$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

$A\vec{v} = \lambda\vec{v}$ をみたす $\vec{v} \neq \vec{0}$ はあるか.

B：2次正方

$B\vec{v} = \vec{0}$ をみたす $\vec{v} \neq \vec{0}$ が存在

$\Leftrightarrow \det B = 0$

$A\vec{v} = \lambda I_2 \vec{v} \iff (\lambda I_2 - A)\vec{v} = \vec{0}$

$\cancel{(\lambda I_2 - B)\vec{v} = \vec{0}}$

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda-1 & -2 \\ -4 & \lambda-3 \end{vmatrix} = 0$$

$$= (\lambda-1)(\lambda-3) - 8$$

$$= (\lambda^2 - 4\lambda - 5) = (\lambda-5)(\lambda+1)$$

$\rightsquigarrow \lambda = 5, -1$　A の固有値

$\underline{\lambda = 5}$　$\begin{pmatrix} 4 & -2 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff 2x - y = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (x \neq 0)$$

固有ベクトル

$\underline{\lambda = -1}$　$\begin{pmatrix} -2 & -2 \\ -4 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff x + y = 0$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -x \end{pmatrix} = x\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (x \neq 0)$$

$$\left(A\binom{x}{y},\binom{x}{y}\right) = \left(A\overbrace{P\,{}^tP}^{I_2}\binom{x}{y},\ P\,{}^tP\binom{x}{y}\right)$$

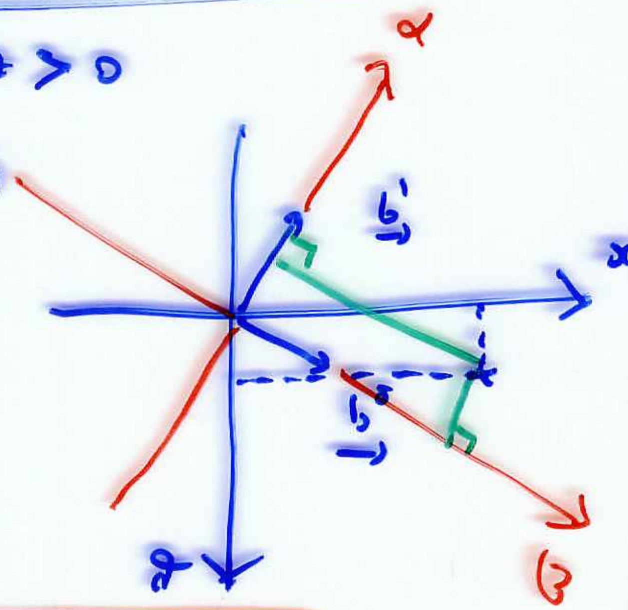
$$= \left({}^tPAP\cdot{}^tP\binom{x}{y},\ {}^tP\binom{x}{y}\right) = \left(\begin{pmatrix}1&0\\0&c\end{pmatrix}\binom{\alpha}{\beta},\binom{\alpha}{\beta}\right) = \alpha^2 + c\beta^2$$

$AP = P\begin{pmatrix}1&0\\0&c\end{pmatrix}$

$I_2 = {}^tPP = P\,{}^tP$

$$\binom{x}{y} = P\binom{\alpha}{\beta} \iff {}^tP\binom{x}{y} = \binom{\alpha}{\beta}$$

$$\boxed{\left(A\binom{x}{y},\binom{x'}{y'}\right) = \left(\binom{x}{y},\ {}^tA\binom{x'}{y'}\right)}$$



${}^tPAP = \begin{pmatrix}1&0\\0&c\end{pmatrix}$

$[illegible]\,x^2 + [illegible]\,xy + [illegible]\,y^2$

$= \alpha^2 + c\beta^2 \geq 0$

$\binom{x}{y} \neq \vec{0} \iff \binom{\alpha}{\beta} \neq \vec{0}$, $\binom{\alpha}{\beta} = {}^tP\binom{x}{y}$

$0 < [illegible]$

b : θ 回転　　　b^{-1} : $(-\theta)$ 回転

$$b = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \Longrightarrow b^{-1} = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= {}^t b.$$

$$b b^{-1} = b^{-1} b = I_2 \Longrightarrow \boxed{b\,{}^t b = {}^t b\, b = I_2}$$

例えば 対称行列と 2次式の対応:

$$A = \begin{pmatrix} 7 & 2 \\ 2 & 5 \end{pmatrix} \longleftrightarrow 7x^2 + 4xy + 5y^2$$

2×2

A の役割

2次形式で.

$$\left(\begin{pmatrix} x \\ y \end{pmatrix}, A \begin{pmatrix} x \\ y \end{pmatrix} \right) = \left(\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} 7 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

$$= \left(\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} 7x + 2y \\ 2x + 5y \end{pmatrix} \right)$$

$$= (7x^2 + 2xy) + (2xy + 5y^2)$$

$$= 7x^2 + 4xy + 5y^2$$

$$Ax^2 + 2Cxy + By^2$$

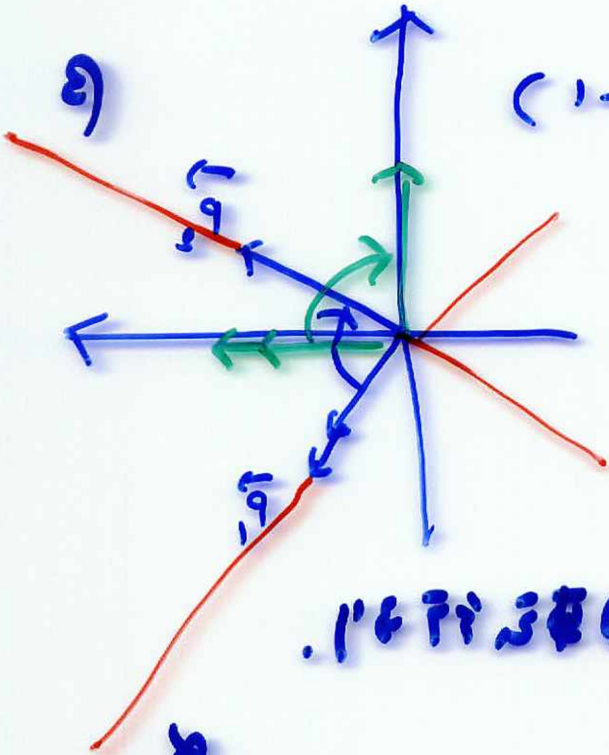
$$= \left(\begin{pmatrix} x \\ y \end{pmatrix}, \underbrace{\begin{pmatrix} A & C \\ C & B \end{pmatrix}}_{\text{対称.}} \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

$$P^{-1}AP = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \rightsquigarrow AP = P\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix} \qquad A = A^T$$

$$|\lambda I_2 - A| = (\lambda - 6)(\lambda - 1)$$



$\underline{\lambda = 1}$ $\quad \vec{p}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\underline{\lambda = 6}$ $\quad \vec{p}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\vec{p}_1 \perp \vec{p}_2$

$P = (\vec{p}_1\ \vec{p}_2)$ は直交行列.

$$\boxed{AP = P\begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \vec{p}_1 + \beta \vec{p}_2 = (\vec{p}_1\ \vec{p}_2)\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= P\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} \rightsquigarrow P\begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = AP\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

[illegible]

$$\begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = P^{-1}AP\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad P^{-1}AP = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$$

$v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad P = (v_1 \ v_2)$

$A v_1 = 2 v_1, \quad A v_2 = -v_2$

$AP = A(v_1 \ v_2)$

$= (A v_1 \ A v_2)$

$= (2 v_1 \ -v_2) = (v_1 \ v_2)\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$

$= P \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$

$\det(P) = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$

$= -1 - 2 = -3$

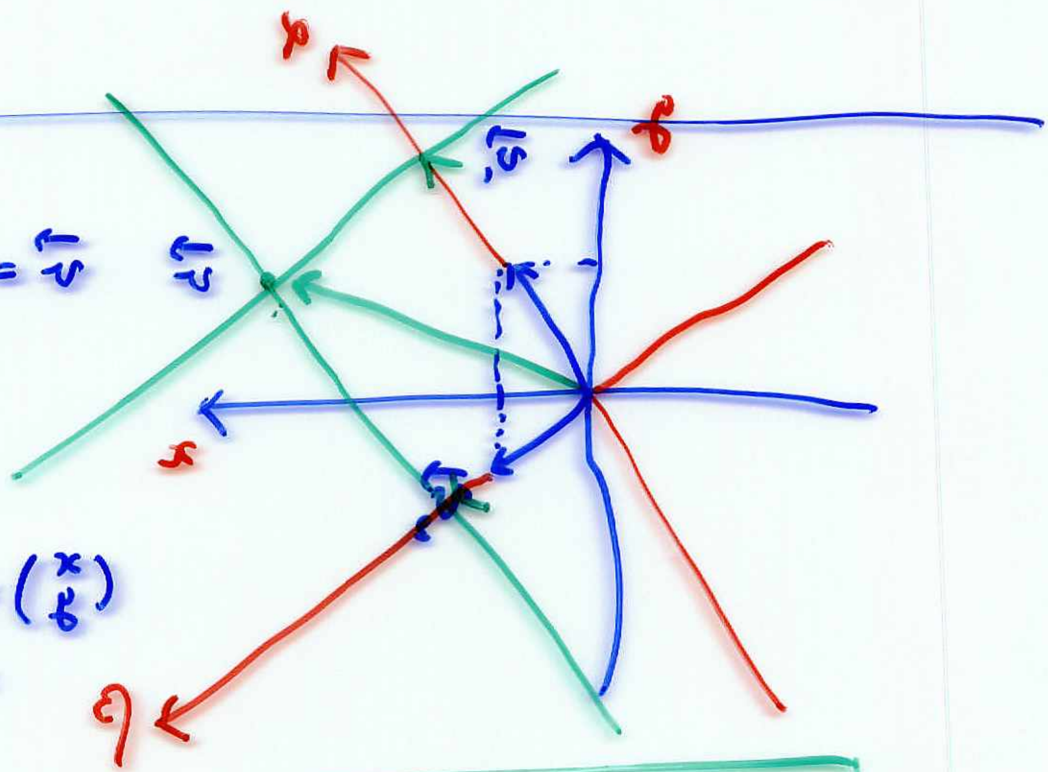
$\neq 0$

P: 正則

$\rightsquigarrow A = P \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} P^{-1}$

Aの対角化.

$$\boxed{(a \ b)\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha a + \beta b}$$



$v = \alpha v_1 + \beta v_2 \quad \Leftrightarrow \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \alpha v_1 + \beta v_2$

$= (v_1 \ v_2)\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$= P \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}$ と $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ の関係式

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = P \begin{pmatrix} \alpha \\ \beta \end{pmatrix}}$$

$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$ 1次変換

$\rightsquigarrow P \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = A P \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightsquigarrow \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} = P^{-1} A P \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

ex 236. $A = \begin{pmatrix} 2 & -2 \\ -2 & -1 \end{pmatrix}$

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda - 2 & 2 \\ 2 & \lambda + 1 \end{vmatrix} = (\lambda - 3)(\lambda + 2)$$

$\underline{\lambda = 3}$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff x + 2y = 0$$

$$\vec{v}_0 = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2y \\ y \end{pmatrix} = y\begin{pmatrix} -2 \\ 1 \end{pmatrix} = -y\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\vec{p}_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$\underline{\lambda = -2}$

$$\begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \vec{0} \iff 2x - y = 0$$

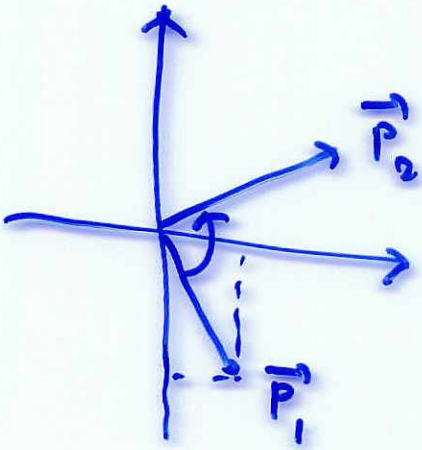
$$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\vec{p}_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 2 \end{pmatrix} \perp \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

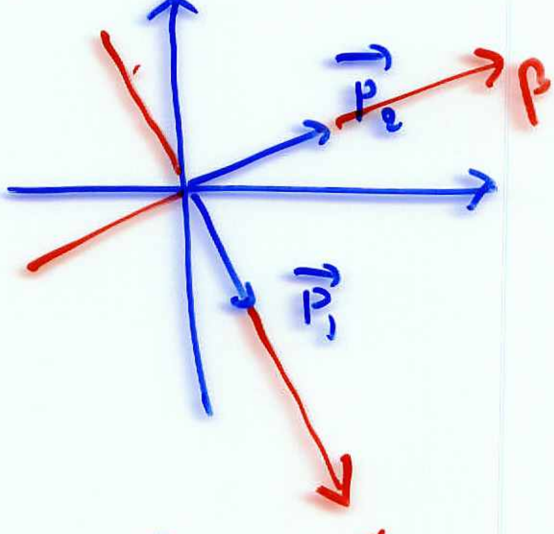
$$P = (\vec{p}_1, \vec{p}_2)$$

$$\cancel{= \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$



$\boxed{\begin{array}{l} A : \text{対称行列.} \quad \alpha \neq \beta \\ \qquad\qquad\qquad\quad \text{固有値} \\ A\vec{v}_1 = \alpha\vec{v}_1 \\ A\vec{v}_2 = \beta\vec{v}_2 \\ \Rightarrow (\vec{v}_1, \vec{v}_2) = 0 \end{array}}$

$$\left(A\binom{x}{y}, \binom{x}{y}\right)$$
$$\parallel$$
$$2x^2 - 4xy - y^2$$



- $\binom{x}{y} = \alpha \vec{p_1} + \beta \vec{p_2}$

  $= P\binom{\alpha}{\beta}$

- ${}^tP\binom{x}{y} = \binom{\alpha}{\beta}$

- $AP = P\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$

$$P^tP = P^tP = I_2$$

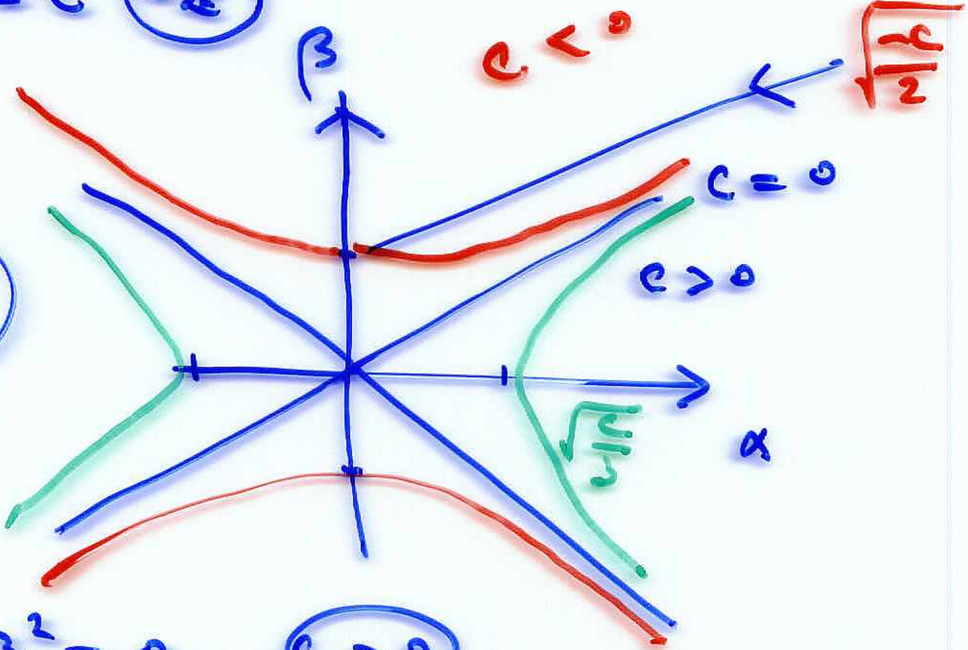
$$\left(A\binom{x}{y}, \binom{x}{y}\right) = \left(AP^tP\binom{x}{y}, \overbrace{P^tP}^{I_2\ \parallel}\binom{x}{y}\right)$$

$$= \left({}^tPAP\cdot\binom{\alpha}{\beta}, \binom{\alpha}{\beta}\right)$$

$$= \left(\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}\binom{\alpha}{\beta}, \binom{\alpha}{\beta}\right)$$

$$= 3\alpha^2 - 2\beta^2.$$

$3\alpha^2 - 2\beta^2 = c$ 一定



双曲線

$c = 0$

$$3\alpha^2 - 2\beta^2 = 0$$

$$\beta = \pm\sqrt{\frac{3}{2}}\,\alpha$$

$c > 0$

$$\beta^2 = \frac{3\alpha^2 - c}{2} = 0$$

$$\alpha = \sqrt{\frac{c}{3}}$$

$c < 0$

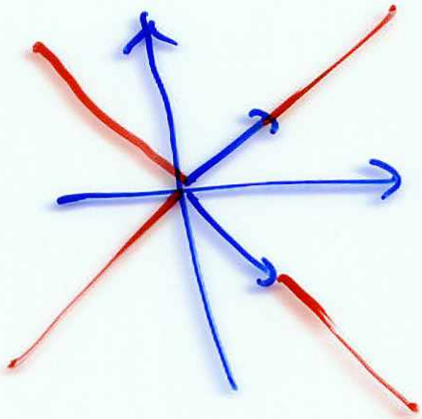
$$\alpha^2 = \frac{2\beta^2 + c}{3} = 0$$

$$\beta = \sqrt{\frac{-c}{2}}$$

$A=\begin{pmatrix}5 & 2\\ 2 & 2\end{pmatrix}$

$$5x^2+4xy+2y^2=C$$

$$=\alpha^2+6\beta^2$$

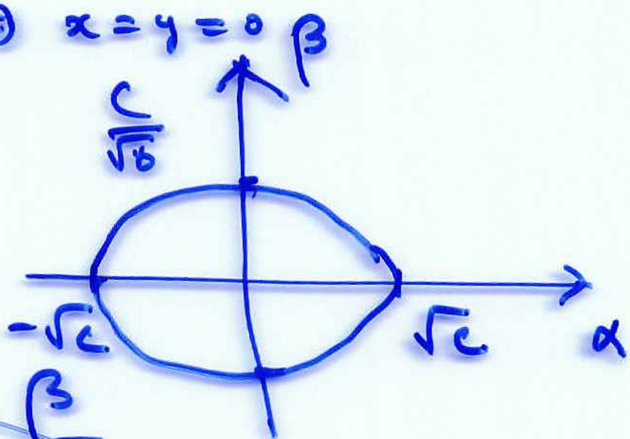


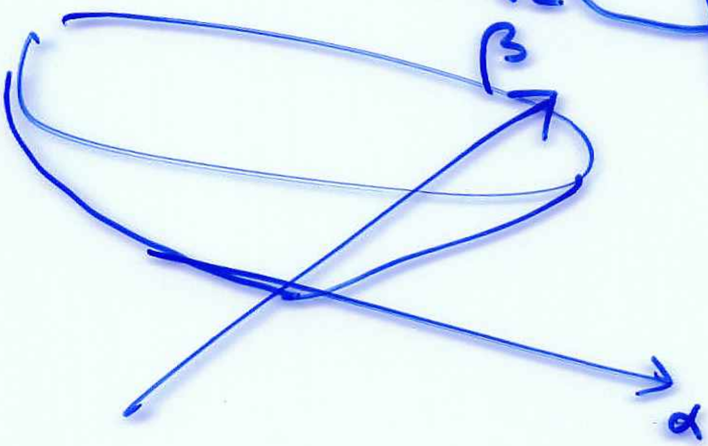
$c<0$ 空集合

$c=0$ $\alpha=\beta=0 \iff x=y=0$

$c>0$

だ円





$z=\alpha^2+6\beta^2$

236 page 対称行列.

$$A = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

$$|\lambda I_2 - A| = \begin{vmatrix} \lambda - a & -c \\ -c & \lambda - b \end{vmatrix}$$

Aの固有多項式

$$= (\lambda - a)(\lambda - b) - c^2$$

$$= \lambda^2 - (a+b)\lambda + (ab - c^2)$$

$$D = \text{判別式} = (a+b)^2 - 4(ab - c^2)$$

$$= a^2 - 2ab + b^2 + 4c^2$$

$$= (a-b)^2 + 4c^2 \geqq 0$$

$$D = 0 \iff a = b,\ c = 0$$

このとき $A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A \neq aI_2$ とすると $D > 0 \rightsquigarrow$ 相異なる実の固有値を持つ.

$\alpha \neq \beta$ 固有値.

$$\left.\begin{array}{l} A\vec{v_1} = \alpha\vec{v_1},\ \vec{v_1} \neq \vec{0} \\ A\vec{v_2} = \beta\vec{v_2},\ \vec{v_2} \neq \vec{0} \end{array}\right\} \longrightarrow (\vec{v_1}, \vec{v_2}) = 0$$

$\vec{p_1} = c_1\vec{v_1}$, $\vec{p_2} = c_2\vec{v_2}$ $\quad \|\vec{p_1}\| = \|\vec{p_2}\| = 1$

$\rightsquigarrow P = (\vec{p_1}\ \vec{p_2})$ 直交行列. $\quad AP = P\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$

......

定理 A: 対称.

P: 回転行列を選べば

$$AP = P\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix}$$

と対角化できる.

$$\left(A\begin{pmatrix}x\\y\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}\right) = \left(\begin{pmatrix}\alpha & 0\\ 0 & \beta\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}, \begin{pmatrix}\xi\\ \eta\end{pmatrix}\right)$$

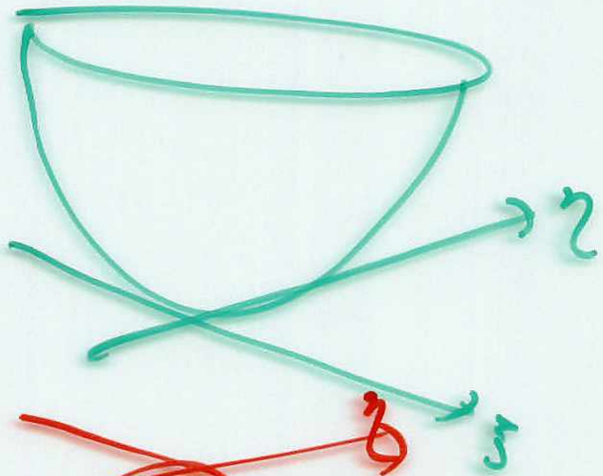
$$\begin{pmatrix}x\\y\end{pmatrix} = P\begin{pmatrix}\xi\\ \eta\end{pmatrix}$$

$$= \alpha\xi^2 + \beta\eta^2.$$

(I) $\alpha, \beta > 0$

$\begin{pmatrix}x\\y\end{pmatrix} \neq \vec{0}$ で

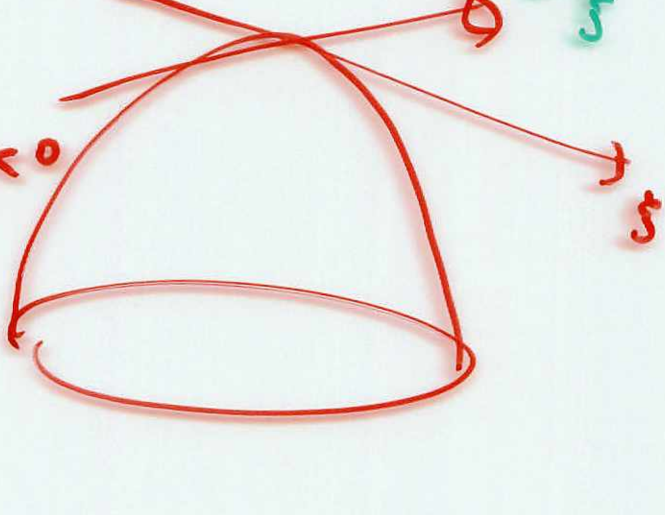
$\left(A\begin{pmatrix}x\\y\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}\right) > 0$



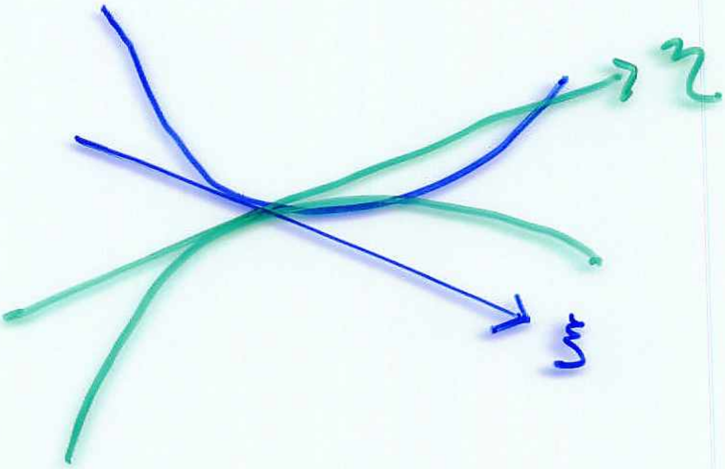
(II) $\alpha, \beta < 0$

$\begin{pmatrix}x\\y\end{pmatrix} \neq 0$ で

$\left(A\begin{pmatrix}x\\y\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix}\right) < 0$



III $\alpha > 0, \beta < 0$



$A = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$ (T 242P あたり).

I $\alpha, \beta > 0 \iff a > 0, \det(A) > 0$

II $\alpha, \beta < 0 \iff a < 0, \det(A) > 0$

III $\alpha \cdot \beta < 0 \iff \det(A) < 0$

---

240 page

x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_N	y_N

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$V(x) = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2$$ 分散

$$V(y) = \frac{1}{N}\sum_{i=1}^{N} (y_i - \bar{y})^2$$

共分散

$$\mathrm{cov}(x, y) = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$

$$\vec{x} = \frac{1}{\sqrt{N}}\begin{pmatrix} x_1 - \bar{x} \\ x_2 - \bar{x} \\ \vdots \\ x_N - \bar{x} \end{pmatrix}, \quad \vec{y} = \frac{1}{\sqrt{N}}\begin{pmatrix} y_1 - \bar{y} \\ \vdots \\ y_N - \bar{y} \end{pmatrix}$$

$$\|\vec{x}\|^2 = V(x), \quad \|\vec{y}\|^2 = V(y)$$

$$\langle \vec{x}, \vec{y} \rangle = \mathrm{Cov}(x, y)$$

$$z = sx + ty$$ s, t : 定数

$$z_i = s x_i + t y_i$$

$$\bar{z} = s\bar{x} + t\bar{y}$$

$$\bar{z} = \frac{1}{N}\sum_{i=1}^{N} (s x_i + t y_i)$$

$$= s\frac{1}{N}\sum_{i=1}^{N} x_i + t\frac{1}{N}\sum_{i=1}^{N} y_i$$

$$= s\bar{x} + t\bar{y}$$

$$\vec{z} = \frac{1}{\sqrt{N}} \begin{pmatrix} z_1 - \bar{z} \\ z_2 - \bar{z} \\ \vdots \\ z_N - \bar{z} \end{pmatrix} = s\frac{1}{\sqrt{N}} \begin{pmatrix} x_i - \bar{x} \\ \\ \\ \end{pmatrix} + t\frac{1}{\sqrt{N}} \begin{pmatrix} y_i - \bar{y} \\ \\ \\ \end{pmatrix}$$

$$= s\vec{x} + t\vec{y}.$$

$$z_i - \bar{z} = (s x_i + t y_i) - (s\bar{x} + t\bar{y})$$

$$= s(x_i - \bar{x}) + t(y_i - \bar{y})$$

$$\boxed{\begin{aligned} &\|\vec{x} + \vec{y}\|^2 \\ &= \|\vec{x}\|^2 + 2(\vec{x}, \vec{y}) \\ &\qquad + \|\vec{y}\|^2 \end{aligned}}$$

$$\begin{aligned} V(z) &= \|\vec{z}\| \\ &= \|s\vec{x} + t\vec{y}\| \\ &= s^2\|\vec{x}\|^2 + 2st(\vec{x}, \vec{y}) + t^2\|\vec{y}\|^2 \\ &= s^2 V(x) + 2st\,\mathrm{Cov}(x, y) + t^2 V(y) \\ &= \underline{\qquad\qquad\qquad} \quad \left(C\begin{pmatrix} s \\ t \end{pmatrix}, \begin{pmatrix} s \\ t \end{pmatrix}\right) \end{aligned}$$

$$C = \begin{pmatrix} V(x) & \mathrm{Cov}(x, y) \\ \mathrm{Cov}(x, y) & V(y) \end{pmatrix}$$

↖ x と y の分散共分散行列.

$s^2 + t^2 = 1$ の下で $V(z)$ を最大化する.

C : 対称, ∃直交行列 P

$$CP = P\begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \qquad \alpha < \beta.$$

$$V(z) = \left(C\begin{pmatrix} s \\ t \end{pmatrix}, \begin{pmatrix} s \\ t \end{pmatrix}\right) = \alpha\xi^2 + \beta\eta^2.$$

$$\begin{pmatrix} s \\ t \end{pmatrix} = P\begin{pmatrix} \xi \\ \eta \end{pmatrix}$$

$s^2+t^2=1$ の下で $V(z) = \left(C\begin{pmatrix} s \\ t \end{pmatrix}, \begin{pmatrix} s \\ t \end{pmatrix}\right)$

$$= \alpha\xi^2 + \beta\eta^2$$

$\begin{pmatrix} s \\ t \end{pmatrix} = P\begin{pmatrix} \xi \\ \eta \end{pmatrix}$

$\alpha < \beta$

P 回転

$\xi^2+\eta^2=1$ の下で $\alpha\xi^2+\beta\eta^2$

$$\alpha\xi^2+\beta\eta^2 = \alpha\xi^2+\beta(1-\xi^2)$$

$$= \beta + \underbrace{(\alpha-\beta)}_{<0}\xi^2 \leq \beta$$

等号が成立する

$\xi=0,\ \eta=\pm1$

$$\begin{pmatrix} s \\ t \end{pmatrix} = (\vec{p}_1\ \vec{p}_2)\begin{pmatrix} 0 \\ \pm1 \end{pmatrix}$$

$$= \pm \vec{p}_2.$$

$\vec{p}_2$: β の固有ベクトル

$\vec{p}_2 = \begin{pmatrix} p \\ q \end{pmatrix}$ $z = px+qy$. 主成分

CT 278 page

$G : (A, B) \longrightarrow \mathbb{R}$　C^2 級.

$c \in (A, B)$

G', G'' 存在

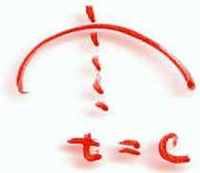
G, G', G'' 連続

$G'(c) = 0,\ G''(c) > 0$

< 0

$\Longrightarrow$ $t = c$ で $G(t)$ は 極小.

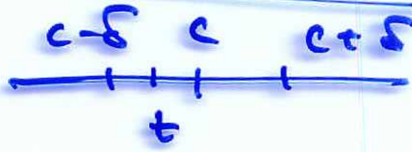
大



G'' : 連続

$G''(c) > 0$

$\rightsquigarrow$ $\delta > 0$ が存在し



$G''(t) > 0 \quad (c-\delta < t < c+\delta)$

Taylorの定理　$t \in (c-\delta, c+\delta)$

$$G(t) = G(c) + \underbrace{G'(c)}_{=0}(t-c) + \frac{G''(\alpha)}{2!}(t-c)^2$$

をみたす α が t と c の間にある.

$t \neq c$

$$= G(c) + \boxed{\frac{G''(\alpha)}{2!}(t-c)^2}$$

$> G(c)$

$\left.\begin{array}{l} t \in (c-\delta, c+\delta) \\ t \neq c \end{array}\right\} \Longrightarrow G(t) > G(c)$

$G(c)$ 極小値.

U : 開 $\subset \mathbb{R}^2$ $\quad f : U \longrightarrow \mathbb{R}$

$(a, b) \in U$ で f が 極大 or 極小

$$\Longrightarrow f_x(a,b) = f_y(a,b) = 0$$

$f : U \longrightarrow \mathbb{R}$ $\quad C^2$ 級.

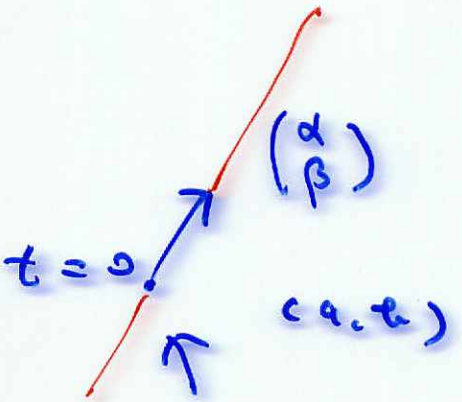
f_x, f_y が存在し

$f_{xx}, f_{xy}, f_{yx}, f_{yy}$ 存在し

$f, f_x, f_y,$ ______ 連続

$\Downarrow$ 定理 8.17

$$(f_x)_y = (f_y)_x$$



$f_x = f_y = 0$

$$F(t) = f(a + \alpha t, b + \beta t)$$

$$F'(t) = f_x(a + \alpha t, b + \beta t) \cdot \alpha + f_y(\quad) \cdot \beta$$

$$F'(0) = f_x(a,b) \cdot \alpha + f_y(a,b)\beta = 0$$

$$F''(t) = \alpha(f_{xx}\cdot\alpha + f_{xy}\cdot\beta)$$
$$+\beta(f_{yx}\cdot\alpha + f_{yy}\cdot\beta)$$
$$= \alpha^2 f_{xx} + 2\alpha\beta f_{xy} + \beta^2 f_{yy}$$

$$H(f) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$ Hesse行列.
対称行列.

$$= \left(H(f)\begin{pmatrix}\alpha\\ \beta\end{pmatrix}, \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \right)$$

III $\det(H(f))(a, b) < 0$

$H(f)(a, b)$ の固有値 p, q

$p > 0$, $q < 0$
$\updownarrow$ $\updownarrow$ 固有ベクトル
$\vec{v}_1$ $\vec{v}_2$

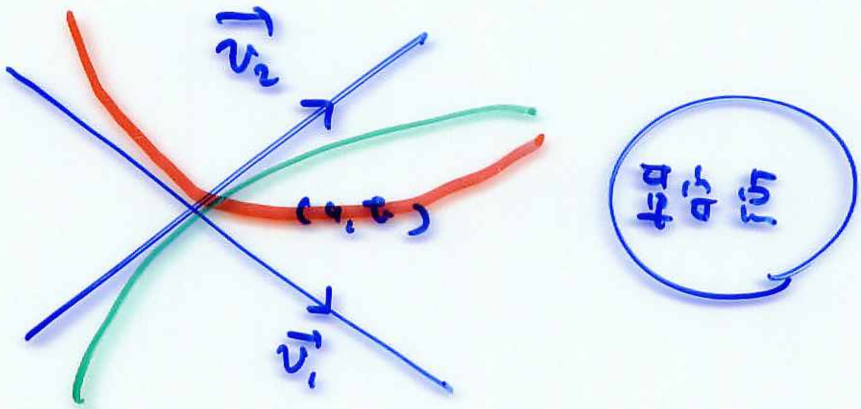
$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \vec{v}_1$

$$F''(0) = (H(f)\vec{v}_1, \vec{v}_1) \qquad H(f)\vec{v}_1 = p\vec{v}_1$$
$$= (p\vec{v}_1, \vec{v}_1) = p\|\vec{v}_1\|^2 > 0$$

$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \vec{v}_2$

$$F''(0) = (q\vec{v}_2, \vec{v}_2) = q\|\vec{v}_2\|^2 < 0$$

結論　(a,b) は極小点でも極大点でもない



Ⅰ　$f_{xx}(a,b) > 0$　$\det(H(f))(a,b) > 0$

$\longrightarrow$ 極小.

Ⅱ　$f_{xx}(a,b) < 0$, $\det(H(f))(a,b) > 0$

$\longrightarrow$ 極大

明日示す.　281 page.

282 p.

434 8.18

$$z = x^3 + y^3 + 6xy$$

$$\begin{cases} z_x = 3x^2 + 6y \cdot 1 \\ \quad = 3(x^2 + 2y) = 0 \\ z_y = 3y^2 + 6x \cdot 1 = 3(y^2 + 2x) = 0 \end{cases}$$

$y = -\frac{1}{2}x^2$ $\qquad$ $\frac{1}{4}x^4 + 2x = 0$

$x = 0$ OR $\qquad$ $x^3 = -8$

$\downarrow$ $\qquad$ $x = -2. \rightarrow y = -2$

0

停留点 $(0, 0)$, $(-2, -2)$

$z_{xx} = 6x$

$z_{xy} = z_{yx} = 6$

$z_{yy} = 6y$

at $(0, 0)$ $\quad \begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix}$ $\quad \det \begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix} = -36 < 0$

極大でも極小でもない

$(-2, -2)$

$\begin{pmatrix} -12 & 6 \\ 6 & -12 \end{pmatrix}$ $\quad \det \begin{pmatrix} -12 & 6 \\ 6 & -12 \end{pmatrix}$

< 0

$$= \left| 6\begin{pmatrix} -2 \\ 1 \end{pmatrix} \; 6\begin{pmatrix} 1 \\ -2 \end{pmatrix} \right|$$

極大

$$= 36 \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \times 36 > 0$$

問 8.19 $\quad z = x^2 + xy + y^2 - 4y + 10x$

$$\begin{cases} z_x = 2x + y + 10 = 0 \\ z_y = x + 2y - 4 = 0 \end{cases}$$

$$\begin{array}{rrr} & 2 & 1 & 10 \\ -) & 2 & 4 & -8 \\ \hline & & -3 & 18 \end{array}$$

$-3y + 18 = 0$

$y = 6$

$x = -8$

$(-8, 6)$ が停留点.

$z_{xx} = 2,\ z_{xy} = z_{yx} = 1,\ z_{yy} = 2$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $\quad \det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 4 - 1 = 3 > 0$

$f_{xx} = 2 > 0 \quad \longrightarrow (-8, 6)$

極小.

CT 269. 問 8.11, 8.12.

$$z = x^2 + xy + y^2 + 10x - 4y$$

$$= (x-\alpha)^2 + (x-\alpha)(y-\beta) + (y-\beta)^2 + \gamma \qquad x = \xi + \alpha, \quad y = \eta + \beta.$$

$$\begin{aligned} = \; & x^2 && + xy && + y^2 \\ & -2\alpha x && -\alpha y - \beta x && -2\beta y \\ & +\alpha^2 && +\alpha\beta && +\beta^2 + \gamma \end{aligned}$$

$= 0$

<u>1次の項</u>

$$10x - 4y$$

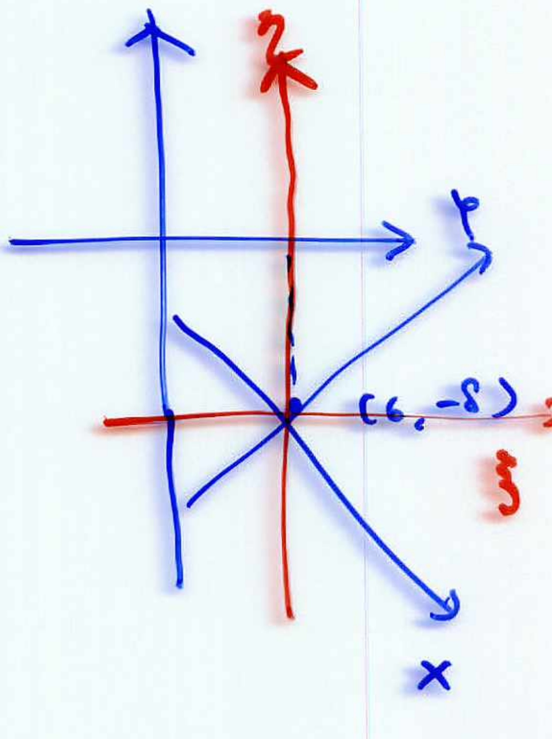
$$= (-2\alpha - \beta)x + (-\alpha - 2\beta)y.$$

$$\begin{cases} 10 = -2\alpha - \beta \\ -4 = -\alpha - 2\beta \end{cases} \qquad (\alpha, \beta) = (6, -8)$$

$$z = \underset{\sim\sim\sim\sim\sim}{\xi^2 + \xi\eta + \eta^2} + \gamma$$

45°回転

$\rightsquigarrow$ $z =$ 一定 だ円



(I) 極大・極小の判定.

(II) Lagrange の未定乗数法 ＜ 2変数の復習.

i) $g(x, y, z) = 0$ の下で

$$w = f(x, y, z)$$

ii) $g_1(x, y, z) = g_2(x, y, z) = 0$ の下で

$$w = f(x, y, z)$$

(III) 2変数のポートフォリオ.