$z = x^2 - xy + y^2$ の $(1, 1, 1)$ における接平面.

$z = f(x, y)$ の $(a, b, f(a, b))$ における接平面.

$$z = f_x(a, b)(x - a) + f_y(a, b)(y - b) + f(a, b)$$

$$z_x = 2x - y \cdot 1 + 0$$
$$= 2x - y.$$

$$z_y = 0 - x \cdot 1 + 2y$$
$$= -x + 2y$$

$\rightsquigarrow$ $z_x(1, 1) = 1$

$z_y(1, 1) = 1$

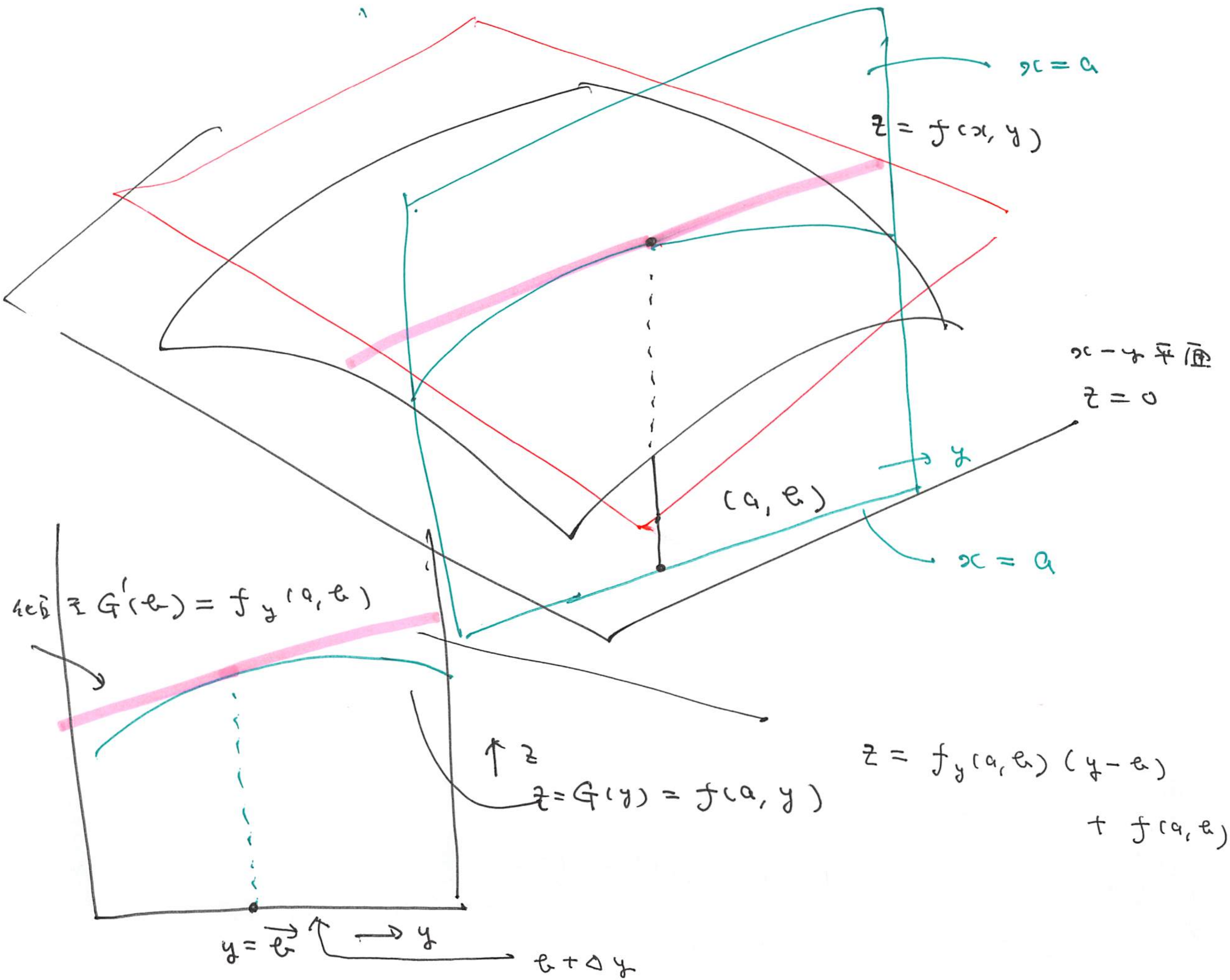
より 接平面は

$$z = 1 \cdot (x - 1) + 1 \cdot (y - 1) + 1$$

すなわち

$$z = (x - 1) + (y - 1) + 1$$

$\rightsquigarrow$ $z = x + y - 1$



x = a
z = f(x, y)
x − y 平面
z = 0
y
(a, b)
x = a
傾き G'(b) = f_y(a, b)
z
z = G(y) = f(a, y)
z = f_y(a, b)(y − b) + f(a, b)
y = b
y
b + Δy

$$z = A(x-a) + B(y-b) + f(a,b)$$

接平面

$$\xrightarrow[x=a\text{ の断面}]{} \quad z = B(y-b) + f(a,b)$$

$$B = g'(b) = f_y(a,b)$$

$$f(a, b+\Delta y) \fallingdotseq f_y(a,b)\Delta y + f(a,b)$$

行列の転置

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \left(\vec{a}_1 \ \vec{a}_2 \right) = \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{pmatrix}$$

列ベクトル　　　行ベクトル

$$A \begin{pmatrix} x \\ y \end{pmatrix} = x \vec{a}_1 + y \vec{a}_2 = x \begin{pmatrix} a \\ c \end{pmatrix} + y \begin{pmatrix} b \\ d \end{pmatrix}$$

$$= \begin{pmatrix} xa + yb \\ xc + yd \end{pmatrix} = \begin{pmatrix} (a \ b) \begin{pmatrix} x \\ y \end{pmatrix} \\ (c \ d) \begin{pmatrix} x \\ y \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \boldsymbol{a}_1 \begin{pmatrix} x \\ y \end{pmatrix} \\ \boldsymbol{a}_2 \begin{pmatrix} x \\ y \end{pmatrix} \end{pmatrix}$$

$$\underbrace{(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n)}_{{}^t\vec{\alpha}} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} = \alpha_1 \beta_1 + \alpha_2 \beta_2 + \cdots + \alpha_n \beta_n$$

$$= \left(\underbrace{\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}}_{\vec{\alpha}} , \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \right)$$

$$(\vec{\alpha}, \vec{\beta}) = {}^t\vec{\alpha} \, \vec{\beta}$$

まとめ　$A = (\vec{a}_1 \ \vec{a}_2) = \begin{pmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \end{pmatrix}$　2×2,　$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

$$A \vec{v} = x \vec{a}_1 + y \vec{a}_2 = \begin{pmatrix} \boldsymbol{a}_1 \vec{v} \\ \boldsymbol{a}_2 \vec{v} \end{pmatrix} \in \mathbb{R}^2$$

転置

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = (\vec{a}_1 \ \vec{a}_2) = \begin{pmatrix} a'_1 \\ a'_2 \end{pmatrix}$$

$${}^tA = \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} {}^ta'_1 & {}^ta'_2 \end{pmatrix} = \begin{pmatrix} {}^t\vec{a}_1 \\ {}^t\vec{a}_2 \end{pmatrix}$$

transposition of A

定理 $\vec{v} \in \mathbb{R}^2 \ (A\vec{v} \in \mathbb{R}^2), \ \vec{w} \in \mathbb{R}^2$

$$(A\vec{v}, \vec{w}) = (\vec{v}, {}^tA\vec{w})$$

$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

(証明) 左 $= (x\vec{a}_1 + y\vec{a}_2, \vec{w})$

$= x(\vec{a}_1, \vec{w}) + y(\vec{a}_2, \vec{w})$

${}^t\vec{a}_1 = (a \ c)$

${}^t\vec{a}_2 = (b \ d)$

$= x \, {}^t\vec{a}_1 \vec{w} + y \, {}^t\vec{a}_2 \vec{w}$

$= \left(\begin{pmatrix} x \\ y \end{pmatrix}, \boxed{\begin{pmatrix} {}^t\vec{a}_1 \vec{w} \\ {}^t\vec{a}_2 \vec{w} \end{pmatrix}} \right)$

$= \begin{pmatrix} {}^t\vec{a}_1 \\ {}^t\vec{a}_2 \end{pmatrix} \vec{w}$

$= {}^tA\vec{w}$

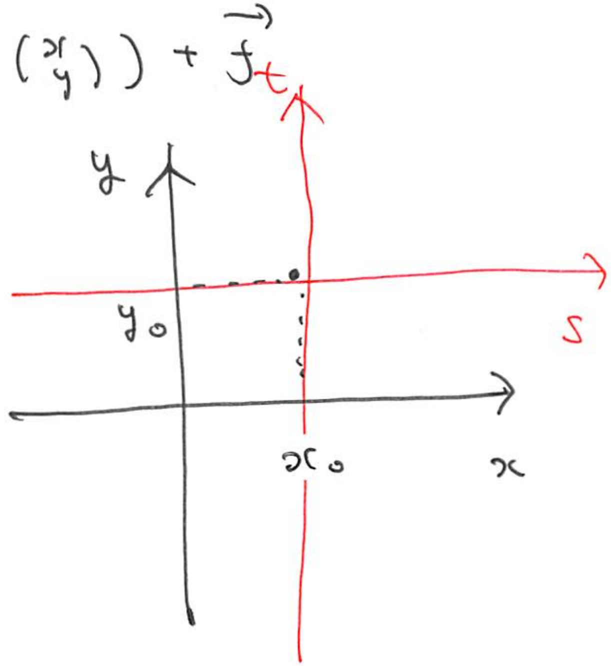
$= (\vec{v}, {}^tA\vec{w}) =$ 右

$A = \begin{pmatrix} a & c \\ c & e \end{pmatrix}$ 対称行列　　$\vec{\beta} = \begin{pmatrix} d \\ e \end{pmatrix}$

$$f(x, y) = ax^2 + 2cxy + ey^2 + dx + ey + f$$

$$= \left(A \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}\right) + \left(\vec{\beta}, \begin{pmatrix} x \\ y \end{pmatrix}\right) + \vec{f}$$

A の 2次形式.

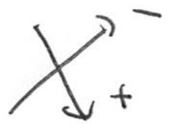


座標変換　　$\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

$$\vec{w} = \vec{v} - \vec{v_0}$$

$$\vec{w} = \begin{pmatrix} s \\ t \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \vec{v_0} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

以下 $|A| = ae - c^2 \neq 0$ とする.



$f(x, y) = (A\vec{v}, \vec{v}) + (\vec{\beta}, \vec{v}) + f.$

$= (A(\vec{w} + \vec{v_0}), \vec{w} + \vec{v_0}) + (\vec{\beta}, \vec{w} + \vec{v_0}) + f$

$\vec{v} = \vec{w} + \vec{v_0}$
を代入

と 各回.

$\boxed{{}^tA \text{ が 必.要}}$

$= (A\vec{w}, \vec{w}) + \boxed{2(A\vec{v_0}, \vec{w}) + (\vec{\beta}, \vec{w})}$
$+ (A\vec{v_0}, \vec{v_0}) + (\vec{\beta}, \vec{v_0}) + f.$

$= (\underbrace{2A\vec{v_0} + \vec{\beta}}_{= \binom{*_1}{*_2}}, \underbrace{\vec{w}}_{= \binom{s}{t}}) = *_1 s + *_2 t.$

$=$ 定数
s, t に依らない

$= a s^2 + 2 s t + c t^2$

$\vec{v_0} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ を $2A\vec{v_0} + \vec{\beta} = \vec{0}$ となるように定める.

$\vec{v_0} = -\frac{1}{2} A^{-1} \vec{\beta}$ とする

まとめ $\vec{v_0} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ を $\vec{v_0} = -\frac{1}{2} A^{-1} \vec{\beta}$ とおくと

$$f(x, y) = \left(A \begin{pmatrix} s \\ t \end{pmatrix}, \begin{pmatrix} s \\ t \end{pmatrix} \right) + f'$$

$$= as^2 + 2cst + et^2$$

但し $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} s \\ t \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \iff \vec{v} = \vec{w} + \vec{v_0}$

$$f' = \underbrace{(A\vec{v_0}, \vec{v_0}) + (\vec{\beta}, \vec{v_0}) + f} = f(x = x_0, y = y_0)$$

$$f(x, y) = (A\vec{v}, \vec{v}) + (\vec{\beta}, \vec{v}) + f.$$

別の計算法.

$$z = x^2 - xy + y^2 - 2x + 4y \boxed{= s^2 - st + t^2 + f'}$$

$$= (x-a)^2 - (x-a)(y-b) + (y-b)^2 + f'$$

常に成立

$x = a = 0$
$y = b$ とすれば
$= -2$

$$((t-a)^n)' = n(t-a)^{n-1} \cdot 1$$

x で偏微分.

$$2x - y - 2 = 2(x-a) - (y-b)$$

y で偏微分.

$$-x + 2y + 4 = -(x-a) + 2(y-b)$$

$4 + 4(-2) = f'$
$= -4$

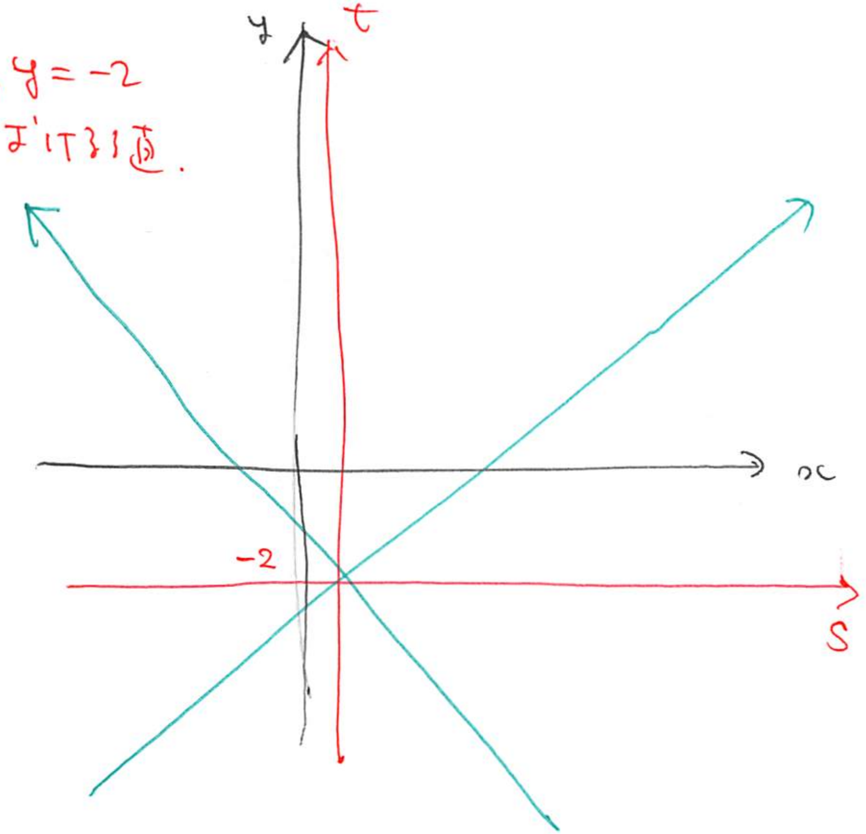
$$\begin{cases} 2a - b - 2 = 0 \\ -a + 2b + 4 = 0 \end{cases} \rightsquigarrow (a, b) = (0, -2)$$

z の $x=0,\ y=-2$ における値.

$$z = s^2 - st + t^2 - 4$$

回転座標変換.

次週.

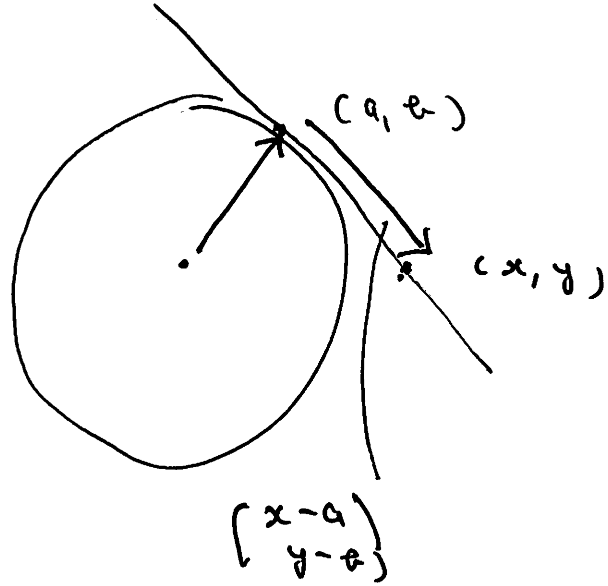


接線

$$g(x,y) = x^2 + y^2 - 1 = 0$$

点 (a, b) における接線

$$ax + by = 1$$



$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix} = 0$$

$$\rightarrow \quad a(x-a) + b(y-b) = 0 \longrightarrow ax + by = 1$$

$$\uparrow$$

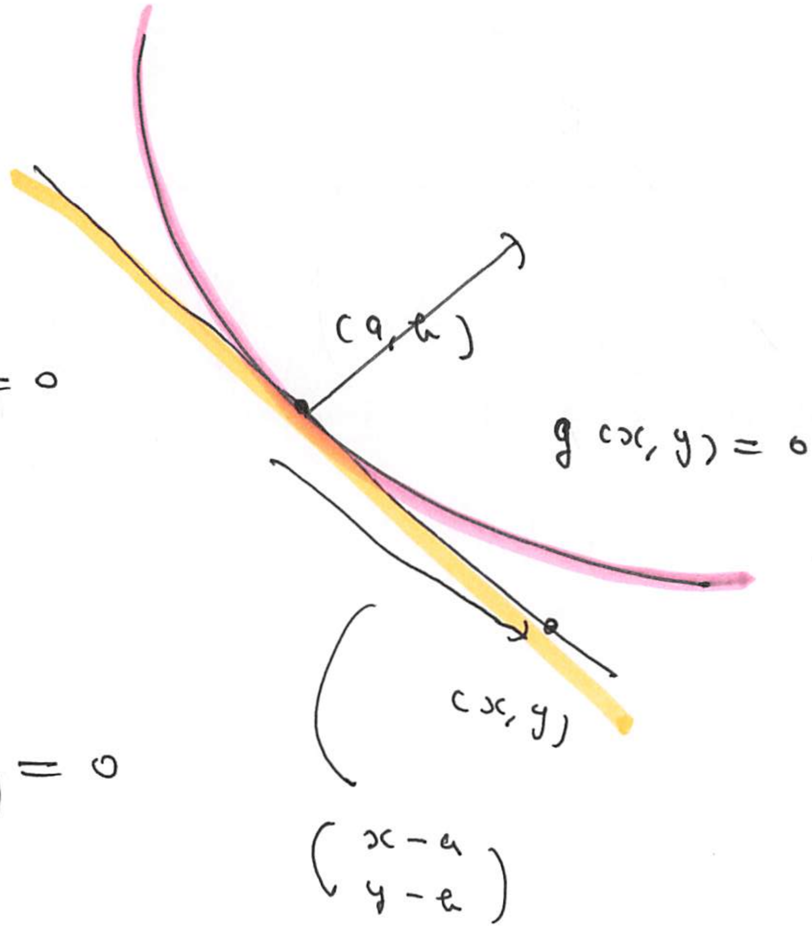
$$a^2 + b^2 = 1$$

接線

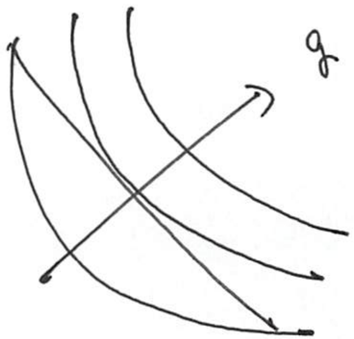
$$g_x(a,b)(x-a) + g_y(a,b)(y-b) = 0$$

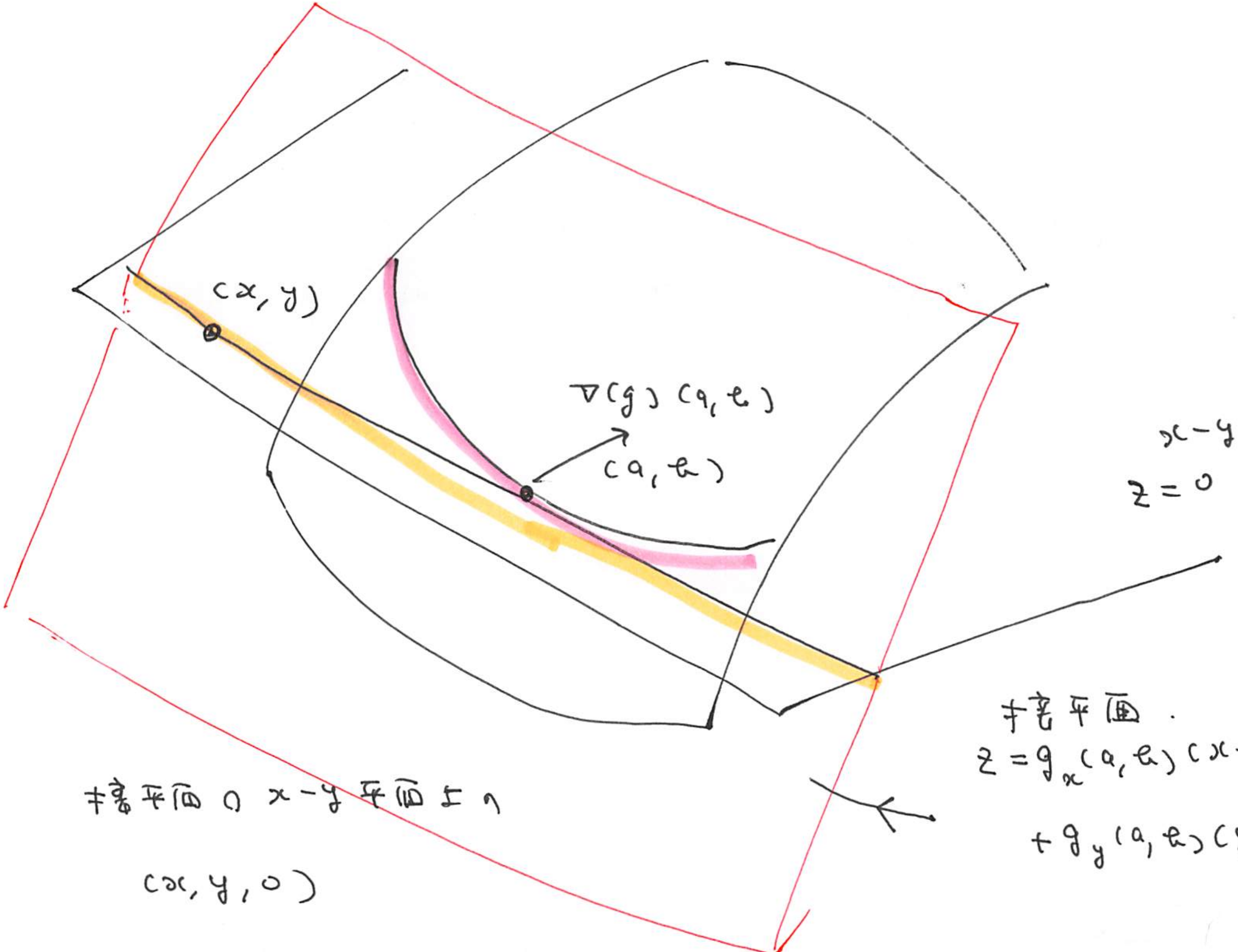
$$\Updownarrow$$

$$\begin{pmatrix} g_x(a,b) \\ g_y(a,b) \end{pmatrix} \cdot \begin{pmatrix} x-a \\ y-b \end{pmatrix} = 0$$



$$\nabla(g)(a,b) = \begin{pmatrix} g_x(a,b) \\ g_y(a,b) \end{pmatrix}$$ 勾配ベクトル





接平面の $x-y$ 平面上の

$(x, y, 0)$

$$\longrightarrow \quad g_x(a, b)(x-a) + g_y(a, b)(y-b) = 0$$

I $x^2 - xy + y^2 - 1 = 0$

の $(0, 1)$ における接線を求める

II $z = x^2 - 3xy + y^2 - 1$ の $(1, 0, 0)$ における

接平面

III 宿題（解答はweb）

$z = x^4 + y^4 + 2x^2 - 4xy + 2y^2$ の停留点

IV 宿題

$x^2 + 4xy + 2y^2 - 6x - 8y$ を平行移動で

簡単に.