CalcNT 2015/12/09.

$g(x, y) = x^2 + 2y^2 - 1 = 0$ の下で

$z = f(x, y) = x + y$

Lagrange の未定乗数法より (x, y) で停留点ならば

接線条件.

$$\begin{cases} \underbrace{\boxed{1}}_{\nabla(f)} + \lambda \cdot \underbrace{\boxed{2x}}_{\nabla(g)} = 0 & \cdots ① \\ \boxed{1} + \lambda \cdot \boxed{4y} = 0 & \cdots ② \\ x^2 + 2y^2 - 1 = 0 & \cdots ③ \end{cases}$$

$$\begin{cases} \boxed{f_x(x, y) + \lambda g_x(x, y) = 0 \\ f_y(x, y) + \lambda g_y(x, y) = 0} \\ g(x, y) = 0 \end{cases}$$

制約条件

① に $\lambda = 0$ を代入すると ① は $1 = 0$ となる $\lambda \neq 0$.

よって ①, ② より

$$x = -\frac{1}{2\lambda}, \quad y = -\frac{1}{4\lambda} \cdots \$$$

これを ③ に代入すると

$$\frac{1}{4\lambda^2} + \frac{1}{8\lambda^2} = 1$$

$$= \frac{1}{\lambda^2}\left(\frac{1}{4} + \frac{1}{8}\right) = \frac{1}{\lambda^2} \cdot \frac{3}{8}$$

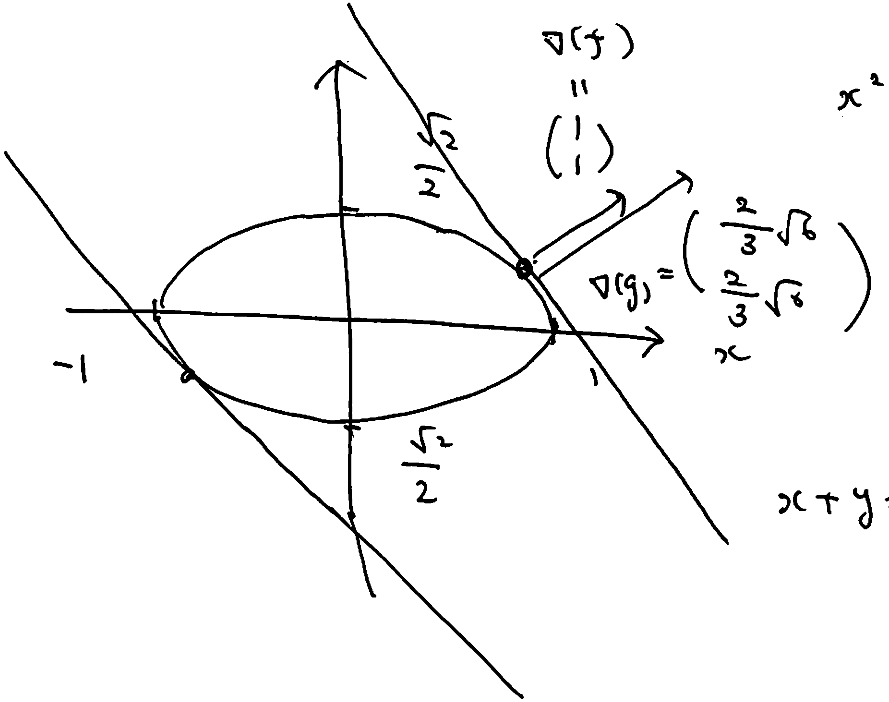
よって $\frac{1}{\lambda} = \pm\sqrt{\frac{8}{3}} = \pm\frac{2\sqrt{2}}{\sqrt{3}} \cdots \#$

$ に代入すると

$$x = \mp\frac{\sqrt{2}}{\sqrt{3}} = \mp\frac{\sqrt{6}}{3}$$

$$y = \mp\frac{\sqrt{2}}{2\sqrt{3}} = \mp\frac{\sqrt{6}}{6}$$

$$\lambda = \pm\frac{\sqrt{3}}{2\sqrt{2}} = \pm\frac{\sqrt{6}}{4}$$



$$x^2 + 2y^2 = 1$$

$$\nabla(f) + \lambda \nabla(g) = 0$$

$$f = x + y$$

$$\nabla(f) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x + y = \frac{\sqrt{6}}{2}$$

$$\nabla(g) = \begin{pmatrix} 2x \\ 4y. \end{pmatrix}$$

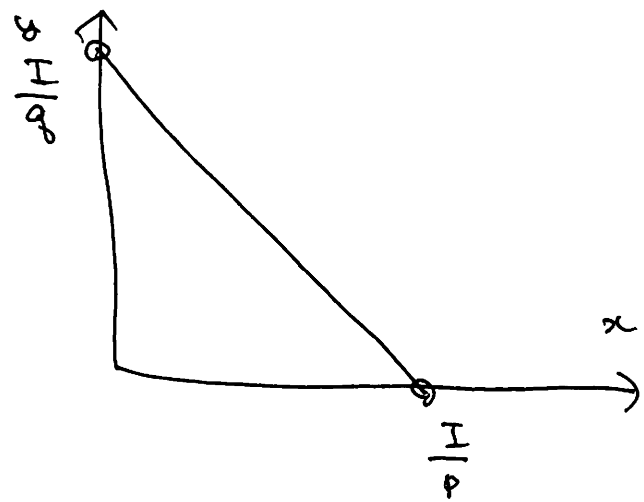
ミクロ経済学 効用最大化. goods

	第1財	第2財		効用.
購入量	x	y	$\longrightarrow$	$u(x,y)$
価格	p	q		$I, p, q, x, y > 0$

$g(x,y) = I - px - qy = 0$

の下で

$u(x,y)$ を最大化.



未定乗数法

$$\begin{cases} u_x(x,y) + \lambda\cdot(-p) = 0 \\ u_y(x,y) + \lambda(-q) = 0 \\ I - px - qy = 0 \end{cases}$$

を満たす λ が存在

$u(x,y) = Cx^{\alpha}y^{\beta}$

$\alpha, \beta > 0$, $C > 0$

Cobb-Douglass 型

$u(x,y) = \sqrt{xy}$

$$\begin{cases} \dfrac{1}{2}\cdot\dfrac{\sqrt{y}}{\sqrt{x}} + \lambda(-p) = 0 \\ \dfrac{1}{2}\cdot\dfrac{\sqrt{x}}{\sqrt{y}} + \lambda(-q) = 0 \\ I - px - qy = 0 \quad \cdots ③ \end{cases}$$

$$(\sqrt{t})' = \frac{1}{2}\cdot\frac{1}{\sqrt{t}}$$

$$\longrightarrow \begin{cases} \dfrac{\sqrt{y}}{\sqrt{x}} = 2\lambda p \quad \cdots ① \\ \dfrac{\sqrt{y}}{\sqrt{x}} = 2\lambda q \quad \cdots ② \end{cases}$$

$$\lambda > 0$$

①にくで×

$$\longrightarrow 1 = 4\lambda^2 pq$$

$$\longrightarrow \lambda = \frac{1}{2\sqrt{pq}}$$

$$\frac{\sqrt{y}}{\sqrt{x}} = 2\cdot\frac{1}{2\sqrt{pq}}\cdot p$$

$$= \sqrt{\frac{p}{q}} \rightsquigarrow \frac{y}{x} = \frac{p}{q} \rightsquigarrow yq = px.$$

③と併せ $\quad yq = xp = \dfrac{I}{2} \rightsquigarrow x = \dfrac{I}{2p},\ y = \dfrac{I}{2q}$ 需要関数.

$$\lambda = \frac{1}{2\sqrt{pq}}$$

所得の限界効用と呼ぶ

$$v(p, q, I) = u(x(p, q, I), y(p, q, I))$$ 間接効用関数

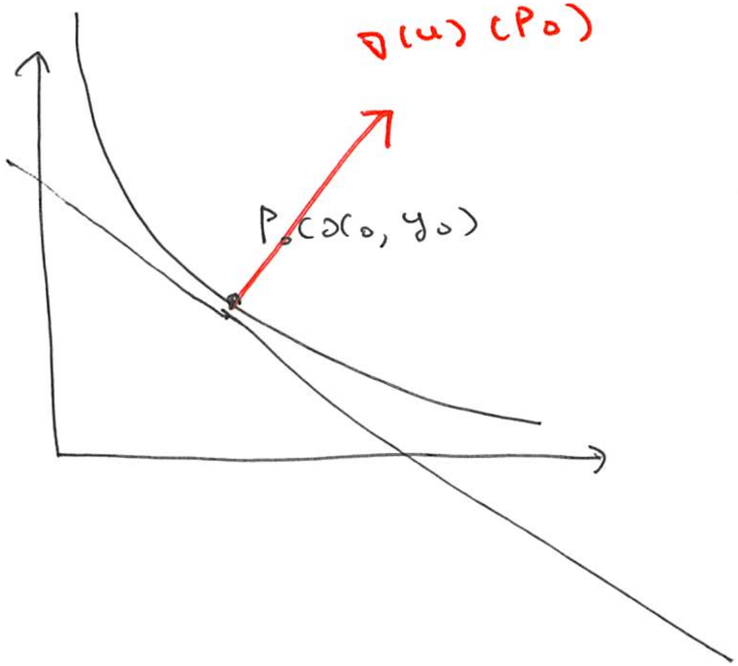
$$= \sqrt{\frac{I}{2p} \cdot \frac{I}{2q}} = \frac{I}{2\sqrt{pq}}$$

$$\frac{\partial v}{\partial I} = \frac{1}{2\sqrt{pq}} = \lambda$$

$$\boxed{\text{一般的に} \quad \frac{\partial v}{\partial I} = \lambda}$$

包絡線定理.

無差別曲線



$$u(x, y) = C$$

例 $u(x, y) = \sqrt{xy} = C$

$$xy = C^2 \to y = \frac{C^2}{x}$$

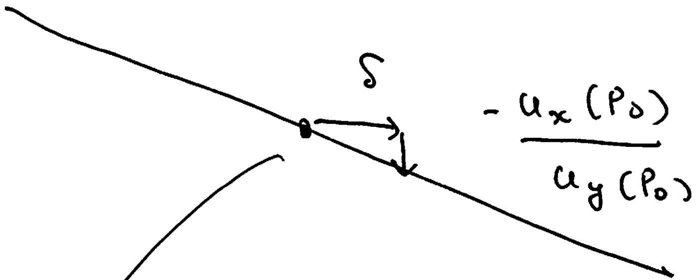
$$u_x(P_0)(x - x_0) + u_y(P_0)(y - y_0) = 0$$

$$\downarrow$$

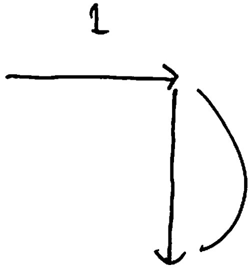
$$y = -\frac{u_x(P_0)}{u_y(P_0)}(x - x_0) + y_0$$

傾き $-\frac{u_x(P_0)}{u_y(P_0)}$

限界的 = 接線で考える　　効用を保つ形で x を増やす.



$$-\frac{u_x(P_0)}{u_y(P_0)}\cdot\delta$$



$$-\frac{u_x(P_0)}{u_y(P_0)}$$

y を減らす量 $\dfrac{u_x(P_0)}{u_y(P_0)}\cdot\delta \longrightarrow$ 率で考えると $\dfrac{u_x(P_0)}{u_y(P_0)}$

限界代替率 (RMS)

Rate of Marginal Substitution

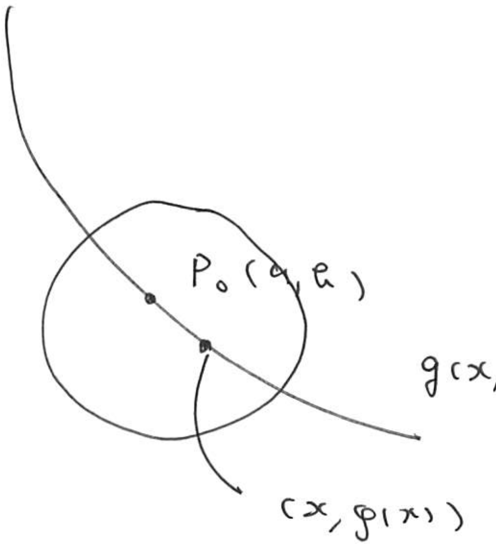
合理的な消費者が選択

$I - px - qy = 0$ の下で u が

ここで 最大化されているとする.

$$\begin{cases} u_x(P_0) + \lambda(-p) = 0 \\ u_y(P_0) + \lambda(-q) = 0 \\ I - px - qy = 0 \end{cases}$$

$$= \frac{\lambda p}{\lambda q} = \frac{p}{q}$$



$g_y(a, b) \neq 0$

$g(x, y) = 0$ は (a, b) の近くで

$y = \varphi(x)$ と書ける.

$g(x, \varphi(x)) \equiv 0$

$$\boxed{\begin{aligned} &\text{chain Rule} \\ &F(t) = f(x(t), y(t)) \\ &F'(t) = f_x(\ )\cdot x'(t) \\ &\qquad\quad + f_y(\ )\cdot y'(t) \end{aligned}}$$

$$g_x(x, \varphi(x))\cdot 1 + g_y(x, \varphi(x))\,\varphi'(x) \equiv 0$$

$$\longrightarrow \boxed{\varphi'(x) = -\frac{g_x(\ )}{g_y(\ )}}$$

$$g_{xx}(\ )\cdot 1 + g_{xy}(\ )\,\varphi' + \varphi'(x)\left(g_{yx}(\ )\cdot 1 + g_{yy}(\ )\,\varphi'(x)\right)$$

$$+ g_y(\ )\cdot \varphi'' \equiv 0$$

$g_y \neq 0$ より

$$\varphi'' = -\frac{1}{g_y(\ )}\left(g_{xx}(\ ) + 2 g_{xy}(\ )\varphi' + g_{yy}(\varphi')^2 \right)$$

$$= -\frac{1}{g_y}\left(g_{xx} - 2 g_{xy}\frac{g_x}{g_y} + g_{yy}\frac{g_x^2}{g_y^2} \right)$$

$$= -\frac{1}{g_y^3}\left(g_{xx} g_y^2 - 2 g_{xy} g_x g_y + g_{yy} g_x^2 \right)$$

$$= \frac{1}{g_y^3}\begin{vmatrix} 0 & g_x & g_y \\ g_x & g_{xx} & g_{xy} \\ g_y & g_{yx} & g_{yy} \end{vmatrix}$$

$$\begin{vmatrix} 0 & p & q \\ p & A & C \\ q & C & B \end{vmatrix} = 2Cpq - Aq^2 - Bp^2$$

← ふちつき行列式

ヘッシアン

I $\quad I, p, q > 0 \quad \alpha, \beta > 0.$

$$g(x, y) = I - px - qy = 0$$

の下で $\quad u(x, y) = \alpha \log x + \beta \log y$

II. $\quad g(x, y) = 2x^2 + y^2 - 1 = 0$ の下で

$$f(x, y) = x^2 y.$$