

$V \subset \mathbb{R}^n$ is a subspace. $u_0 \in V$ and $u \in \mathbb{R}^n$ is given.

$$\textcircled{+} \|u - v\|^2 \geq \|u - u_0\|^2 \quad (\forall v \in V)$$

is the minimum error. \Rightarrow and

$$u - u_0 \perp V$$

is proved.

$$\begin{aligned} \|u - v\|^2 &= \|u - u_0 + u_0 - v\|^2 \\ &= \|u - u_0\|^2 + 2(u - u_0, u_0 - v) \\ &\quad + \|u_0 - v\|^2 \end{aligned}$$

is the same as $\textcircled{+}$ is

$$2(u - u_0, u_0 - v) + \|u_0 - v\|^2 \geq 0 \quad (\forall v \in V)$$

is the same as $\textcircled{+}$ is. $\forall w \in V$ is given.

$$2(u - u_0, u_0 - w) + \|u_0 - w\|^2 \geq 0$$

is the same as $\textcircled{+}$ is. $\forall w \in V$ is given. $w = u_0 - v$ is the same as $v \in V$ is the same as $\textcircled{+}$ is.

$$2(u - u_0, v) + \|v\|^2 \geq 0$$

is the same as $\textcircled{+}$ is. \Rightarrow $t \in \mathbb{R}$ is given. $t v \in V$ is the same as $\textcircled{+}$ is.

$$2(u - u_0, tv) + \|tv\|^2 \geq 0$$

is the same as

$$\textcircled{+} t^2 \|v\|^2 + 2t(u - u_0, v) \geq 0$$

is the same as $\textcircled{+}$ is. $t \in \mathbb{R}$ is given.

$$v = 0 \text{ is the same as } (u - u_0, v) = 0 \text{ for } v = 0 \text{ is the same as}$$

$$v \neq 0 \text{ is the same as } \|v\|^2 > 0 \text{ is the same as}$$

もし $(\vec{g} - \vec{v}_0, \vec{v}) \neq 0$ ならば $\textcircled{\#}$ の法線は t の関数

として t の関数として $\textcircled{\#}$ と

一致する。すると

$$(\vec{g} - \vec{v}_0, \vec{v}) = 0$$

が成り立つ。

よって \vec{v}

$$\vec{g} - \vec{v}_0 \perp \vec{v}$$

が成り立つ。

