(1) $\displaystyle\int_0^1 t\,e^{2t}\,dt = \int_0^1 t\left(\frac{1}{2}e^{2t}\right)' dt$

$$\boxed{(e^{2t})' = 2e^{2t} \text{ より } \left(\frac{1}{2}e^{2t}\right)' = e^{2t}}$$

$$= \left[t\cdot\frac{1}{2}e^{2t}\right]_0^1 - \frac{1}{2}\int_0^1 1\cdot e^{2t}\,dt$$

$$= \frac{1}{2}e^2 - \frac{1}{2}\left[\frac{1}{2}e^{2t}\right]_0^1$$

$$= \frac{1}{2}e^2 - \frac{1}{2}\cdot\frac{1}{2}(e^2-1)$$

$$= \frac{1}{4}(e^2+1)$$

(2) $\displaystyle\int_0^1 t^2 e^{-2t}\,dt = \int_0^1 t^2\left(-\frac{1}{2}e^{-2t}\right)' dt$

$$= -\frac{1}{2}\left[t^2 e^{-2t}\right]_0^1 + \frac{1}{2}\int_0^1 2t\cdot e^{-2t}\,dt$$

$$= -\frac{1}{2}e^{-2} + \int_0^1 t\left(-\frac{1}{2}e^{-2t}\right)' dt$$

$$= -\frac{1}{2}e^{-2} - \frac{1}{2}\left[t e^{-2t}\right]_0^1 + \frac{1}{2}\int_0^1 e^{-2t}\,dt$$

$$= -\frac{1}{2}e^{-2} - \frac{1}{2}e^{-2} + \frac{1}{2}\left[-\frac{1}{2}e^{-2t}\right]_0^1$$

$$= -e^{-2} - \frac{1}{4}(e^{-2}-1) = \frac{1-5e^{-2}}{4}$$

$$= \frac{e^2-5}{4e^2}$$

(3)

$$\int_1^{e^2} x \log x \, dx = \int_1^{e^2} \left(\frac{x^2}{2}\right)' \log x \, dx$$

$$= \left[\frac{x^2}{2} \log x\right]_1^{e^2} - \int_1^{e^2} \frac{x^2}{2} \cdot \frac{1}{x} \, dx$$

$$= \frac{e^4}{2} \cdot \log e^2 - \frac{1}{2} \int_1^{e^2} x \, dx$$

$$= e^4 - \frac{1}{2} \left[\frac{x^2}{2}\right]_1^{e^2}$$

$$= e^4 - \frac{1}{4}(e^4 - 1) = \frac{3e^4 + 1}{4}$$

(4)

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{2}} = -(0 - 1) = 1$$

$(\cos x)' = -\sin x$ より $(-\cos x)' = \sin x$

(5)

$$\int_0^{\frac{\pi}{2}} x \sin x \, dx = \int_0^{\frac{\pi}{2}} x (-\cos x)' \, dx$$

$$= -\left[x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 1 \cdot \cos x \, dx$$

$$= -(0 - 0) + \left[\sin x\right]_0^{\frac{\pi}{2}} = 1 - 0 = 1$$