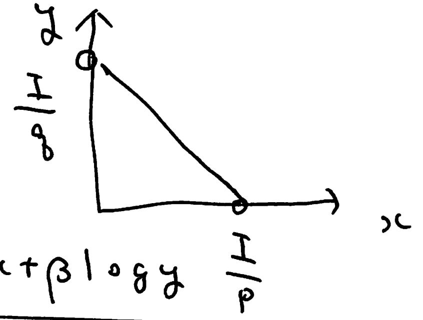
I　　$I,\ p,\ q > 0$,　$\alpha,\ \beta > 0$.

$g(x, y) = I - px - qy = 0$ の下で

$z = f(x, y) = \alpha \log x + \beta \log y$



(x, y) で 極大(小)

$$\Longrightarrow \begin{cases} f_x(x, y) + \lambda g_x(x, y) = 0 \\ f_y(x, y) + \lambda g_y(x, y) = 0 \\ g(x, y) = 0 \end{cases}$$

$$\begin{cases} \dfrac{\alpha}{x} + \lambda(-p) = 0 \quad \cdots ① \\ \dfrac{\beta}{y} + \lambda(-q) = 0 \quad \cdots ② \\ I - px - qy = 0 \quad \cdots ③ \end{cases}$$

$\lambda = 0$ とすると

$\frac{\alpha}{x} = 0$

から $\alpha = 0$ となるので impossible.

① から　$x = \dfrac{\alpha}{\lambda p}$,　$y = \dfrac{\beta}{\lambda q}$.

と ②

これらを ③ に 代入

$$0 = I - \frac{\alpha}{\lambda} - \frac{\beta}{\lambda} = I - \frac{\alpha + \beta}{\lambda}$$

よって　$\dfrac{1}{\lambda} = \dfrac{I}{\alpha + \beta}$

$\lambda = \dfrac{\alpha + \beta}{I}$

$$x = \frac{\alpha}{\alpha+\beta} \cdot \frac{I}{p}, \quad y = \frac{\beta}{\alpha+\beta} \cdot \frac{I}{q}$$

需要関数

無差別曲線 $\alpha \log x + \beta \log y = C$

$$= \log(x^{\alpha} y^{\beta})$$

$$\longrightarrow \quad x^{\alpha} y^{\beta} = e^{C}$$

$F(x, y) = x^{\alpha} y^{\beta}$ コブ・ダグラス型.

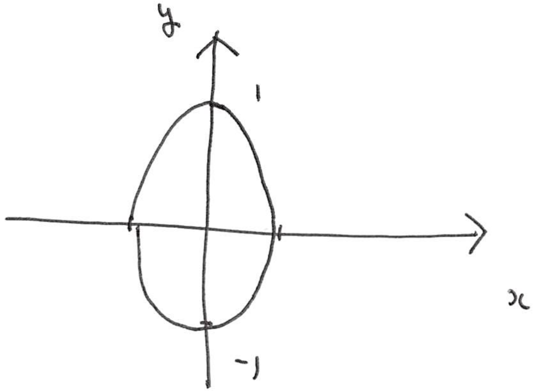
需要関数を計算すると一致する.

N.B. 無差別曲線の形が一致

II.

$g(x, y) = 2x^2 + y^2 - 1 = 0$ の下で

$f(x, y) = x^2 y$



$$\begin{cases} 2xy + \lambda \cdot 4x = 0 & \cdots ① \\ x^2 + \lambda \cdot 2y = 0 & \cdots ② \\ 2x^2 + y^2 - 1 = 0 & \cdots ③ \end{cases}$$

① ⇔ $x = 0$ OR $y + 2\lambda = 0$

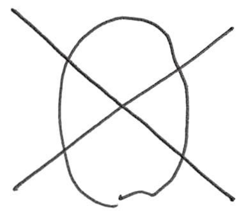
(i) $x = 0$ のとき.

③より $y = \pm 1$, ②に $x = 0$ を代入 $\lambda y = 0$, $y \neq 0$ より $\lambda = 0$

$(x, y, \lambda) = (0, \pm 1, 0)$

(ii) $y = -2\lambda$ のとき.

$2\lambda = -y$ を②に代入 $x^2 - y^2 = 0$



③に $x^2 = y^2$ を代入 $3x^2 = 1$ より $y = \pm\frac{1}{\sqrt{3}}$, $x = \pm\frac{1}{\sqrt{3}}$.

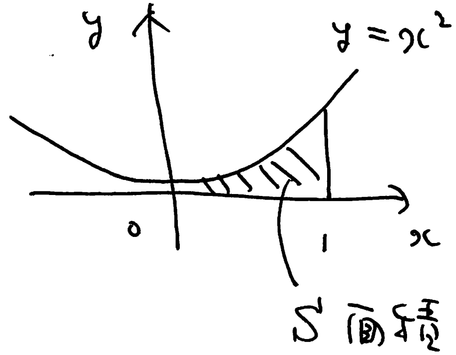
$\lambda = -\frac{y}{2} = \mp\frac{1}{2\sqrt{3}}$

$(x, y, \lambda) = \left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \mp\frac{1}{2\sqrt{3}}\right)$ yとλは複号同順

15.2 p.

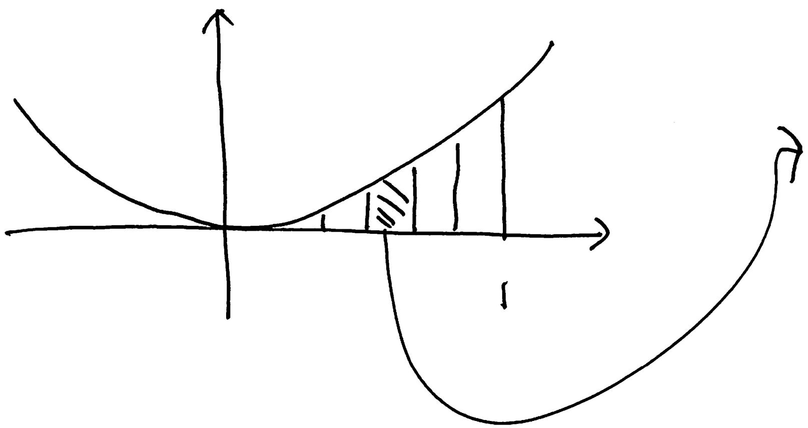
積分 $\int_0^1 x^2 dx. = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}.$

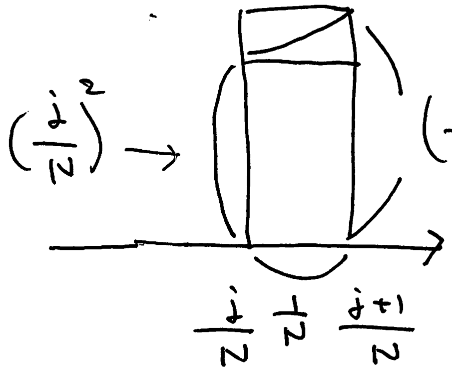
$$\left(\frac{x^3}{3}\right)' = x^2$$



正しい積分のイメージ.

[0, 1] を N 等分.





$$\sum_{j=0}^{N-1} \frac{1}{N}\left(\frac{j}{N}\right)^2 \leq S \leq \sum_{j=0}^{N-1} \frac{1}{N}\cdot\left(\frac{j+1}{N}\right)^2$$

$j+1=k$

$$= \frac{1}{N^3}\sum_{j=1}^{N-1} j^2$$

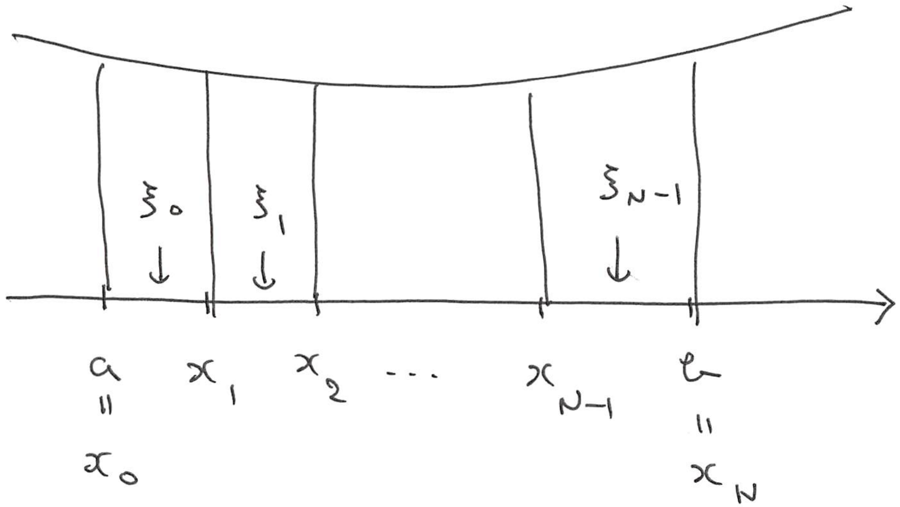
$$= \frac{1}{N^3}\cdot\frac{(N-1)N(2N-1)}{6}$$

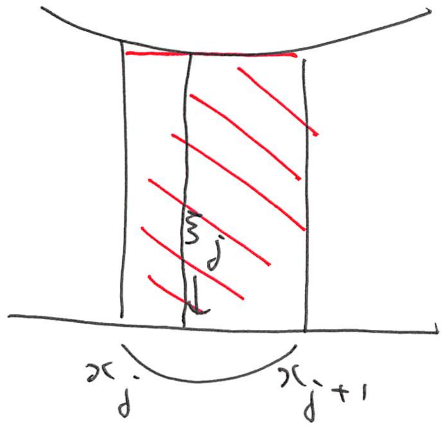
$$= \frac{1}{N^3}\sum_{k=1}^{N} k^2$$

$$= \frac{1}{N^3}\cdot\frac{N(N+1)(2N+1)}{6}$$

$$\underbrace{\frac{1}{6}\left(1-\frac{1}{N}\right)\left(2-\frac{1}{N}\right)}_{\substack{\downarrow \\ \frac{1}{3}}} \leq \underset{\substack{\parallel \\ \frac{1}{3}}}{S} \leq \underbrace{\frac{1}{6}\left(1+\frac{1}{N}\right)\left(2+\frac{1}{N}\right)}_{\substack{\downarrow \\ \frac{1}{3}}}$$

$$\boxed{\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta = ?}$$





$$\int_a^b f(x)\,dx \fallingdotseq \sum_{j=0}^{N-1} f(\xi_j)(x_{j+1}-x_j)$$

ある条件の下で

$$\downarrow \qquad\qquad \max_j (x_{j+1}-x_j) \to 0$$

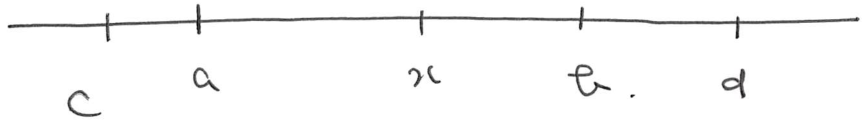
$$\int_a^b f(x)\,dx.$$

$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$. は何故.

$f : (c, d) \longrightarrow \mathbb{R}$

$F : (c, d) \longrightarrow \mathbb{R}$

$F' = f$ とする



$$\Longrightarrow \int_a^b f(t)\,dt = \left[F(t)\right]_a^b$$

微分積分学の基本定理　$f : (c, d) \longrightarrow \mathbb{R}$ 連続

$$F_0(x) = \int_a^x f(t)\,dt$$

$$\Longrightarrow F_0'(x) = f(x)$$

$(F_0 - F)' = f - f \equiv 0.$

$\boxed{\text{定理}\quad g : (c, d) \to \mathbb{R}\ \text{微分可能}\quad g' \equiv 0 \Rightarrow g \equiv C\ \text{(定数)}}$

定理を使うと $\quad F_0 - F \equiv C \quad$ (定数)

$F_0(a) = 0$ $\quad$ ↑ $x = a$ を代入 $\quad F_0(a) - F(a) = C$

$$F_0(a) = 0$$

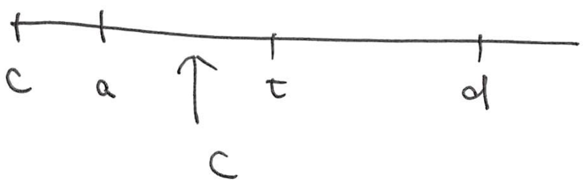
まとめ $\quad C = -F(a)$

$F_0(b) = \int_a^b f(x)\,dx.$ $\quad$ $x = b$ を代入

$$F_0(b) - F(b) = C$$

$$F_0(b) = F(b) + C = F(b) - F(a) = [F]_a^b$$

平均値の定理



$g(t) - g(a) = \overbrace{g'(c)}^{0}(t - a) \rightarrow g(t) = g(a)$

をみたす c が t と a の間に存在

169P.

① $\int_0^1 t^4 dt = \left[\frac{1}{5}t^5\right]_0^1 = \frac{1}{5}$

$(t^5)' = 5t^4$

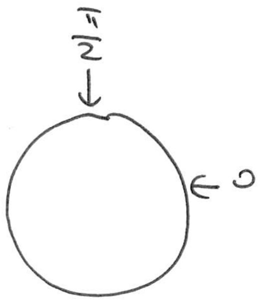
$\left(\frac{1}{5}t^5\right)' = t^4$

② $\int_0^{\frac{\pi}{2}} \sin t \, dt = [-\cos t]_0^{\frac{\pi}{2}}$

$= (-0) - (-1) = 1$

$(\sin t)' = \cos t$

$(\cos t)' = -\sin t$



$(-\cos t)' = \sin t$.

③ $\int_0^1 e^{2t} dt$

$\parallel$

$\left[\frac{1}{2}e^{2t}\right]_0^1 = \frac{1}{2}(e^2 - 1)$

$(e^{2t})' = 2e^{2t}$

$(e^{ct})' = ce^{ct}$

$\left(\frac{1}{2}e^{2t}\right)' = e^{2t}$

④ $\int_2^3 \frac{1}{t} dt = [\log t]_2^3$

$= \log 3 - \log 2 = \log \frac{3}{2}$

$(\log t)' = \frac{1}{t}$

$(t\sqrt{t})' = \left(t^{\frac{3}{2}}\right)' = \frac{3}{2}t^{\frac{1}{2}}$

⑤ $\int_0^1 \sqrt{t} \, dt = \left[\frac{2}{3}t\sqrt{t}\right]_0^1 = \frac{2}{3}$

$\left(\frac{2}{3}t\sqrt{t}\right)' = \sqrt{t}$

I (1) $\int_0^1 x^2\sqrt{x}\,dx$

(2) $\int_1^2 \frac{1}{\sqrt{x}}\,dx$

(3) $\int_0^1 e^{-2t}\,dt$

(4) $\int_1^2 \frac{1}{x^2}\,dx.$

(5) $\int_0^{\frac{\pi}{2}} \sin 2t\,dt$

$(\cos 2t)' = ?$

II. $p, q, I > 0$ とする $x, y > 0.$

$g(x, y) = I - px - qy = 0$

の下で

$z = f(x, y) = \sqrt[3]{xy}.$