I $x^2 - xy + y^2 - 1 = 0$ の $(0,1)$ における接線.

Calc NT 2015/10/21

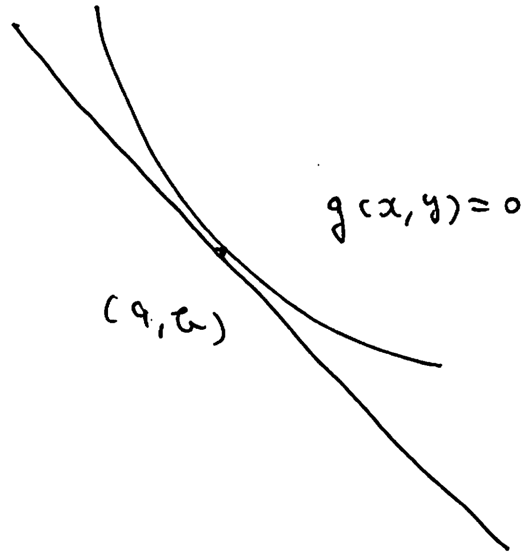
曲線

$$g(x,y) = 0$$

の上の点 (a,b) における接線

$$g_x(a,b)(x-a) + g_y(a,b)(y-b) = 0$$

$$g(x,y) = x^2 - xy + y^2 - 1$$



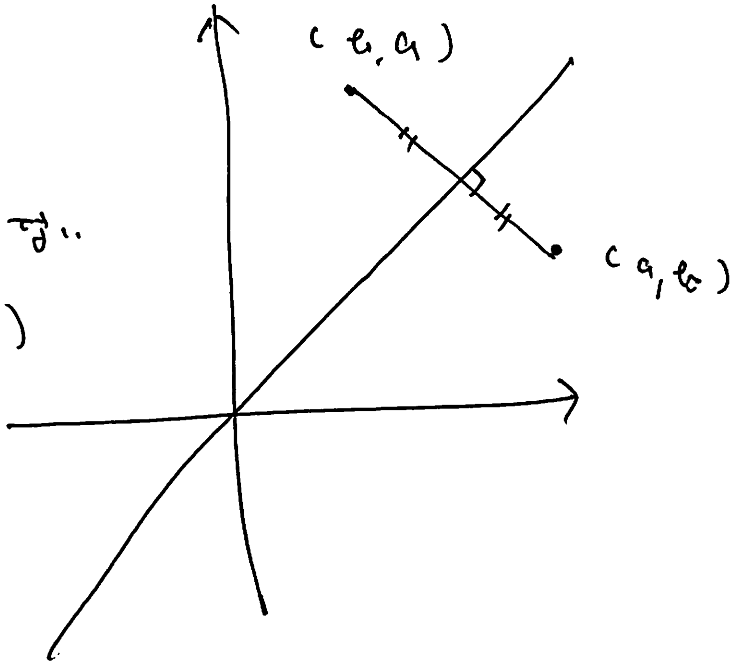
$$\begin{cases} g_x = 2x - y \\ g_y = -x + 2y. \end{cases}$$

$$g_x(0,1) = -1,\ g_y(0,1) = 2$$

より

$$-x + 2(y-1) = 0$$

$$g(x, y) = x^2 - xy + y^2 - 1 = 0$$



45°線 $y = x$ で折り返しても変わらない

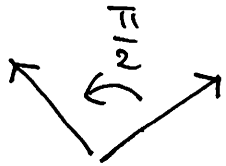
（45°線について対称）

$g(a, b) = a^2 - ab + b^2 - 1 = 0$ とする

$$g(b, a) = b^2 - ba + a^2 - 1$$
$$= a^2 - ab + b^2 - 1 = 0$$

座標変換.

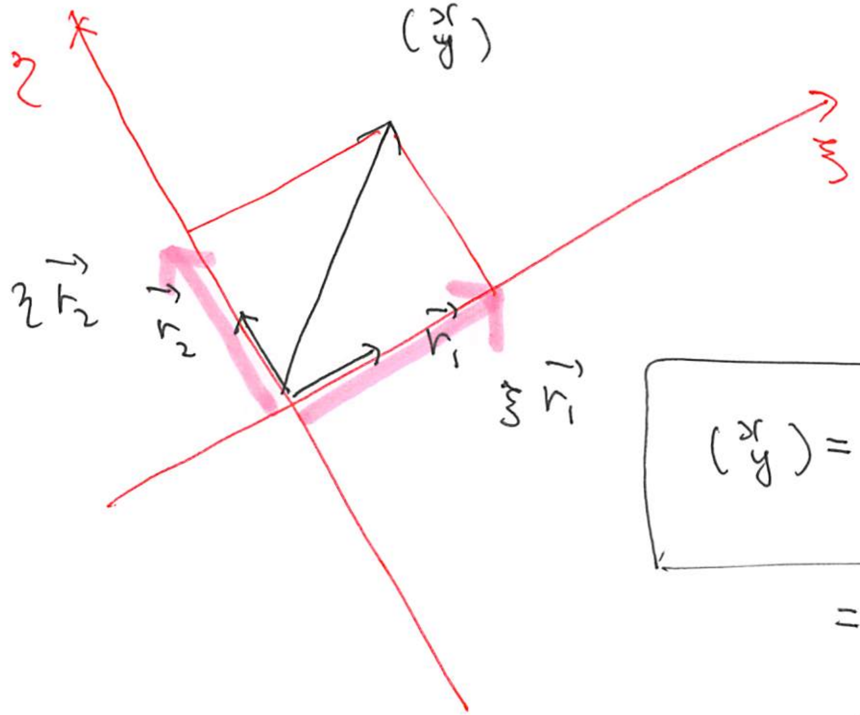
$$\vec{r}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

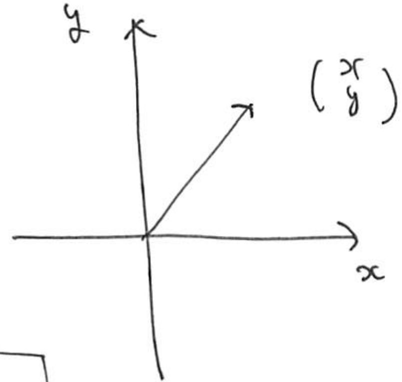


$$\vec{r}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$R = (\vec{r}_1\ \vec{r}_2)$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$$

回転行列.





$$\boxed{\begin{pmatrix}x\\y\end{pmatrix} = \xi\,\vec{r}_1 + \eta\,\vec{r}_2}$$

$$= \left(\vec{r}_1\ \ \vec{r}_2\right)\begin{pmatrix}\xi\\\eta\end{pmatrix}$$

$$= R\begin{pmatrix}\xi\\\eta\end{pmatrix}$$

$$\boxed{\begin{pmatrix}x\\y\end{pmatrix} = R\begin{pmatrix}\xi\\\eta\end{pmatrix}}$$

相補的.

$$\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}\xi\\\eta\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\xi-\eta\\\xi+\eta\end{pmatrix}$$

ξ-η 座標で $g(x,y) = x^2 - xy + y^2 - 1 = 0$ はどう表せるか?

$x^2 - xy + y^2 = (x+y)^2 - 3xy.$

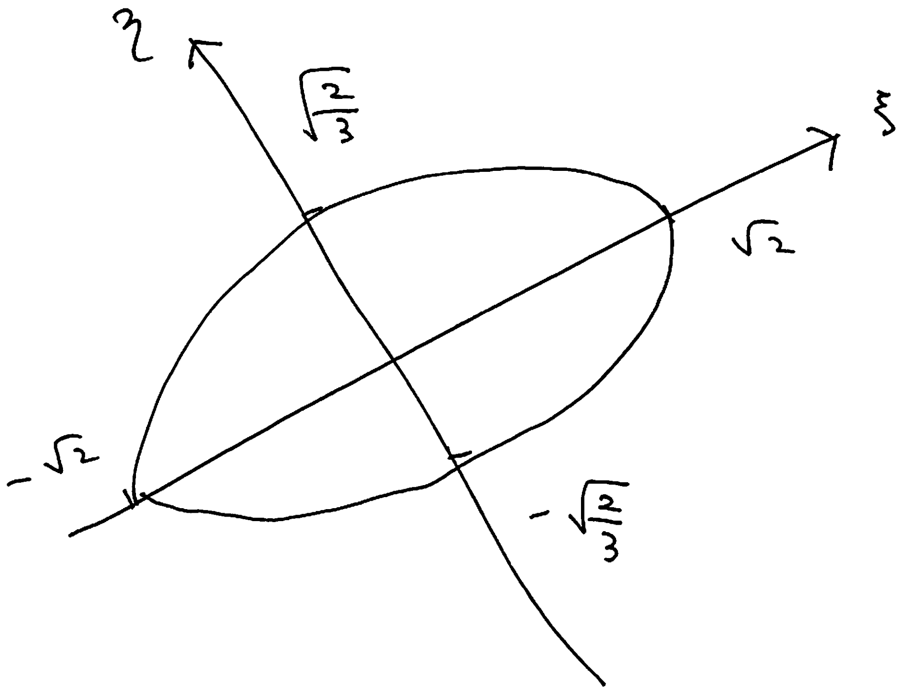
$x + y = \frac{1}{\sqrt{2}} \cdot 2\xi = \sqrt{2}\xi$

$xy = \frac{1}{2}(\xi^2 - \eta^2)$

$= (\sqrt{2}\xi)^2 - \frac{3}{2}(\xi^2 - \eta^2)$

$= \frac{1}{2}\xi^2 + \frac{3}{2}\eta^2$

$\longrightarrow \quad \frac{1}{2}\xi^2 + \frac{3}{2}\eta^2 - 1 = 0$



$\eta = 0$ のとき

$\xi^2 = 2$

$\xi = \pm\sqrt{2}$

$\xi = 0$ のとき.

$\eta^2 = \frac{2}{3}$

$\eta = \pm\sqrt{\frac{2}{3}}$

II $z = x^2 - 3xy + y^2 - 1$ の上の点 $(1, 0, 0)$ における接平面

曲面

$z = f(x, y)$ 上の点 $(a, b, f(a, b))$ における接平面.

$$z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$$

$z_x = 2x - 3y$, $z_y = -3x + 2y$

$z_x(1, 0) = 2$, $z_y(1, 0) = -3$ より

$$\boxed{z = 2(x - 1) - 3y}$$

問

45°線の座標 (ξ, η) でどう表わせるか?

135° =

$$z = (x + y)^2 - 5xy - 1 = \cdots$$

Ⅲ $z = x^4 + y^4 + 2x^2 - 4xy + 2y^2$ の停留点

$$\begin{cases} z_x = 4x^3 + 4x - 4y = 0 & \cdots ① \\ z_y = 4y^3 - 4x + 4y = 0 & \cdots ② \end{cases}$$

$$\Longleftrightarrow \begin{cases} x^3 + x - y = 0 & \cdots ①' \\ y^3 - x + y = 0 & \cdots ②' \end{cases}$$

$P \Rightarrow Q$ が成立してる

$\boxed{P \Longleftrightarrow P \text{ and } Q}$

①'+②' より $x^3 + y^3 = 0$ が必要.

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

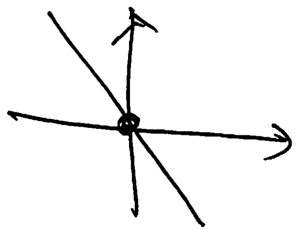
$\boxed{\begin{aligned} &A \cdot B = 0 \\ \Longleftrightarrow\ &A = 0 \text{ or } B = 0 \end{aligned}}$

$x^3 + y^3 = 0 \Longleftrightarrow x + y = 0$ OR $\boxed{x^2 - xy + y^2 = 0.}$

$x^2 - xy + y^2 = (x - \frac{1}{2}y)^2 + \frac{3}{4}y^2$
$\quad \| $
$\quad 0$ とすると

$\boxed{\begin{aligned} &A, B \geqq 0 \text{ とすると} \\ &A + B = 0 \Longrightarrow A = B = 0 \end{aligned}}$

$\leadsto x - \frac{1}{2}y = y = 0$ より $x = y = 0$.



$\Longleftrightarrow x + y = 0.$

$x+y=0$ のとき ①' と ②' は $y=-x$ を代入すると.

$$x^3+2x=-x^3-2x=0$$

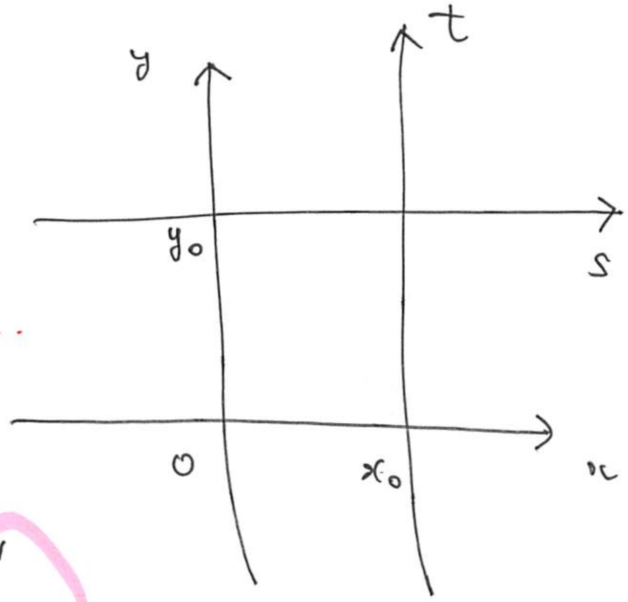
で $x^3+2x=x(x^2+2)$ において $x^2+2>0$ より $x=0$.

$y=-x=0$ より $(x,y)=(0,0)$

例 $z=x^4+y^4-2x^2+4xy-2y^2$ の停留点

IV $\quad f(x,y) = x^2 + 4xy + 2y^2 - 6x - 8y.$

$$\begin{cases} s = x - x_0 \\ t = y - y_0 \end{cases}$$



1次が消えるように。

$$x^2 + 4xy + 2y^2 - \underline{6x - 8y}$$

$= (x-x_0)^2 + 4(x-x_0)(y-y_0) + 2(y-y_0)^2 + f'$

両辺を x, y で偏微分

$$((t-a)^n)' = n(t-a)^{n-1}$$

$$\begin{cases} 2x + 4y - 6 = 2(x-x_0) + 4(y-y_0) \\ 4x + 4y - 8 = 4(x-x_0) + 4(y-y_0) \end{cases}$$

$$\longrightarrow \begin{cases} 2x_0 + 4y_0 = 6 \\ 4x_0 + 4y_0 = 8 \end{cases} \longrightarrow x_0 = y_0 = 1.$$

に $x = x_0 = 1$, $y = y_0 = 1$ を代入 $-7 = f'$

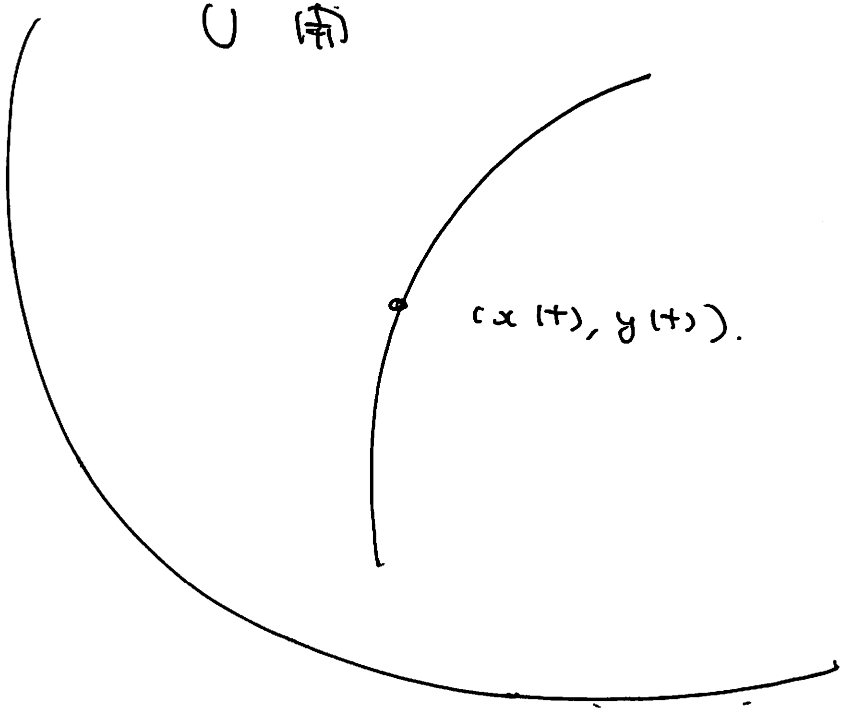
$z = f(x, y)$ は　$z = s^2 + 4st + 2t^2 - 7$ とする

$$\Big\downarrow \text{回転座標変換}$$

$$z = *_1 X^2 + *_2 Y^2 - 7.$$

Chain Rule　CT275p〜.

パラメーター表示された曲線



$$f : U \longrightarrow \mathbb{R}.$$

$$(x(t), y(t)) \in U$$

$$(t_0 < t < t_1)$$

$$F(t) = f(x(t), y(t))$$

Chain Rule

$$F'(t) = f_x(x(t), y(t))\, x'(t) + f_y(x(t), y(t))\, y'(t).$$

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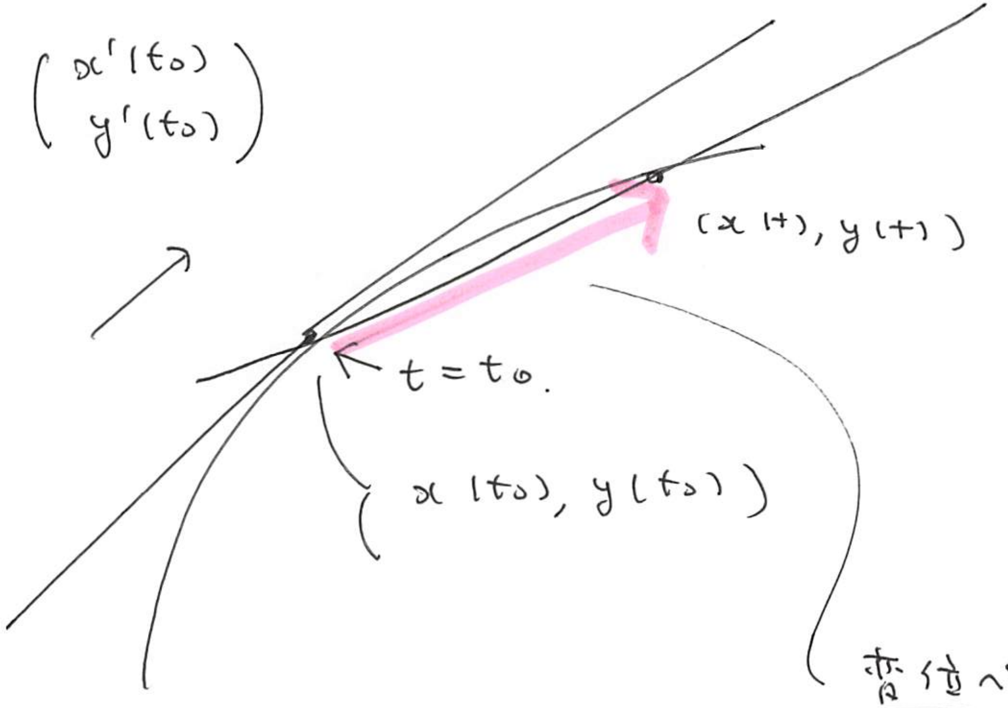
f 1変数のとき

$$(f(x(t)))' = f'(x(t)) \cdot x'(t).$$

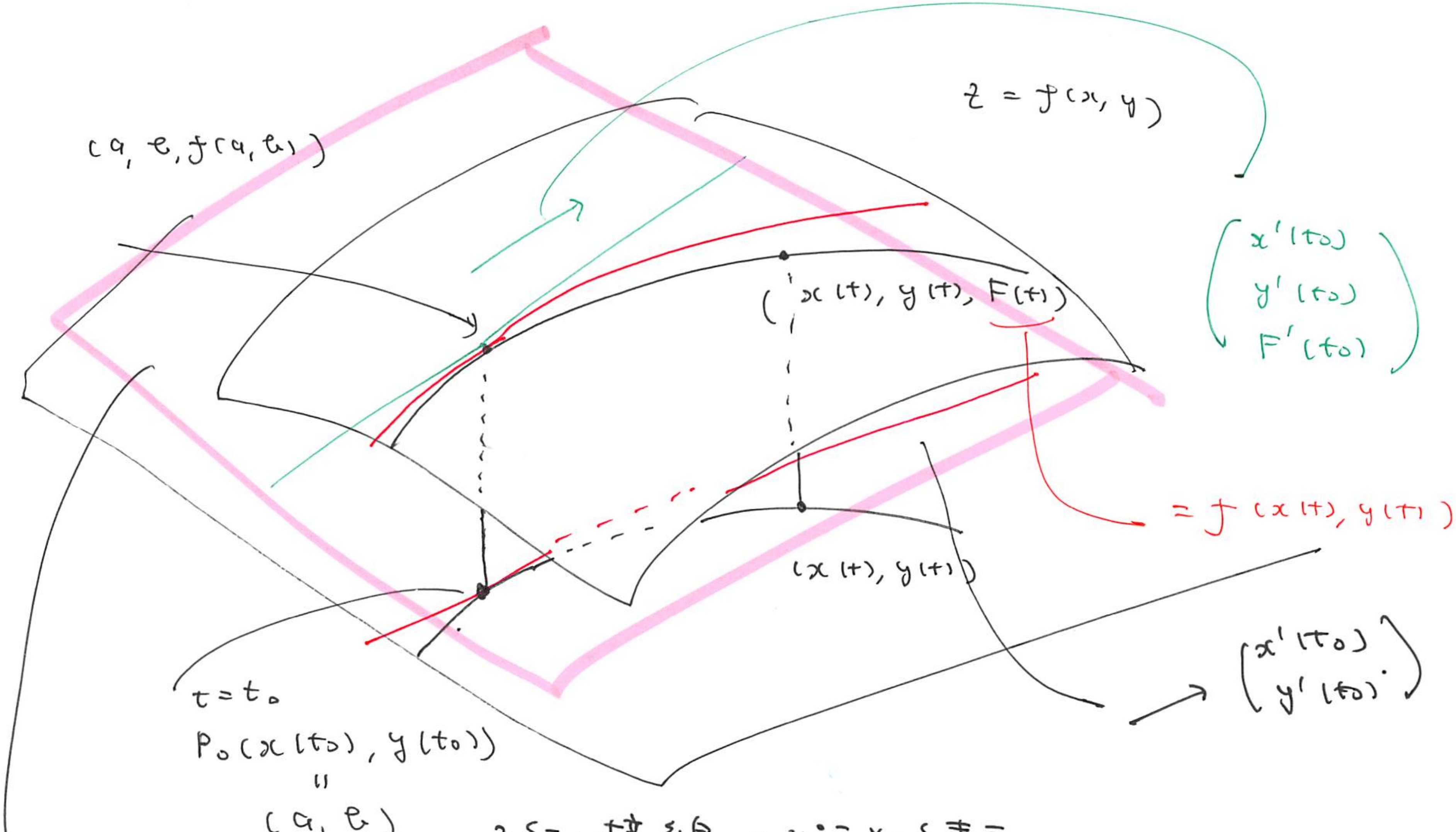
合成関数の微分



$$\underbrace{\frac{1}{t-t_0}\overbrace{\begin{pmatrix}x(t)-x(t_0)\\ y(t)-y(t_0)\end{pmatrix}}^{\text{変位ベクトル}}}_{\text{速度ベクトル}}$$

速度ベクトル
単位時間あたりの変化

$$= \begin{pmatrix}\dfrac{x(t)-x(t_0)}{t-t_0}\\[2ex] \dfrac{y(t)-y(t_0)}{t-t_0}\end{pmatrix} \longrightarrow \begin{pmatrix}x'(t_0)\\ y'(t_0)\end{pmatrix}$$



3次元の接線のパラメータ表示.

$$z = f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

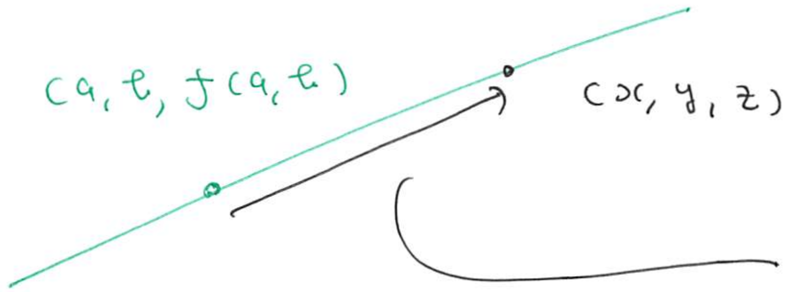
$$x = a + x'(t_0)(t-t_0)$$

$$y = b + y'(t_0)(t-t_0)$$

$$z = f(a,b) + x'(t_0)(t-t_0) + y'(t_0)(t-t_0)$$

$f_x(a,b)$

$f_y(a,b)$



$$\begin{pmatrix} x-a \\ y-b \\ z-f(a,b) \end{pmatrix}$$

$$\begin{pmatrix} x'(t_0) \\ y'(t_0) \\ F'(t_0) \end{pmatrix} \qquad = (t-t_0)\boxed{\begin{pmatrix} x'(t_0) \\ y'(t_0) \\ f_x(a,b)\,x'(t_0) + f_y(a,b)\,y'(t_0) \end{pmatrix}}$$

は $\begin{pmatrix} x'(t_0) \\ y'(t_0) \\ F'(t_0) \end{pmatrix}$ に平行

$$\longrightarrow \quad F'(t_0) = f_x(a,b)\,x'(t_0) + f_y(a,b)\,y'(t_0)$$

$$z = x^3 + y^3 - 9xy + 27$$

の極値点を求めよ.