frac{f_z(P_0)}{g_z(P_0)}\right)} g_y(P_0) = 0 \end{cases}$$

$$\lambda = -\frac{f_z(P_0)}{g_z(P_0)}$$ とおくと

$$
(\#)\begin{cases} f_x(P_0) + \lambda g_x(P_0) = 0 \\ f_y(P_0) + \lambda g_y(P_0) = 0 \\ f_z(P_0) + \lambda g_z(P_0) = 0 \end{cases} \Longleftrightarrow \nabla(f)(P_0) + \lambda \nabla(g)(P_0) = 0
$$

接点条件.

定理　$g(x, y, z) = 0$ の下で $w = f(x, y, z)$ を考える

$$
\begin{cases} g_z(a, b, c) \neq 0 \\ g(a, b, c) = 0 \end{cases}
$$
を仮定する.

(a, b, c) で極値をとるならば $(\#)$ を満たす λ が存在.

문34 CT 331p.

$$g(x, y, z) = x^2 + y^2 + z^2 - 1 = 0. \quad \cdots ①$$

∘ 조건

$$w = f(x, y, z) = 3x - y + 2z.$$

$$\begin{cases} 3 + \lambda \cdot 2x = 0 \\ -1 + \lambda \cdot 2y = 0 \\ 2 + \lambda \cdot 2z = 0 \end{cases}$$

즉

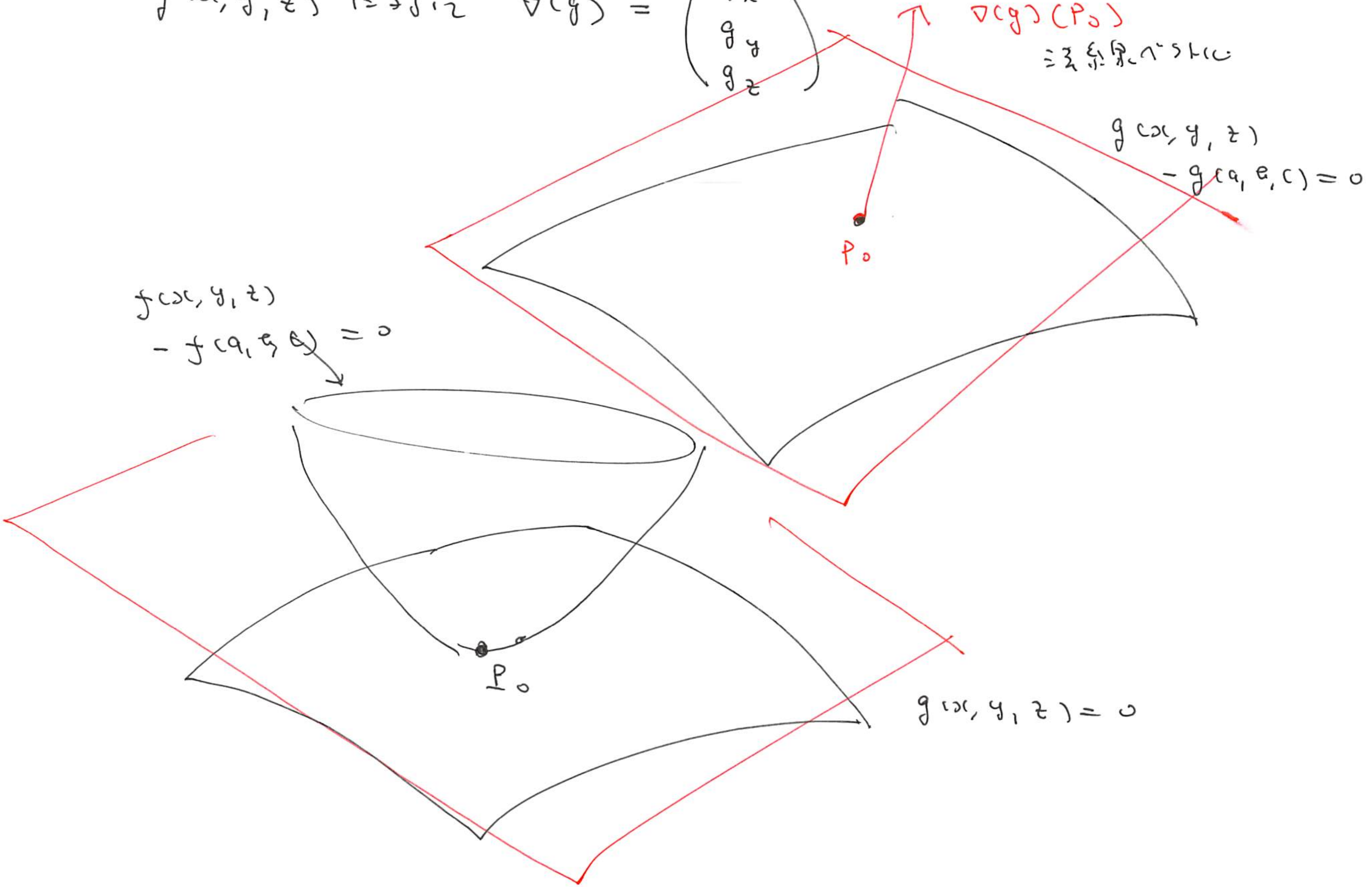
$$\left.\begin{aligned} x &= -\frac{3}{2\lambda} \\ y &= \frac{1}{2\lambda} \\ z &= -\frac{1}{\lambda} \end{aligned}\right\} \text{① 에 대입}$$

$$1 = \frac{9}{4\lambda^2} + \frac{1}{4\lambda^2} + \frac{4}{4\lambda^2} \qquad \lambda^2 = \frac{14}{4} \qquad \lambda = \pm\frac{\sqrt{14}}{2}$$

$$\frac{1}{\lambda} = \pm\frac{2}{\sqrt{14}}$$

$$x = \mp\frac{3}{\sqrt{14}}, \quad y = \pm\frac{1}{\sqrt{14}}, \quad z = \mp\frac{2}{\sqrt{14}}$$

$g(x, y, z)$ に対して $\nabla(g) = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix}$



$g(x, y, z) = x + y + z - 1 = 0$ の下で

$w = f(x, y, z) = x^2 + 4y^2 + 9z^2$ を考える.

停留点を求めよ. 未定乗数も求めよ.