I (1) $\int_0^1 (1+t^2)^4 t = \frac{1}{2}\int_0^1 (1+t^2)^4 (1+t^2)' dt$

$= \frac{1}{2}\int_1^2 x^4 \, dx$

$\uparrow$

$x = 1+t^2$

| $t$ | $0 \nearrow 1$ |
|---|---|
| $x$ | $1 \nearrow 2$ |

$= \frac{1}{2}\left[\frac{1}{5}x^5\right]_1^2 = \frac{1}{10}(32-1) = \frac{31}{10}$

(2) $\int_0^1 (1-t)^6 dt = -\int_0^1 (1-t)^6 (1-t)' dt$

$= -\int_1^0 x^6 dx$

$\uparrow$

$x = 1-t$

| $t$ | $0 \nearrow 1$ |
|---|---|
| $x$ | $1 \searrow 0$ |

$= -\left[\frac{x^7}{7}\right]_1^0 = -\frac{1}{7}(0-1) = \frac{1}{7}$

別解

$I = \int_0^1 (1-x)^6 dx$

$1-x = t$ とおく。ように $x = \varphi(t) = 1-t$ とする.

$\varphi'(t) = -1$.

| $t$ | $1 \searrow 0$ |
|---|---|
| $x$ | $0 \nearrow 1$ |

$I = \int_1^0 t^6 (-1) \, dt = \cdots$

(3) $I = \int_0^1 t(1-t)^5\, dt =$

$\{(1-t)^6\}' = -6(1-t)^5$ より $\left\{-\frac{1}{6}(1-t)^6\right\}' = (1-t)^5$

$$I = \int_0^1 t\left\{-\frac{1}{6}(1-t)^6\right\}' dt$$

$$= -\frac{1}{6}\left[t(1-t)^6\right]_0^1 + \frac{1}{6}\int_0^1 (1-t)^6 dt$$

$$= \frac{1}{6}\cdot\frac{1}{7} = \frac{1}{42}$$

(別解)

$$I = \int_0^1 x(1-x)^5\, dx$$

$t = 1-x$ となるように

$x = \varphi(t) = 1-t$ とおく.

$\varphi'(t) = -1$

| $t$ | $1 \searrow 0$ |
|---|---|
| $x$ | $0 \nearrow 1$ |

$$= \int_1^0 (1-t)t^5(-1)\, dt$$

$$= \int_0^1 (t^5 - t^6)\, dt$$

$$= \left[\frac{t^6}{6} - \frac{t^7}{7}\right]_0^1 = \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$$

(4) $I = \int_0^1 (2x+1)^4\, dx$

$2x+1 = t$ となるように $x = \varphi(t) = \frac{1}{2}(t-1)$

とおく. $\varphi'(t) = \frac{1}{2}$.

| $t$ | $1 \nearrow 3$ |
|---|---|
| $x$ | $0 \nearrow 1$ |

$= \frac{121}{5}$

$$I = \int_1^3 t^4 \frac{1}{2}\, dt = \frac{1}{2}\left[\frac{1}{5}t^5\right]_1^3 = \frac{1}{10}(3^5 - 1) = \frac{242}{10}$$

(5) $$\int_0^1 \frac{t}{(1+t^2)^2}\,dt = \frac{1}{2}\int_0^1 \frac{(1+t^2)'}{(1+t^2)^2}\,dt$$

$$= \frac{1}{2}\int_1^2 \frac{1}{x^2}\,dx = \frac{1}{2}\left[-\frac{1}{x}\right]_1^2 = \frac{1}{4}$$

II $g(x,y) = x^2+xy+y^2-1=0$ のとき.

$g_x = 2x+y,\quad g_y = x+2y$

$g_{xx}=2,\quad g_{xy}=g_{yx}=1,\quad g_{yy}=2$

より

$$\varphi''(x) = \frac{1}{(x+2y)^3}\begin{vmatrix} 0 & 2x+y & x+2y \\ 2x+y & 2 & 1 \\ x+2y & 1 & 2 \end{vmatrix}$$

$$\varphi''(0) = \frac{1}{2^3}\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \frac{1}{8}\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & -3 & 0 \end{vmatrix} = -\frac{1}{8}\begin{vmatrix} 1 & 2 \\ -3 & 0 \end{vmatrix} = -\frac{3}{4}$$

↑

3ト+=2ト×(−2)

(参考) $x^2+x\varphi(x)+\varphi(x)^2-1=0$

$x$で微分すると

① $2x+x\varphi'(x)+\varphi(x)+2\varphi(x)\varphi'(x) \equiv 0$

より

$$\varphi'(x) = -\frac{2x+\varphi(x)}{x+2\varphi(x)}$$

$$\varphi'(0) = -\frac{1}{2}$$

①を $x$ で微分すると

$$2+\varphi'(x)+x\varphi''(x)+\varphi'(x)+2(\varphi'(x))^2$$

$$+2\varphi(x)\varphi''(x)=0$$

より

$$\varphi''(x)=-\frac{2+2\varphi'(x)+2(\varphi'(x))^2}{x+2\varphi(x)}$$

これを用いても解くことができます