$A = \begin{pmatrix} 5 & 3 \\ 3 & -3 \end{pmatrix}$ 対称行列. ${}^tA = A$

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda-5 & -3 \\ -3 & \lambda+3 \end{vmatrix} = (\lambda-5)(\lambda+3) - 9$$

$$= \lambda^2 - 2\lambda - 24$$

$$= (\lambda-6)(\lambda+4)$$

固有値は $\lambda = 6, -4$

$\underline{\lambda = 6}$ $\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff x = 3y$

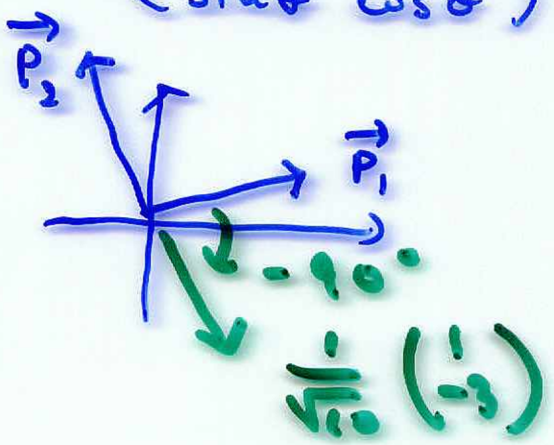
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3y \\ y \end{pmatrix} = y\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$\vec{p_1} = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} = 0$

$\underline{\lambda = -4}$ $\begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff 3x + y = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -3x \end{pmatrix} = x\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \vec{p_1} & \vec{p_2} \end{pmatrix}$$




$\uparrow$

どう選ぶか、

$\vec{p_2} = \frac{1}{\sqrt{10}}\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

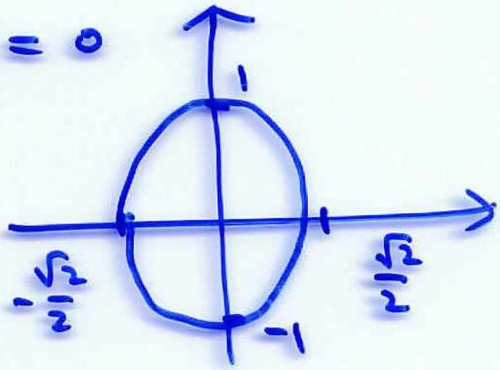
# Lagrange の未定乗数法　　Euler

290 p. 8.13

$$g(x, y) = 2x^2 + y^2 - 1 = 0$$

の下で

$$z = f(x, y) = x^2 y$$




$$\begin{cases} 2xy = \lambda \cdot 4x & \cdots ① \\ x^2 = \lambda \cdot 2y & \cdots ② \\ 2x^2 + y^2 = 1 & \cdots ③ \end{cases}$$

① $\iff$ $x = 0$ OR $y = 2\lambda$

$$\boxed{\begin{cases} \nabla(f) = \lambda \nabla(g) \\ g = 0 \end{cases}}$$

Ⅰ $x = 0$ のとき. ③に代入すると $y = \pm 1$

② に $x = 0$, $y = \pm 1$ を代入.

$$0 = \lambda(\pm 2) \rightsquigarrow \lambda = 0$$

Ⅱ $y = 2\lambda$ のとき　$y = 2\lambda$ を ② に代入.

$$x^2 = 4\lambda^2$$

$$x = \pm 2\lambda$$

$x = \pm 2\lambda$, $y = 2\lambda$ を ③ に代入

$$8\lambda^2 + 4\lambda^2 = 1 \qquad \lambda^2 = \frac{1}{12}$$

$$\lambda = \pm \frac{1}{2\sqrt{3}}$$

$$x = \pm\frac{1}{\sqrt{3}},\quad y = \pm\frac{1}{2\sqrt{3}}$$ （4通り）

---

問4 8.24, 8.25 を解け.

$x+y=1$ の下で　$g = x+y-1 = 0$

$$z = f(x, y) = x^{\frac{1}{2}}\, y^{\frac{1}{4}} \quad (x, y > 0)$$

---

$$\begin{cases} \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{4}} = \lambda\cdot 1 & \cdots ① \\ \frac{1}{4}x^{\frac{1}{2}}y^{-\frac{3}{4}} = \lambda & \cdots ② \end{cases}$$

$$x+y = 1 \quad \cdots ③$$

①×②

$$\frac{1}{8}y^{-\frac{1}{2}} = \lambda^2$$

$$y^{\frac{1}{2}} = \frac{1}{8\lambda^2} \rightsquigarrow y = \frac{1}{64\lambda^4}.$$

$$\hookrightarrow y^{\frac{1}{4}} = \frac{1}{2\sqrt{2}\lambda}$$

$$\boxed{\frac{1}{2}x^{-\frac{1}{2}}\cdot\frac{1}{2\sqrt{2}\lambda} = \lambda}$$

$$x^{-\frac{1}{2}} = 4\sqrt{2}\lambda^2 \qquad \frac{1}{x} = 32\lambda^4$$

$$x = \frac{1}{32\lambda^4}$$

$$x+y = \frac{1}{32\lambda^4} + \frac{1}{64\lambda^4} = \frac{3}{64\lambda^4} = 1$$

$$\lambda^4 = \frac{3}{64},\quad x = \frac{2}{3},\quad y = \frac{1}{3}.$$

**定理** $f : U \longrightarrow \mathbb{R}$, $C^2$級. $(a,b) \in U$

$f_x(a,b) = f_y(a,b) = 0$

$<0$

$f_{xx}(a,b) > 0$, ~~$f_y(a,$~~

$\det(H(f))(a,b) > 0$

$\Longrightarrow$ $(a,b)$ で極小.

大

---

$$H(f) = \begin{pmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{pmatrix}$$

$=$

$C^2$級 $\rightsquigarrow$ $f_{xy} = f_{yx}$ 対称.

---

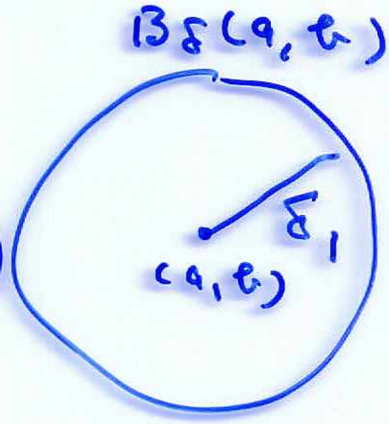
$C^2$級 $\rightsquigarrow$ 特に $f_{xx}, f_{xy}, f_{yy}$ 連続.

$f_{xx}(a,b) > 0$, $f_{xx}$: 連続

$\Longrightarrow$ ある $\delta_1 > 0$ をとると

$f_{xx}(x,y) > 0$

$\left( (x,y) \in B_{\delta_1}(a,b) \right)$



$\det(H(f))(a, b)$

$= f_{xx}(a, b)\, f_{yy}(a, b) - f_{xy}(a, b)^2$

$\det(H(f))(x, y)$ : 連続

$\rightsquigarrow$

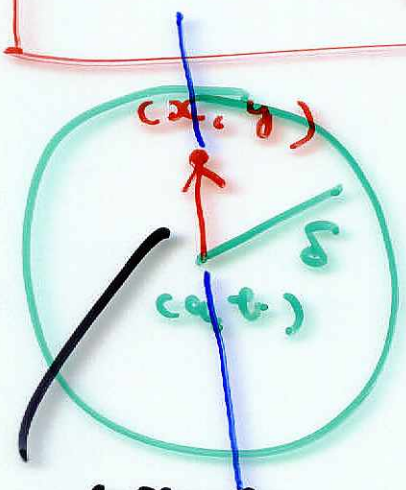
$\det(H(f))(x, y) > 0$

$((x, y) \in B_{\delta_2}(a, b))$

$\boxed{\begin{array}{l} \varphi, \psi : \text{連続} \\ \Longrightarrow \varphi \cdot \psi : \text{連続} \\ \quad \varphi \pm \psi : \text{—} \end{array}}$

$\delta = \min(\delta_1, \delta_2)$ とおくと

$\boxed{\begin{array}{l} f_{xx}(x, y) > 0,\ \det H(f)(x, y) > 0 \\ \qquad ((x, y) \in B_\delta(a, b)) \end{array}}$




cT 281 page.

$(x, y) \neq (a, b),\ (x, y) \in B_\delta$

$F(t) = f(a + (x-a)t,$
$\qquad b + (y-b)t)$

$\begin{pmatrix} x-a \\ y-b \end{pmatrix}$

$F(0) = f(a, b)$

$F(1) = f(x, y)$

Taylor
~~平均値~~の
定理

$F(1) - F(0) = \cancel{F'(c)(1-0)}$

$\cancel{= F'(c)}$ を満たす c が

$= \dfrac{F'(0)}{1}\cancel{t} + \dfrac{F''(c)}{2!}\cancel{t}^2$

0と1の間に在る

$$f(x,y) - f(a,b)$$

$$= \frac{F''(c)}{2!} \cancel{t^2} 1^2 > 0 \Bigg) \rightsquigarrow f(x,y) > f(a,b)$$

極小

$$F'(t) = f_x(a+(x-a)t,\ b+(y-b)t)\cdot(x-a)$$

$$+ f_y(\quad)\cdot(y-b)$$

$t=0$

$$F'(0) = \underbrace{f_x(a,b)}_{\overset{\shortparallel}{0}}(x-a) + \underbrace{f_y(a,b)}_{\overset{\shortparallel}{0}}(y-b)$$

$$= 0$$

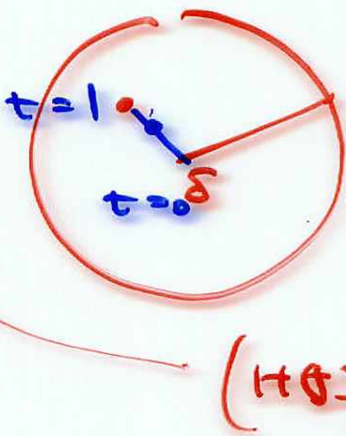
$$F''(t) = f_{xx}(\quad)(x-a)^2$$

$$+ 2f_{xy}(\quad)(x-a)(y-b)$$

$$+ f_{yy}(\ )\cdot(y-b)^2$$

$$\boxed{\begin{array}{c} f_{xx}(x,y) > 0,\ \det H(f)(x,y) > 0 \\ (x,y) \in B_\delta \end{array}}$$

$$\downarrow$$




$$\left(H(f)(x,y)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}, \begin{pmatrix}\alpha\\ \beta\end{pmatrix}\right)$$

$$> 0$$

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \neq 0$$

$$\left(H(f)\begin{pmatrix}x-a\\ y-b\end{pmatrix}, \begin{pmatrix}x-a\\ y-b\end{pmatrix}\right) > 0$$

$$(a+(x-a)c,\ b+(y-b)c)$$

26 9r. 4311 8.11

$$z = f(x, y) = x^2 + 4xy + 2y^2 - 6x - 8y$$

$$\begin{cases} z_x = 2x + 4y \cdot 1 - 6 \\ \quad = 2x + 4y - 6 = 0 \\ z_y = 4x \cdot 1 + 4y - 8 = 0 \end{cases}$$

$$\begin{cases} x + 2y = 3 \quad \cdots ① \\ x + y = 2 \quad \cdots ② \end{cases} \qquad (x, y) = (1, 1)$$

---

$$z_{xx} = 2, \quad z_{xy} = z_{yx} = 4, \quad z_{yy} = 4$$

$$\begin{vmatrix} 2 & 4 \\ 4 & 4 \end{vmatrix} = 8 - 16 < 0$$

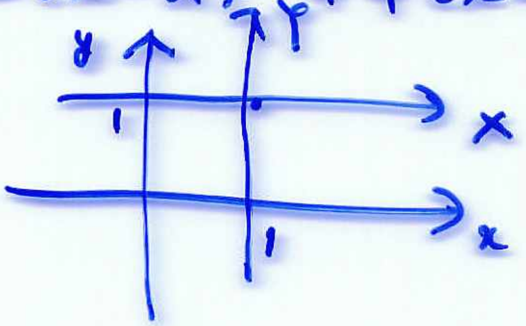
$(1, 1)$は
極大でも
極小でもな‥

---

$$z = (x-1)^2 + 4(x-1)(y-1) + 2(y-1)^2 + -7$$

$$z = x^2 + 4xy + 2y^2 - 6x - 8y$$

$$= (x-\alpha)^2 + 4(x-\alpha)(x-\beta) + 2(y-\beta)^2 + \gamma$$



$$z = x^2 + 4xy + 2y^2 - 7$$

$$= \left(A\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix}\right) - 7$$

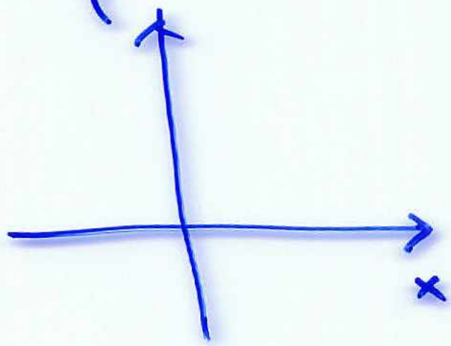
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

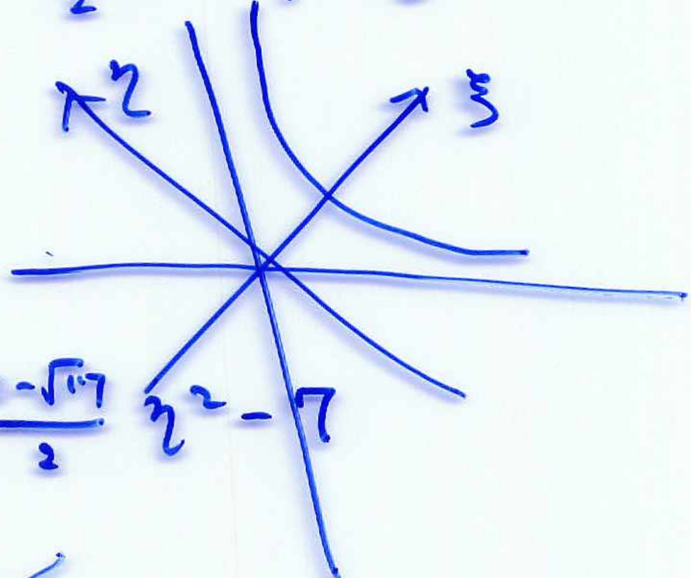
$$|\lambda I_2 - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda-1)(\lambda-2) - 4$$

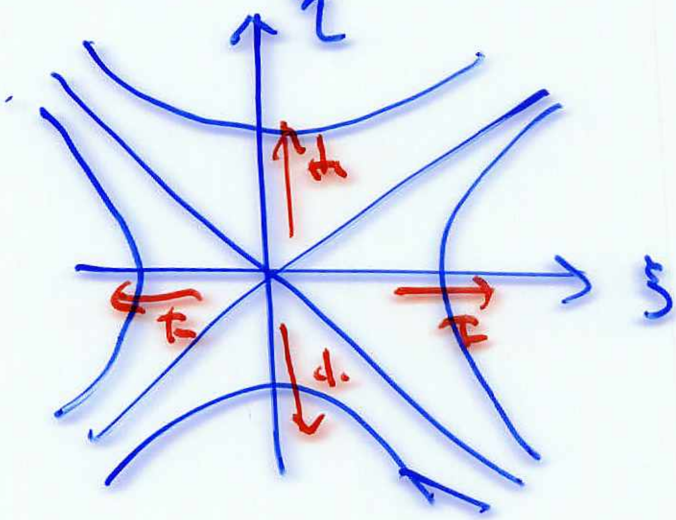
$$= \lambda^2 - 3\lambda - 2 \qquad \lambda = \frac{3 \pm \sqrt{17}}{2}$$

$$\frac{3+\sqrt{17}}{2} > 0, \quad \frac{3-\sqrt{17}}{2} < 0$$




$$z = \frac{3+\sqrt{17}}{2}\xi^2 + \frac{3-\sqrt{17}}{2}\eta^2 - 7$$



4311 8.12

$$z = x^3 - xy - y^2$$

$$\begin{cases} z_x = 3x^2 - y = 0 & \cdots ① \\ z_y = -x - 2y = 0 & \cdots ② \end{cases}$$

$x = -2y$ を①に代入する.

$12y^2 - y = 0 \iff y = 0$ OR $y = \frac{1}{12}$.

$x = 0,\ -\frac{1}{6}$

$(x, y) = (0, 0),\ \left(-\frac{1}{6}, \frac{1}{12}\right)$

---

$z_{xx} = 6x,\quad z_{xy} = z_{yx} = -1$

$z_{yy} = -2$

$(0, 0)$ $\begin{vmatrix} 0 & -1 \\ -1 & -2 \end{vmatrix} = -1 < 0$ 極大でも極小でもない.

$\left(-\frac{1}{6}, \frac{1}{12}\right)$ $\begin{vmatrix} -1 & -1 \\ -1 & -2 \end{vmatrix} = 2 - 1 = 1 > 0$

$f_{xx} = -1 < 0$

$\rightarrow$ 極大.

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \qquad |\lambda I_2 - A| = (\lambda - 5)(\lambda + 1)$$

$$P = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$ にすると

$$AP = P\begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$$

---

$$\underbrace{\frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}}_{\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}} = A\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x(t) + 2y(t) \\ 4x(t) + 3y(t) \end{pmatrix}$$

---

$$B(t) = \begin{pmatrix} b_1(t) & c_1(t) \\ b_2(t) & c_2(t) \end{pmatrix} \quad B'(t) = \begin{pmatrix} b_1' & c_1' \\ b_2' & c_2' \end{pmatrix}$$

$$\left(B(t)\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}\right)' = B'(t)\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + B(t)\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix}$$

---

$$\left(P^{-1}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}\right)' = \underbrace{(P^{-1})'}_{O_2}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + P^{-1}\frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$= P^{-1}A\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$= \underbrace{P^{-1}AP}_{\begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}} \cdot \underbrace{P^{-1}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}}_{\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix}} \qquad \begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = P^{-1}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} X'(t) \\ Y'(t) \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \quad \frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = A \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$P^{-1}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} X(t) \\ Y(t) \end{pmatrix}$$

$$\begin{pmatrix} X'(t) \\ Y'(t) \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} 5X(t) \\ -Y(t) \end{pmatrix}$$

$$\rightsquigarrow \begin{cases} X'(t) = 5X(t) \\ Y'(t) = -Y(t) \end{cases}$$

$$\boxed{f'(t) = \alpha f(t) \Longrightarrow f(t) = f(0)e^{\alpha t} \qquad \alpha \in \mathbb{R}}$$

$$X(t) = X(0)e^{5t}$$
$$Y(t) = Y(0)e^{-t}$$

$$\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \underline{P^{-1}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}} = \begin{pmatrix} X(0)e^{5t} \\ Y(0)e^{-t} \end{pmatrix}$$

$$= \begin{pmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{pmatrix}\begin{pmatrix} X(0) \\ Y(0) \end{pmatrix}$$

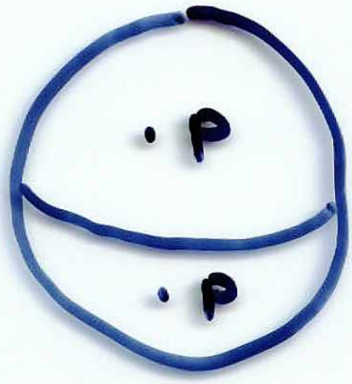
$$= \underline{\begin{pmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{pmatrix} P^{-1}\begin{pmatrix} x(0) \\ y(0) \end{pmatrix}}$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = P\begin{pmatrix} e^{5t} & 0 \\ 0 & e^{-t} \end{pmatrix} P^{-1}\begin{pmatrix} x(0) \\ y(0) \end{pmatrix}$$

S:

$$x^2+y^2+z^2-1=0$$

$P(a,b,c)\in S$.



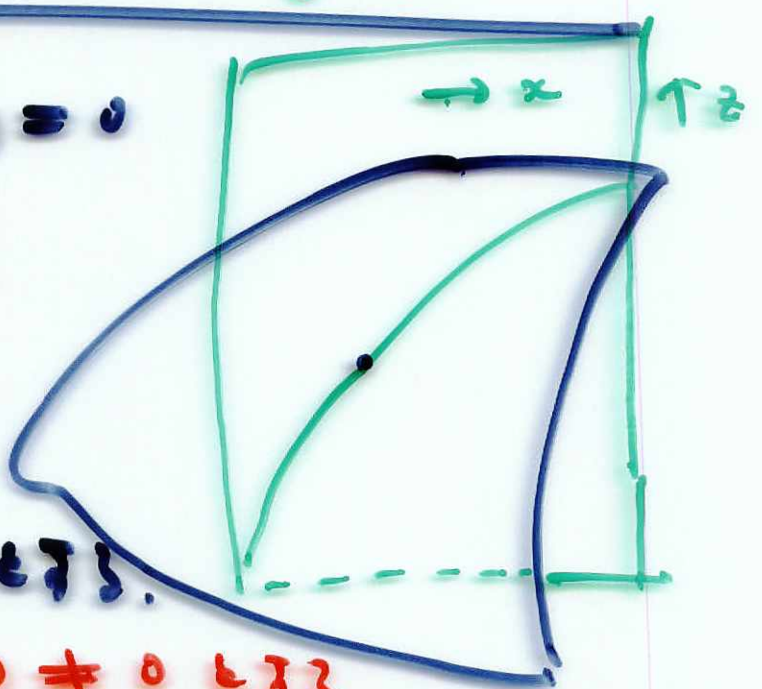
$c>0$ $\quad z=\sqrt{1-x^2-y^2}$

とPの近くで表示.

$c<0$ $\quad z=-\sqrt{1-x^2-y^2}$

---

S: $g(x,y,z)=0$




$(a,b,c)\in S$

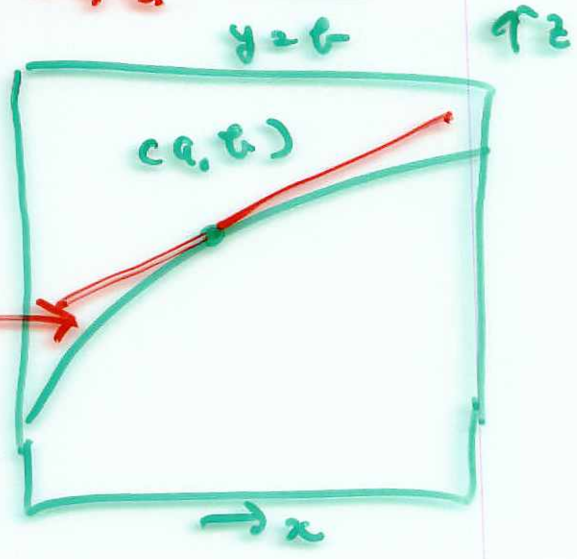
の近くで

$z=\varphi(x,y)$

とSの表示ができるか.

$g_z(a,b,c)\neq 0$ とする.

$\varphi_x,\ \varphi_y = ?$




$g(x,b,z)=0$

$z=\varphi(x,b)$

$G(x,z)=0$

$$G(x,z) = g(x,b,z)$$

$(x,z)=(a,c)$ における接線

$$G_x(a,c)(x-a) + G_z(a,c)(z-b) = 0$$

$$G_x(a,c) = \lim_{x\to a} \frac{G(x,c)-G(a,c)}{x-a}$$

$$= \lim_{x\to a} \frac{g(x,b,c)-g(a,b,c)}{x-a}$$

$$= g_x(a,b,c)$$

同様に $G_z(a,c) = g_z(a,b,c) \neq 0$.

$$z = \boxed{-\frac{g_x(a,b,c)}{g_z(a,b,c)}}(x-a) + b.$$

$$= \varphi_x(a,b)$$

<u>まとめ</u>

$$\varphi_x(a,b) = -\frac{g_x(a,b,c)}{g_z(a,b,c)}$$

$$\varphi_y(a,b) = -\frac{g_y(a,b,c)}{g_z(a,b,c)}$$

$(a,b,c)$ において

$S$: $g(x,y,z)=0$ implicit

$z = \varphi(x,y)$ explicit

$$w = f(x, y, z)$$

$$x = X(s,t),\quad y = Y(s,t),\quad z = Z(s,t)$$

$$\to\ w = f(X(s,t), Y(s,t), Z(s,t))$$

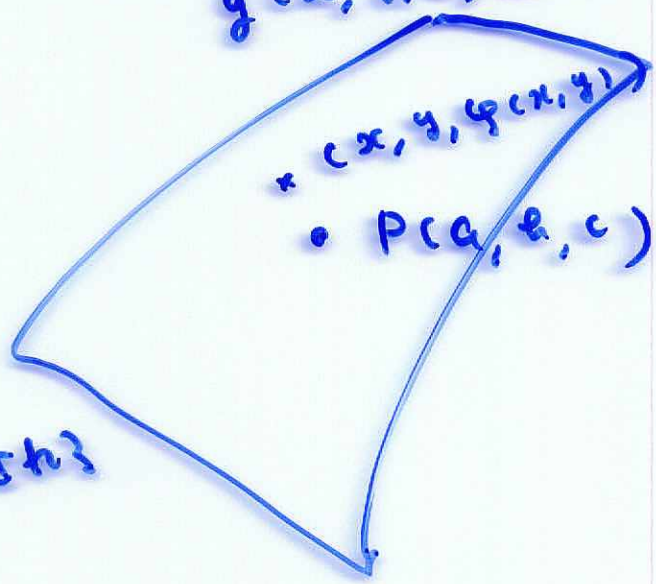
$$= F(s,t)$$

$$\frac{\partial F}{\partial s} = f_x(\quad)\cdot\frac{\partial X}{\partial s} + f_y\cdot\frac{\partial Y}{\partial s} + f_z\frac{\partial Z}{\partial s}$$

$$\frac{\partial F}{\partial t} = f_x\frac{\partial X}{\partial t} + f_y\cdot\frac{\partial Y}{\partial t} + f_z\cdot\frac{\partial Z}{\partial t}.$$

---

$$g(x, y, z) = 0$$




Pの近くで

$z = \varphi(x, y)$

と表示される

$$\to\ g(x, y, \varphi(x,y)) \equiv 0$$

$(x,y) = (a,b)$の近くで

$$g(s,t,\varphi(s,t)) \equiv 0$$

$$(s,t) \sim (a,b)$$

$\frac{\partial}{\partial s}$

$$g_x \cdot 1 + g_y \cdot 0 + g_z \cdot \frac{\partial \varphi}{\partial s} \equiv 0$$

$\frac{\partial}{\partial t}$

$$g_x \cdot 0 + g_y \cdot 1 + g_z \cdot \frac{\partial \varphi}{\partial t} \equiv 0$$

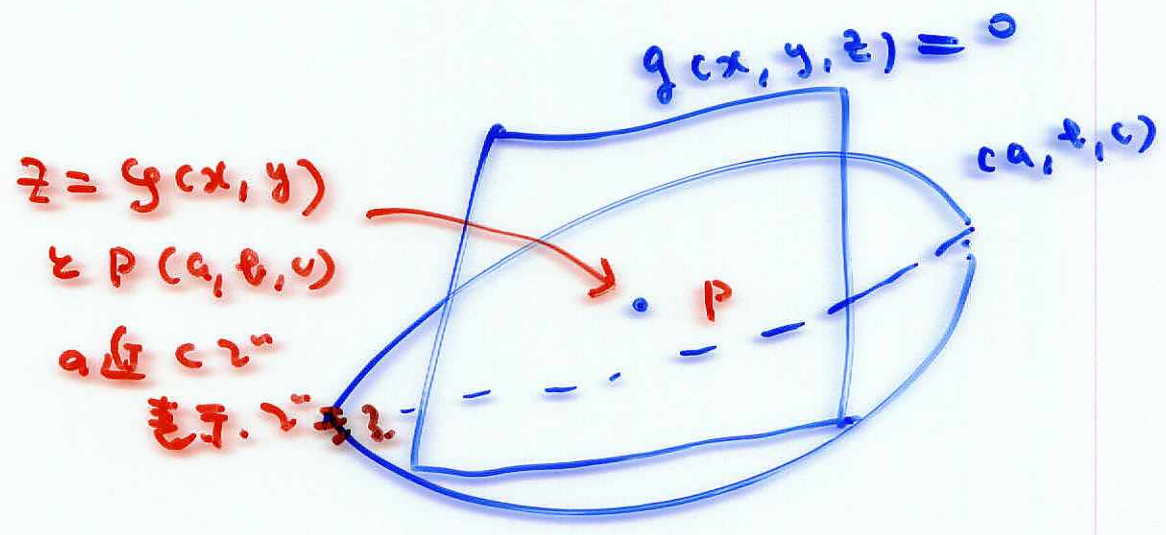
$g_z(a,b,c) \neq 0$ $\quad$ < $g_z$ : 連続だ.

$(a,b,c)$ の近くで

$$g_z \neq 0$$

$$\left\{ \begin{aligned} \frac{\partial \varphi}{\partial s} &= -\frac{g_x(\quad)}{g_z(s,t,\varphi(t))} \\ \frac{\partial \varphi}{\partial t} &= -\frac{g_y(\quad)}{g_z(\quad)} \end{aligned} \right.$$

---



$$z = \underbrace{\varphi_x(a,b)}_{= -\frac{g_x(a,b,c)}{g_z(a,b,c)}}(x-a) + \underbrace{\varphi_y(a,b)}_{= -\frac{g_y(a,b,c)}{g_z(a,b,c)}}(y-b) + c$$

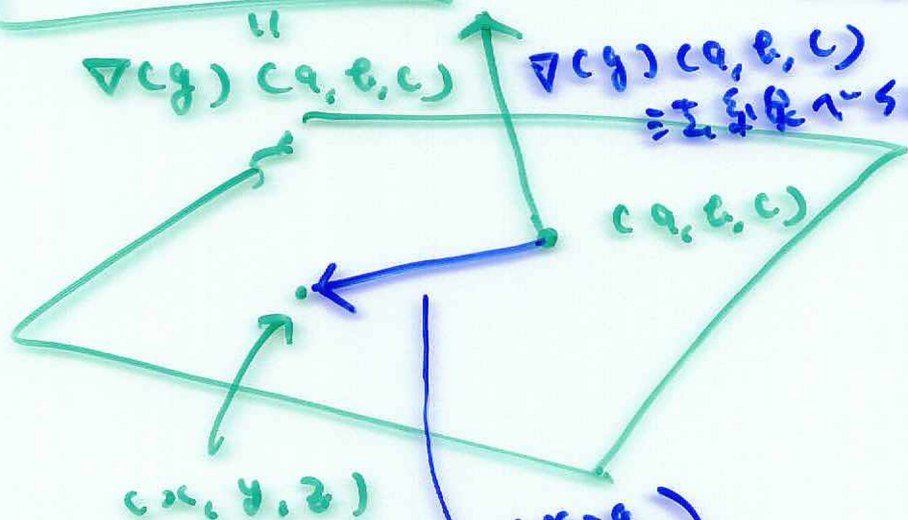
$$\Rightarrow g_x(a,b,c)(x-a) + g_y(a,b,c)(y-b) + g_z(a,b,c)(z-c) = 0$$

$$\Rightarrow \underbrace{\boxed{\begin{pmatrix} g_x(a,b,c) \\ g_y(\quad) \\ g_z(\quad) \end{pmatrix}}}_{= \nabla(g)(a,b,c)} \cdot \begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix} = 0$$

とみる.

$\nabla(g)(a,b,c)$

= 法線ベクトル




$$\begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix}$$

$S: g(x,y,z)=0$ 上に $(a,b,c)$ がある

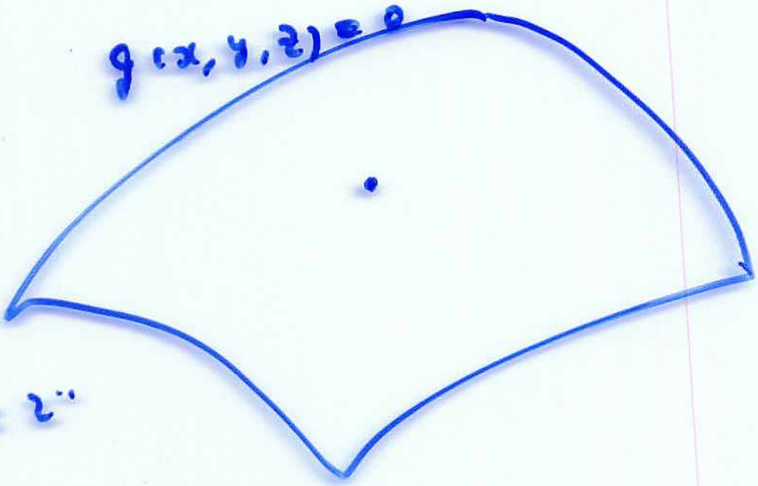
$g_z(a,b,c) \neq 0$

$\downarrow$

$z = \varphi(x,y)$

と $(a,b,c)$ の近くで

表示できる　（陰関数定理）




---

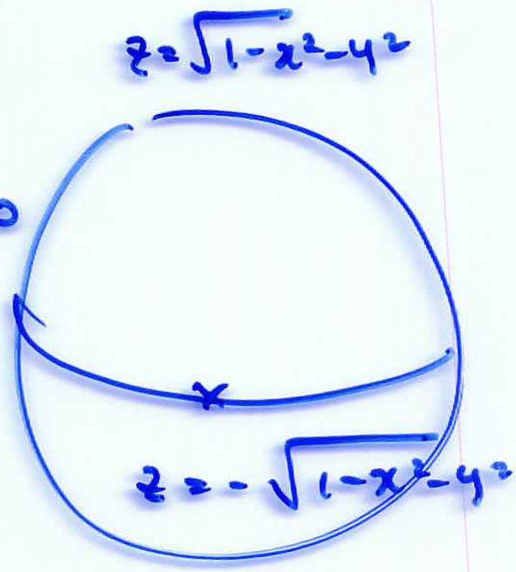
$g: C^1 \rightsquigarrow \varphi: C^1$

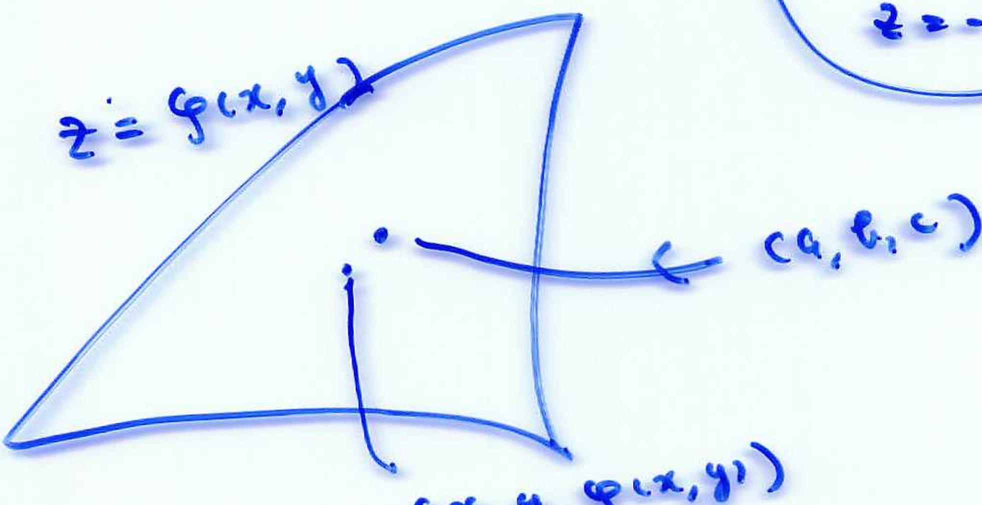
$\quad C^2 \qquad \varphi: C^2$

例　$S$:

$g = x^2+y^2+z^2-1=0$

$g_z = 2z$



Lagrangeの未定乗数法

$(a,b,c)$の近くで $z=\varphi(x,y)$ と表示する.

$$g(x,y,z)=0$$

の下で $w=f(x,y,z)$

$(a,b,c)$で極値をとるとする

$$\begin{cases} g(a,b,c)=0 \\ g_z(a,b,c)\neq 0 \end{cases}$$

$$\Phi(s,t)=f(s,t,\varphi(s,t))$$

が $(s,t)=(a,b)$で極値をとる

$$\frac{\partial\Phi}{\partial s}(a,b)=\frac{\partial\Phi}{\partial t}(a,b)=0$$

$$\frac{\partial\Phi}{\partial s}=f_x\cdot 1+f_y\cdot 0+f_z\frac{\partial\varphi}{\partial s}$$

$$\frac{\partial\Phi}{\partial t}=f_x\cdot 0+f_y\cdot 1+f_z\frac{\partial\varphi}{\partial t}$$

$$\begin{cases} f_x(a,b,c)+f_z(a,b,c)\cdot\boxed{\frac{\partial\varphi}{\partial s}(a,b)}=0 \\ f_y(a,b,c)+f_z(a,b,c)\boxed{\frac{\partial\varphi}{\partial t}\cdot(a,b)}=0 \end{cases}$$

$$-\frac{g_x(a,b,c)}{g_z(\ )}=\frac{\partial\varphi}{\partial s}(a,b)$$

$$\frac{\partial\varphi}{\partial t}(a,b)=-\frac{g_y(\ )}{g_z(\ )}$$

$$f_x(P_0) - \boxed{f_z(P_0)\frac{g_x(P_0)}{g_z(P_0)}} = 0$$

$$\boxed{\begin{matrix} P_0 \\ \shortparallel \\ (a,b,c) \end{matrix}} \quad f_y(P_0) - \boxed{f_z(P_0)\frac{g_y(P_0)}{g_z(P_0)}} = 0$$

$$\lambda = \frac{f_z(P_0)}{g_z(P_0)}$$ とおく.

$$\left\{\begin{aligned} f_x(P_0) &= \lambda g_x(P_0) \\ f_y(P_0) &= \lambda g_y(P_0) \\ f_z(P_0) &= \lambda g_z(P_0) \end{aligned}\right.$$

$\nabla(f)(P_0) = \lambda \nabla(g)(P_0)$

を満たす $\lambda$ が存在

134 9.13

$g = x^2 + y^2 + z^2 - 1 = 0$ の下で

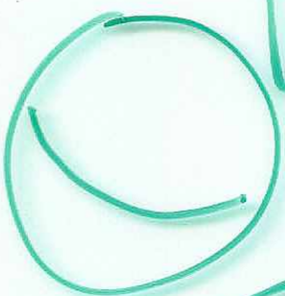
$w = 3x - y + 2z = f(x, y, z)$

---

$g_x = 2x, \quad g_y = 2y, \quad g_z = 2z$

$f_x = 3, \quad f_y = -1, \quad f_z = 2.$

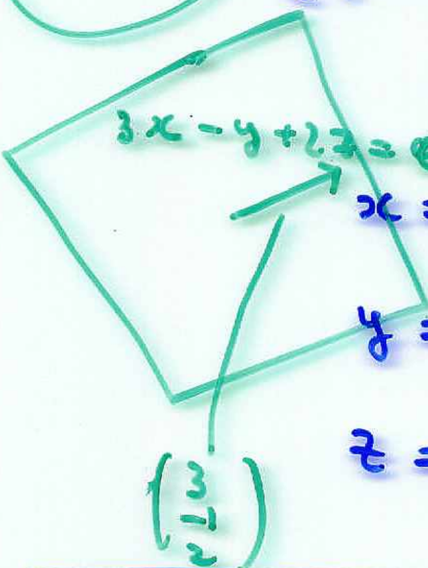
$$\boxed{\begin{aligned} 3 &= \lambda \cdot 2x \\ -1 &= \lambda \cdot 2y \\ 2 &= \lambda \cdot 2z \end{aligned}} \rightarrow \begin{aligned} x &= \frac{3}{2} \cdot \frac{1}{\lambda} \\ y &= -\frac{1}{2} \frac{1}{\lambda} \\ z &= 1 \cdot \frac{1}{\lambda}. \end{aligned}$$



$$\left(\frac{9}{4} + \frac{1}{4} + 1\right) \frac{1}{\lambda^2} = 1$$

$$\lambda^2 = \frac{14}{4} \qquad \lambda = \pm \frac{\sqrt{14}}{2}$$

$$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$




$$x = \pm \frac{3}{2} \cdot \frac{2}{\sqrt{14}} = \pm \frac{3}{\sqrt{14}}$$

$$y = \mp \frac{1}{2} \cdot \frac{2}{\sqrt{14}} = \mp \frac{1}{\sqrt{14}}$$

$$z = \pm \frac{2}{\sqrt{14}}$$

（同順）

---

演

$x^2 + y^2 + z^2 = 1$ の下で

$w = x + y - z.$

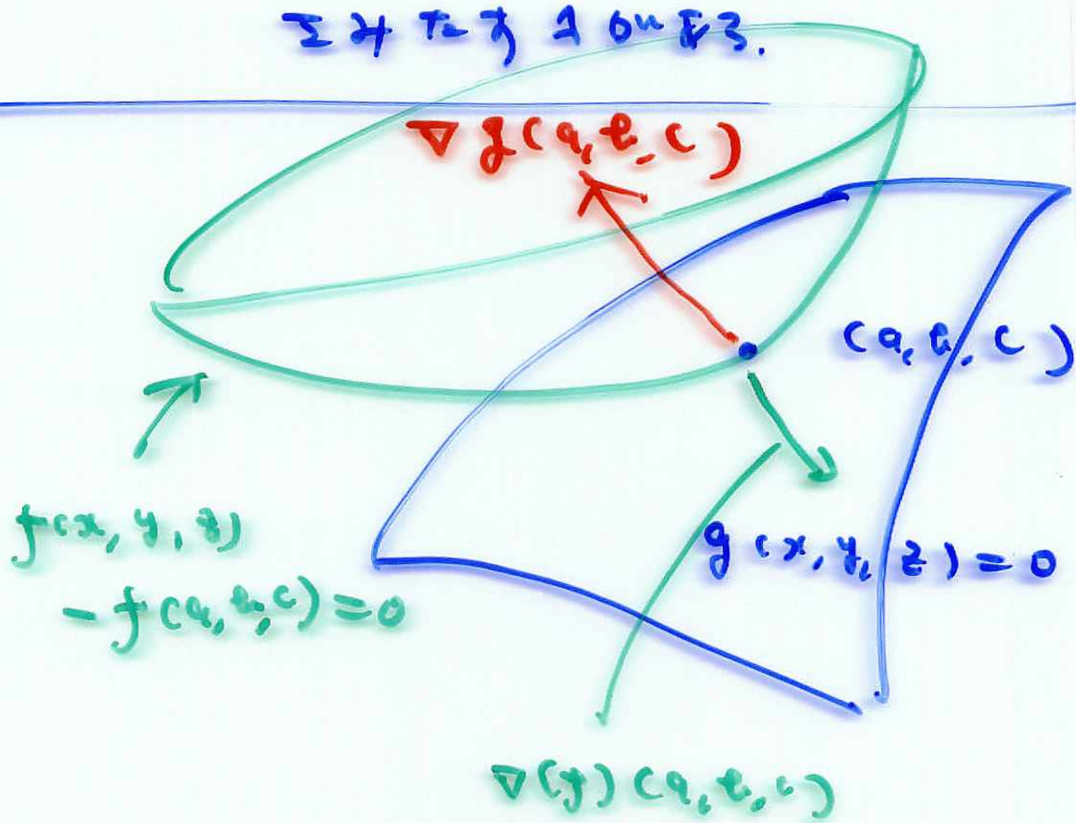
$$\boxed{\begin{array}{c} g(x,y,z)=0 \text{ の下で} \\ w=f(x,y,z) \end{array}}$$

$(a,b,c)$ で極値

$\Longrightarrow$ $\nabla(f)(a,b,c)$

$= \lambda\, \nabla(g)(a,b,c)$

となる $\lambda$ がある.



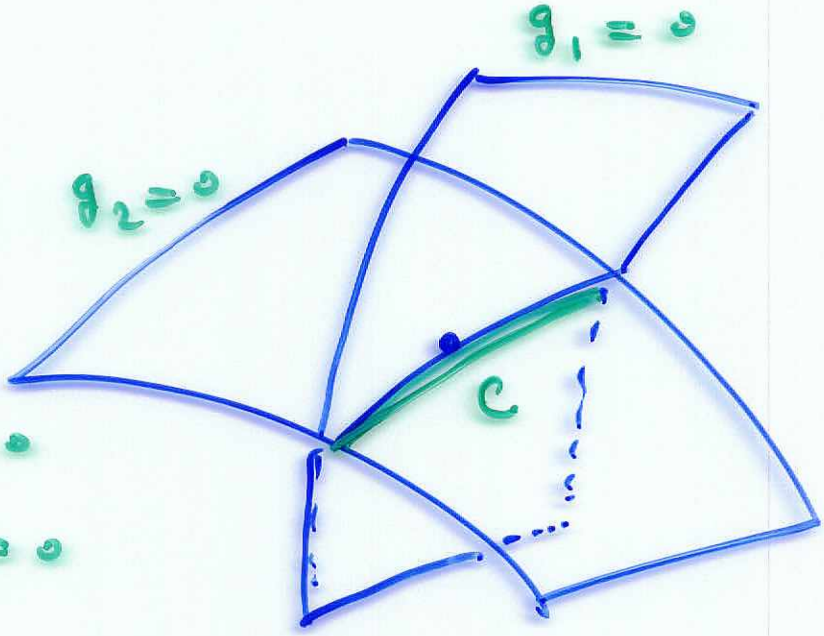

$\longrightarrow \left\{f(x,y,z)=f(a,b,c)\right\}$

と $S$: $g(x,y,z)=0$

が $(a,b,c)$ で接している.

$C$:

$$\begin{cases} g_1(x,y,z)=0 \\ g_2(x,y,z)=0 \end{cases}$$




曲線 $C$ が

曲面 $S_1 : g_1 = 0$

$S_2 : g_2 = 0$

の交わりで書けている.

$C$ の下で $w = f(x,y,z)$ を考える.

---

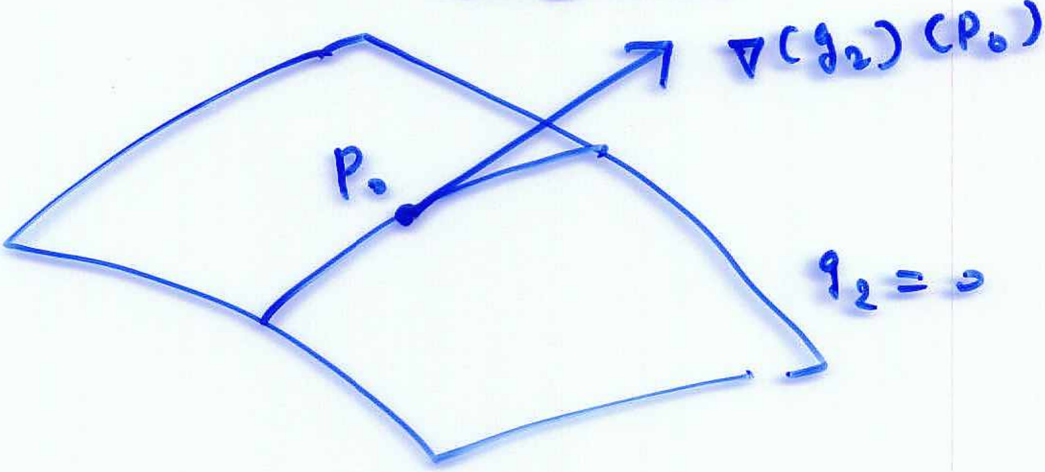
$C$ 上の点 $\underset{= P_0}{(a,b,c)}$ で極値をとるとする.

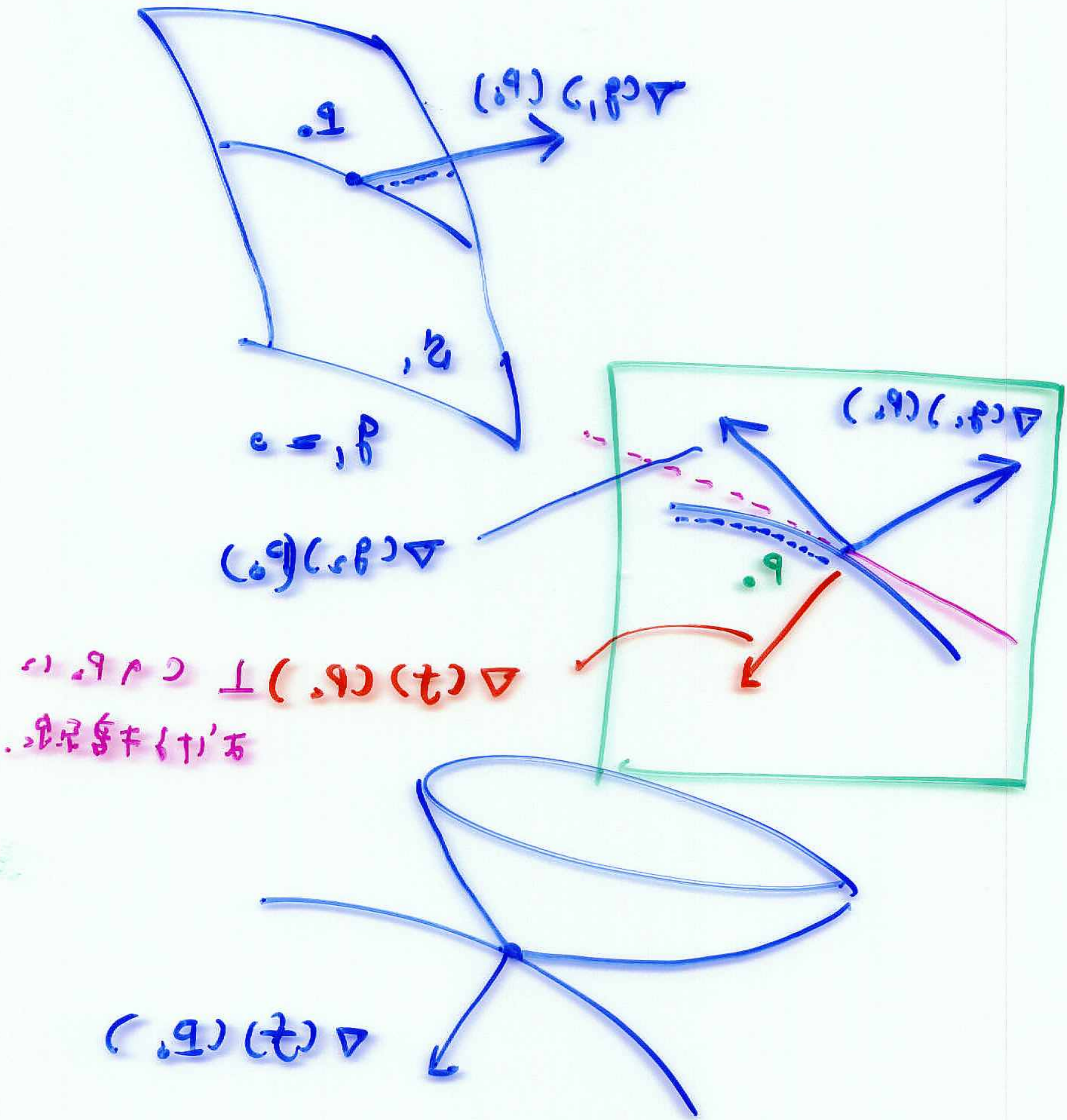
$$\nabla(f)(P_0) = \lambda_1 \nabla(g_1)(P_0) + \lambda_2 \nabla(g_2)(P_0)$$

を満たす $\lambda_1, \lambda_2$ が存在する.

---

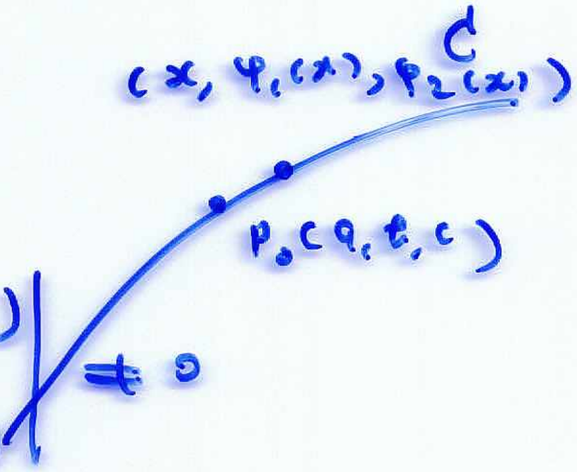
335P. 定理 9.13.





$\cancel{g_1(a,b,c) = 0}$

$$\left.\begin{aligned} g_1(x,y,z) &= 0 \\ g_2(x,y,z) &= 0 \end{aligned}\right\} C$$




$$\begin{vmatrix} g_{1y}(P_0) & g_{2y}(P_0) \\ g_{1z}(P_0) & g_{2z}(P_0) \end{vmatrix} \neq 0$$

$$\Downarrow$$

C は $P_0$ の近くで

$$y = \varphi_1(x),\quad z = \varphi_2(x)$$

と書ける.

---

$$F(t) = f(t, \varphi_1(t), \varphi_2(t))$$

$F'(a) = 0$ を考える.

I $A = \begin{pmatrix} 3 & -4 \\ -4 & 3 \end{pmatrix}$ を回転の行列で対角化する.

極値を求める.

II. (1) $x^2 - xy + y^2 - 4x - y$

(2) $xy(2 - x - y)$

(3) $xy\, e^{x-y}$

---

III (1) $x^2 + xy + y^2 = 1$ の下で

$$z = xy$$

---

IV (1) $A = \begin{pmatrix} 1 & 12 \\ 3 & 1 \end{pmatrix}$ (2) $A^n$ を計算する

$$\frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = A \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

を解く.

V $x^2 + y^2 + z^2 = 1$

$$w = x + y - z$$