2016/01/06

I (1)

$$\int_0^1 x^2\sqrt{x}\,dx = \left[\frac{2}{7}x^{\frac{7}{2}}\right]_0^1 = \frac{2}{7}$$

$$\left(x^{\frac{7}{2}}\right)' = \frac{7}{2}x^{\frac{5}{2}} \rightsquigarrow \left(\frac{2}{7}x^{\frac{7}{2}}\right)' = x^{\frac{5}{2}}$$

(2)

$$\int_1^2 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_1^2 = 2\left(\sqrt{2}-1\right)$$

$$\left(\sqrt{x}\right)' = \frac{1}{2}\cdot\frac{1}{\sqrt{x}} \rightsquigarrow \left(2\sqrt{x}\right)' = \frac{1}{\sqrt{x}}$$

(3)

$$\int_0^1 e^{-2t}\,dt = \left[-\frac{1}{2}e^{-2t}\right]_0^1 = -\frac{1}{2}\left(e^{-2}-1\right) = \frac{e^2-1}{2e^2}$$

$$'' \ \frac{1}{e^2}$$

$$\left(e^{-2t}\right)' = -2e^{-2t} \longrightarrow \left(-\frac{1}{2}e^{-2t}\right)' = e^{-2t}$$

(4)

$$\int_1^2 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2 = -\left(\frac{1}{2}-1\right) = \frac{1}{2}.$$

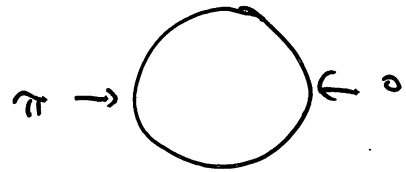
$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

(5) $\int_0^{\frac{\pi}{2}} \sin 2t \, dt = \left[-\frac{1}{2} \cos 2t \right]_0^{\frac{\pi}{2}} = -\frac{1}{2} \left(\cos \pi - \cos 0 \right)$

$= -\frac{1}{2} (-1 - 1) = 1$

$(\cos 2t)' = - \sin 2t \cdot 2$

$$\boxed{\begin{aligned} (\cos t)' &= - \sin t \\ (\sin t)' &= \cos t \end{aligned}}$$

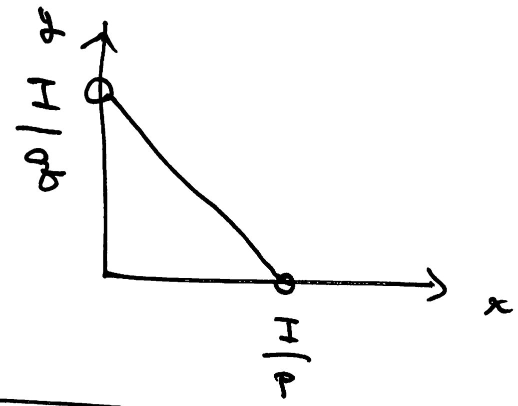


II　$p, q, I > 0$　　$x, y > 0$

$$g(x, y) = I - px - qy = 0$$

の下で　$z = \sqrt[3]{xy} = f(x, y)$

$$= x^{\frac{1}{3}} y^{\frac{1}{3}}$$



$g_x = -p$,　$g_y = -q$.

$$f_x = \frac{1}{3} x^{-\frac{2}{3}} y^{\frac{1}{3}}, \quad f_y = \frac{1}{3} x^{\frac{1}{3}} y^{-\frac{2}{3}}$$

なので Lagrange の未定乗数法から

$$\begin{cases} f_x + \lambda g_x = 0 \\ f_y + \lambda g_y = 0 \\ g = 0 \end{cases}$$

$$\begin{cases} \dfrac{1}{3} \dfrac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} + \lambda(-p) = 0 & \cdots ① \\ \dfrac{1}{3} \cdot \dfrac{x^{\frac{1}{3}}}{y^{\frac{2}{3}}} + \lambda(-q) = 0 & \cdots ② \\ I - px - qy = 0 & \cdots ③ \end{cases}$$

$$\frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} = 3\lambda p \quad \cdots ①'$$

$$\frac{x^{\frac{1}{3}}}{y^{\frac{2}{3}}} = 3\lambda q. \quad \cdots ②'$$

$$\lambda = \frac{y^{\frac{1}{3}}}{x^{\frac{2}{3}}} \cdot \frac{1}{3p} = \frac{x^{\frac{1}{3}}}{y^{\frac{2}{3}}} \cdot \frac{1}{3q} \rightsquigarrow \quad qy = px \text{ となる.}$$

③から　$px = qy = \frac{I}{2}$　となるので

$$x = \frac{I}{2p}, \quad y = \frac{I}{2q}$$

---

①' と ②' を用いて

$(①') \times (②')^2$　　$\frac{1}{y} = 27\lambda^3 p q^2$

$(①')^2 \times ②'$　　$\frac{1}{x} = 27\lambda^3 p^2 q$

$px = qy = \frac{1}{27\lambda^3 pq}$　よって ③を用いると　$px = qy = \frac{I}{2}$

$x = \cdots \quad y = \cdots$

---

③に代入すると

$$I - \frac{2}{27\lambda^3 pq} = 0, \quad \lambda^3 = \frac{2}{27pqI}$$

より $\lambda = \frac{1}{3}\sqrt[3]{\frac{2}{pqI}}$　← 所得の限界効用.

$\alpha, \beta > 0$

$g(x, y) = I - px - qy = 0$, $z = F(x, y)$

$= \alpha \log x + \beta \log y$

$= \log(x^{\alpha} y^{\beta})$

$\longrightarrow \quad x = \dfrac{\alpha}{\alpha+\beta} \cdot \dfrac{I}{p}, \quad y = \dfrac{\beta}{\alpha+\beta} \cdot \dfrac{I}{q}$

$\alpha = \beta = \frac{1}{3}$ の場合.

$x = \dfrac{1}{2} \cdot \dfrac{I}{p}, \quad y = \dfrac{1}{2} \cdot \dfrac{I}{q}$ 需要関数

$f(x, y) = x^{\frac{1}{3}} y^{\frac{1}{3}}$

$F(x, y) = \log(x^{\frac{1}{3}} y^{\frac{1}{3}})$

$f(x_1, y_1) < f(x_2, y_2)$

$\Updownarrow$

$F(x_1, y_1) < F(x_2, y_2)$

$(\log t)' = \dfrac{1}{t} > 0$

$\log(t_1) < \log(t_2)$

$\Updownarrow$

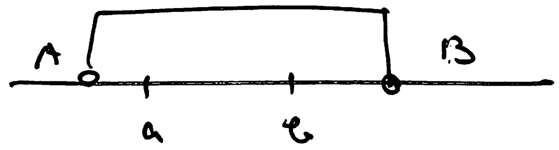
$t_1 < t_2$

効用の相対的な大小は

f と F で変わらない.

スケール変換で導かれる需要関数は不変

171p. 部分積分.



$$f: (A, B) \longrightarrow \mathbb{R}$$

$$g: (A, B) \longrightarrow \mathbb{R}$$

$$F' = f, \quad G' = g. \quad \Longleftrightarrow \quad F = \int f, \quad G = \int g.$$

---

fG の不定積分 $\quad \int fG \Longleftrightarrow (\quad)' = fG.$

↑ ここが欲しい

Leibniz の公式

$$(FG)' = F'G + FG' = fG + Fg.$$

$$fG = (FG)' - Fg. = (FG)' - \left(\int Fg\right)'$$

$$= \left(FG - \int Fg\right)'$$

$$\int fG = FG - \int Fg.$$

$$\int_a^b fG\,dx = [FG]_a^b - \int_a^b Fg\,dx.$$

$F \to f \quad f \to f'$

$G \to g \quad g \to g'$

公式

1) $\int f'g = fg - \int fg'$

2) $\int_a^b f'g = [fg]_a^b - \int_a^b fg'$

---

① $\int t e^t\,dt = \int t(e^t)'\,dt = te^t - \int 1\cdot e^t = te^t - e^t + C$

$(e^t)' = e^t$

② $\int \overset{1\cdot}{} \log t\,dt = \int (t)' \log t\,dt$

$= t\log t - \int t(\log t)'\,dt$

$= t\log t - \int \overbrace{t\cdot\frac{1}{t}}^{1}\,dt$

$= t\log t - t + C$

117 2p.

① $\int_0^1 t^2 e^t\,dt = \int_0^1 t^2 (e^t)'\,dt$

$= [t^2 e^t]_0^1 - \int_0^1 (t^2)' e^t\,dt$

$= e - \int_0^1 2t e^t\,dt = e - 2\int_0^1 t\,(e^t)'\,dt$

$= e - 2[te^t]_0^1 + 2\int_0^1 1\cdot e^t\,dt$ $\leftarrow (t)'$ $\qquad e = 2.7\cdots$

$= e - 2e + 2[e^t]_0^1 = -e + 2(e-1) = e - 2 > 0$

② $\int_1^e x^2 \log x\,dx = \int_1^e \left(\frac{x^3}{3}\right)' \log x\,dx$

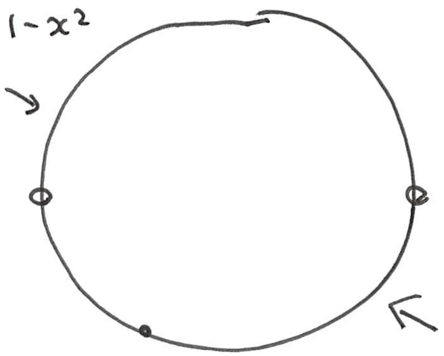
$= \left[\frac{x^3}{3}\log x\right]_1^e - \int_1^e \frac{x^3}{3}\cdot(\log x)'\,dx$ $\quad = \frac{1}{x}.$ $\qquad \log e = 1$, $\log 1 = 0.$

$= \frac{e^3}{3} - \frac{1}{3}\int_1^e x^2\,dx = \frac{e^3}{3} - \frac{1}{3}\left[\frac{x^3}{3}\right]_1^e$

$= \frac{e^3}{3} - \frac{1}{9}(e^3 - 1) = \frac{2e^3+1}{9}$

陰関数の2階微分

$\varphi = \sqrt{1-x^2}$



$g = x^2 + y^2 - 1 = 0$

$g_y = 2y \neq 0$

$\varphi = -\sqrt{1-x^2}$

$(x, \varphi(x))$

問

$\varphi'' = \cdots$

例

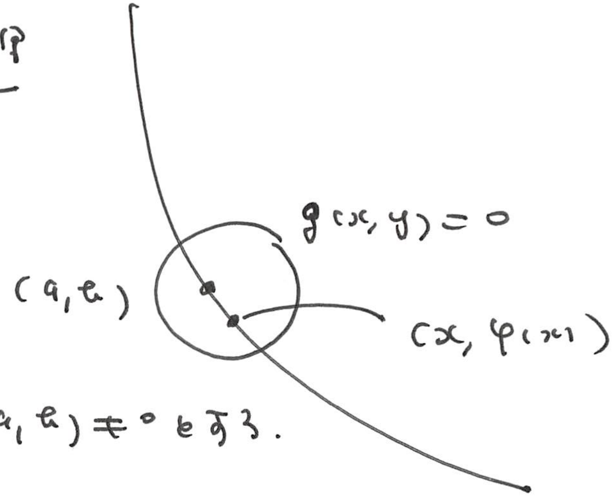
$$x^2 + \varphi(x)^2 - 1 \equiv 0$$

微分

$$2x + 2\varphi(x) \cdot \varphi'(x) \equiv 0$$

$$\varphi'(x) = -\frac{x}{\varphi(x)}.$$

定理



$g_y(a,b) \neq 0$ とする.

(a,b) の近くで

$g(x,y)=0$ は $y=\varphi(x)$ と解ける.

$u = \varphi(x)$

$$\frac{du^2}{dx} = \frac{du^2}{du} \cdot \frac{du}{dx} = 2u \cdot \varphi' = 2\varphi \cdot \varphi'.$$

もう1回微分.

Leibnitz rule

$(fg)' = f'g + fg'$

$$2 + 2(\varphi' \cdot \varphi' + \varphi \cdot \varphi'') \equiv 0$$

$$\varphi'' = -\frac{(\varphi')^2 + 1}{\varphi} = -\frac{\frac{x^2}{\varphi^2} + 1}{\varphi} = -\frac{x^2 + \varphi^2}{\varphi^3}$$

$$= -\frac{1}{\varphi^3}$$

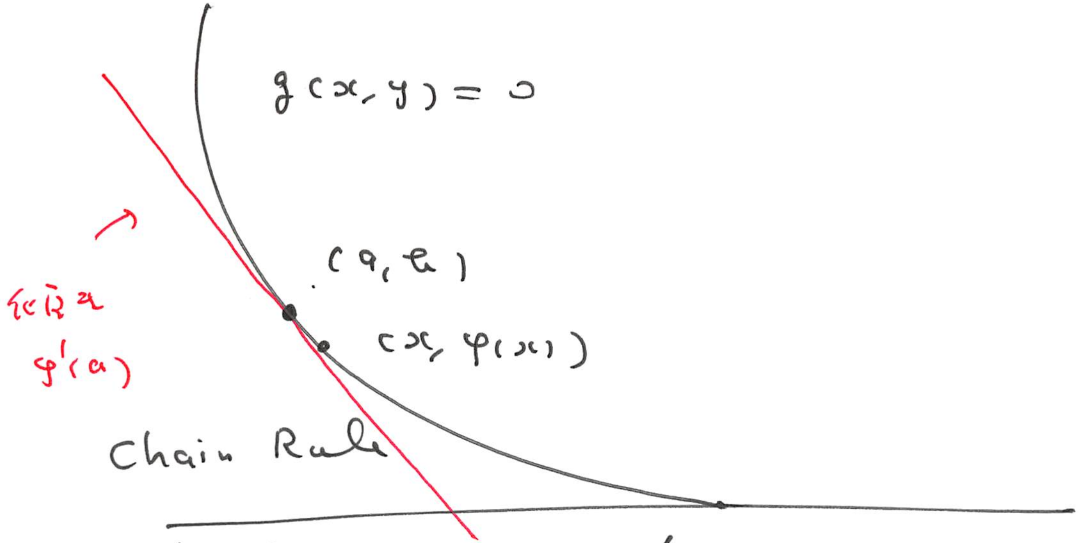
$g_y(a,b) \neq 0$ とする

(a,b) の近くで $g=0$ は

$$y = \varphi(x)$$

と書ける.

$$\longrightarrow \quad g(x, \varphi(x)) \equiv 0.$$



Chain Rule

$$\left(g(x(t), y(t))\right)' = g_x(x(t), y(t)) \cdot x'(t) + g_y(\quad) \cdot y'(t)$$

x で微分

$$g_x(x, \varphi(x)) \cdot 1 + g_y(x, \varphi(x)) \cdot \varphi'(x) \equiv 0$$

$$\longrightarrow \quad \varphi'(x) = -\frac{g_x(\quad)}{g_y(\quad)}$$

特に

$$\varphi'(a) = -\frac{g_x(a,b)}{g_y(a,b)}$$

$$g_{xx}(\ )\cdot 1 + g_{xy}(\ )\cdot \varphi' +$$

$$\varphi'\left(g_{yx}(\ )\cdot 1 + g_{yy}(\ )\varphi'\right) + g_y(\ )\varphi'' \equiv 0$$

$$\varphi'' = -\frac{1}{g_y}\left(g_{xx} + 2g_{xy}\varphi' + g_{yy}(\varphi')^2\right)$$

$$= -\frac{1}{g_y}\left(g_{xx} - 2g_{xy}\frac{g_x}{g_y} + g_{yy}\frac{g_x^2}{g_y^2}\right)$$

$$= -\frac{1}{g_y^3}\left(g_{xx}g_y^2 - 2g_{xy}g_xg_y + g_{yy}g_x^2\right)$$

$$= \frac{1}{g_y^3}\begin{vmatrix} 0 & g_x & g_y \\ g_x & g_{xx} & g_{xy} \\ g_y & g_{yx} & g_{yy} \end{vmatrix}$$

$$\begin{vmatrix} 0 & \alpha & \beta \\ \alpha & A & C \\ \beta & C & B \end{vmatrix} = -\left(A\beta^2 - 2C\alpha\beta + B\alpha^2\right)$$

(1) $\int_0^1 t e^{2t} dt$ $\qquad (e^{2t})' = 2e^{2t}$

(2) $\int_0^1 t^2 e^{-2t} dt$ $\qquad (e^{-2t})' = -2e^{2t}$

(3) $\int_1^{e^2} x \log x \, dx$ $\qquad \left(\frac{x^2}{2}\right)' = x$

(4) $\int_0^{\frac{\pi}{2}} \sin x \, dx$

(5) $\int_0^{\frac{\pi}{2}} x \sin x \, dx$ $\qquad \leftarrow (\cos x)' = -\sin x$